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# NONLINEAR MATHEMATICAL MODELS OF WEAKLY LOADED CONTACT JOINTS

Janusz Skrodzewicz

*Manufacturing Engineering Institute, Technical University of Szczecin*
*e-mail: skrod@safona.tuniv.szczecin.pl*

Paweł Gutowski

*Mechanical Engineering Faculty, Technical University of Szczecin*
*e-mail: ptgut@safona.tuniv.szczecin.pl*

The paper presents nonlinear mathematical models describing the properties of dry and lubricated contact joints loaded in normal direction. The energy dissipation coefficient $\psi$ is expressed as a function of the load amplitude. The structure of the models is defined on the basis of the experimentally determined spectral-response characteristics of relative displacements of the contact surfaces, and of contact load in the normal direction. The form of the functional factors of models is estimated of the linear regression analysis.

*Key words:* contact joint, energy dissipation, nonlinear models

## 1. Introduction

Contact joints loaded weakly in the normal direction occur frequently in various technical applications. For instance, they are commonly found in connections of elements of the technological equipment. The physical phenomena that describe the motion in the vicinity of such joints are highly nonlinear (Petrulli, 1983; Andrew et al., 1967). The application of linear models for description of these phenomena in many cases is too simplified, what becomes a source of many errors, e.g. in the modeling technique. Therefore, there is a need for construction of the mathematical models, which describe more precisely the properties of the contact. The structure of this model and its parameters should be estimated on the basis of the experimentally obtained results.

## 2. Estimation of nonlinear mathematical models of contact joints

### 2.1. The test stand

The experimental tests were carried out on a specially designed and constructed stand, which made possible the realizations of the static and dynamic loading in the normal direction, and which enabled an accurate measurement of relative displacement of the nominal contact surfaces. The block diagram of this stand is given in Fig.1. The samples were made of steel 45.

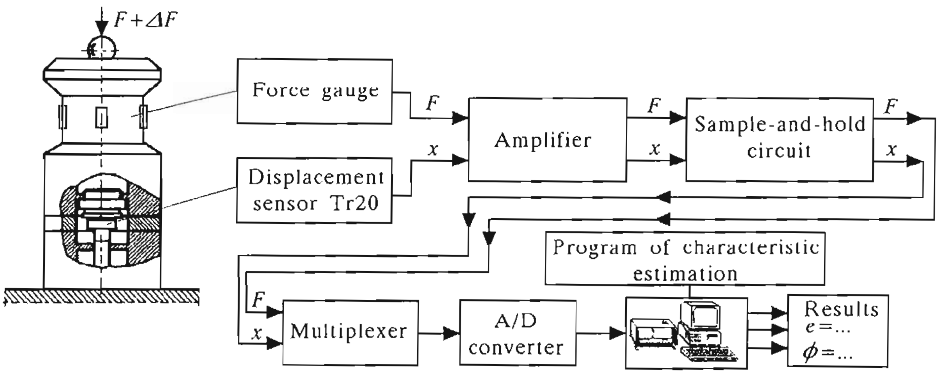


Fig. 1. Block diagram of the system for measuring, recording and processing the signals

The constant load was assumed according to that existing in the slideway joints of machine tools.

It was equal to $\sigma_0 = 1\,\mathrm{MPa}$. The surfaces of contact were ground ($R_a = 0.67\ \mu\mathrm{m}$) and turned ($R_a = 4.8\ \mu\mathrm{m}$). The dry and lubricated contact joints were examined. The nominal area of contact was equal to $A = 51\,\mathrm{cm}^2$. The relative normal displacements of contact joints were measured by inductive sensor Tr-20, which was placed centrally in the axis of a specially designed sample. The shape of this sample was designed numerically, with the use of finite elements method, to obtain a uniform distribution of the normal pressure on the whole area of contact. The difference between the minimal and maximal value of the pressure distribution on the contact surface was less than 2%.

The measurements of the force, both its constant and variable components, were made directly in the upper part of the sample that was designed as a force gauge.

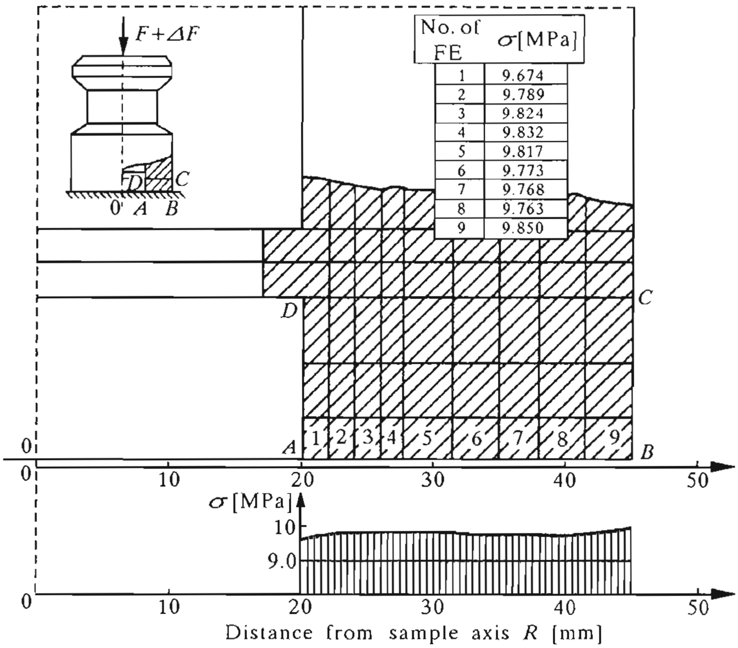


Fig. 2. Load distribution in near-surface zone of contact layer for normal load $F = 50\,\mathrm{kN}$

### 2.2. Mathematical model estimation

One of the main purposes of the tests was to estimate the mathematical model, which describes the relationship between the relative displacements of the couple of elements in the zone of their contact, and the load of this contact. For this purpose, the input signal in a form close to the harmonic one, was applied. The input signal can be described by the following relationship

$$\sigma(t) = \sigma_0 + \sigma_m \sin(\omega t) \tag{2.1}$$

fulfilling the condition $\sigma_0 > \sigma_m$, which provides joint stability of the tested elements.

In such cases, the relative displacement of the two contacting surfaces is represented by the response signal of a periodic form. The character of the response signal depends on the parameters of the input signal, and this is caused by nonlinearity of the examined phenomena. One of the methods that enables the determination of the structure of nonlinearity is the description of the input and response signals using the same elementary functions. In the case discussed it will be harmonic functions. The change of the periodic response signal to the sum of harmonic signals is possible, using the Fourier transform. With the use of this transformation, the nonlinear form of the response signal can be written as a sum of harmonics

$$x(t) = \sum_{k=0}^{\infty} a_k \sin(k\omega t + \delta_k) \tag{2.2}$$

for $\sigma_0, \sigma_m, \omega = \text{const}$.

In the case of variable input signal parameters, the response signal can be a function of all these parameters. The equation (2.2) will become nonlinear since coefficients $a_k$ will become the functions dependent on the parameters of the input signal (for simplicity it is assumed that $\delta_k$ does not depend on these parameters). So the equation (2.2) can be written as

$$x(\sigma_0, \sigma_m, \omega, t) = \sum_{k=0}^{\infty} a_k(\sigma_0, \sigma_m, \omega) \sin(k\omega t + \delta_k) \tag{2.3}$$

After the analysis of the signals obtained from the preliminary tests, the number of independent variables in the core tests plan was reduced to one $(\sigma_m)$. The next step consisted in determination of such a mathematical model, which guaranteed a sufficient accuracy of description of the tested phenomenon at a possibly small number $k$ of series elements. As a result, the structure of the mathematical model, which expresses the relative displacements of the contact joint surfaces in terms of with its load in the form of nonlinear parametric equations, was obtained

$$\begin{aligned} \sigma(t) &= \sigma_0 + \sigma_m \sin(\omega t) \\ x(\sigma_m, t) &= a_1(\sigma_m) \sin(\omega t + \varphi) + a_2(\sigma_m)[\cos 2(\omega t + \varphi) - 1] + \\ &\quad - a_3(\sigma_m) \sin 3(\omega t + \varphi) \end{aligned} \tag{2.4}$$

assuming for simplicity that $a_0 = 0$.

To determine the form of the function represented by the component

$$a_k(\sigma_m) \tag{2.5}$$

the methods of linear regression analysis were used. As a result of the analysis (considering simple mathematical models), the best result for the component $a_i$ was obtained for the model of the form

$$a_1(\sigma_m) = \alpha_{11} + \alpha_{12}\sigma_m + \alpha_{13}\sigma_m^2 \tag{2.6}$$

For the remaining factors the best consistency appeared for the multiplicative model. So the component (2.5) can by given as the relationship

$$a_k(\sigma_m) = \alpha_k \sigma_m^{\beta_k} \tag{2.7}$$

for $\sigma_0 = \text{const}$, where $k = 2, 3$.

As it is seen from the equations (2.6) and (2.7), for the number of series elements $k = 3$ the number of the estimated parameters is equal to 7. Assuming for simplicity that $\beta_k = k\beta_1$ and $\beta_1 = \pi/2$, the equation (2.7) takes the form

$$a_k(\sigma_m) = \alpha_k \sigma_m^{k\frac{\pi}{2}} \tag{2.8}$$

Thus it was possible to reduce the number of estimated parameters to 5.

### 2.3. Determination of the energy dissipation coefficient

In the case of nonlinear contact joints, the most convenient measure of the energy dissipation seems to be the nondimensional coefficient of energy dissipation $\psi$. It is defined as the ratio of the dissipated energy $E_h$ to the potential energy $E_p$. This definition is given and described by Skrodzewicz (1999) in integral form. If in the experiment the periodic input signal is applied

$$F(t) = \sum_{k=0}^{n} F_k \sin(k\omega t + \phi_k) \tag{2.9}$$

then the response signal is obtained, in which the tested nonlinear object can be described also in the periodic form

$$x(t) = \sum_{k=0}^{n} x_k \sin(k\omega t + \delta_k) \tag{2.10}$$

Integrating the parametric equations, the area of curvilinear trapezoid can be calculated according to the following relationship

$$S = \int_{t_1}^{t_2} y(t)\,dt \tag{2.11}$$

where

$$y(t) = F(t)\frac{dx(t)}{dt} \tag{2.12}$$

and energies $E_h$ and $E_p$ are represented by the corresponding areas of the plot of relative displacements of the surfaces which constitute the contact as a function of its load. Assuming the integration limits in the equation (2.11) as $0 \div 2\pi$, it is possible to calculate the value of potential energy $E_p$ and the value of dissipation energy $E_h$. Energy dissipation is represented in nonlinear model (2.4) by the coefficient of phase angle $\varphi$.

## 3. Experimental results

In the described experiment, the value of the contact preload was equal to $\sigma_0 = 1\,\mathrm{MPa}$. Dividing this value into 10 parts, the 9 values of input signal amplitudes were obtained and therefore, the range for variable $\sigma_m$ value was defined as $\sigma_{m1} = \Delta\sigma_m = 0.1\sigma_0$. This gives on the 9th level the value $\sigma_{m9} = 0.9\sigma_0$. As a result, a 9 point experimental model was obtained. Therefore the value of contact joint load for all the discussed cases can be written in the following form

$$\sigma(t) = 1 + \sigma_m \sin(\omega t)$$

where $\sigma_m = (0.1 - 0.9)\sigma_0$.

Design of the experiment for nonlinear problem in contact joints was described by Chmielewski and Skrodzewicz (1999). As the result of the estimation for turned and ground, dry and lubricated surfaces, the following values of model parameters given by formula (2.4) were obtained:

— dry, ground surface

$$a_1 = 0.021 + 0.205\sigma_m + 0.27\sigma_m^2 \qquad a_2 = 0.15\sigma_m^{\pi}$$

$$a_3 = 0.038\sigma_m^{\frac{3\pi}{2}} \qquad \varphi = -0.015 \text{ rad}$$

— lubricated, ground surface

$$a_1 = -0.021 + 0.29\sigma_m + 0.105\sigma_m^2 \qquad a_2 = 0.089\sigma_m^{\pi}$$

$$a_3 = 0.015\sigma_m^{\frac{3\pi}{2}} \qquad \varphi = -0.045 \text{ rad}$$

— dry, turned surface

$$a_1 = 0.03 + 1.275\sigma_m + 0.54\sigma_m^2 \qquad a_2 = 0.31\sigma_m^{\pi}$$

$$a_3 = 0.067\sigma_m^{\frac{3\pi}{2}} \qquad \varphi = -0.01 \text{ rad}$$

— lubricated, turned surface

$$a_1 = 0.0357 + 1.13\sigma_m + 0.576\sigma_m^2 \qquad a_2 = 0.295\sigma_m^{\pi}$$

$$a_3 = 0.067\sigma_m^{\frac{3\pi}{2}} \qquad \varphi = -0.033 \text{ rad}$$

The graphs of the estimated nonlinear mathematical models, describing the properties of contact joints under normal load in the form of deformability characteristics, are given in Fig.3.

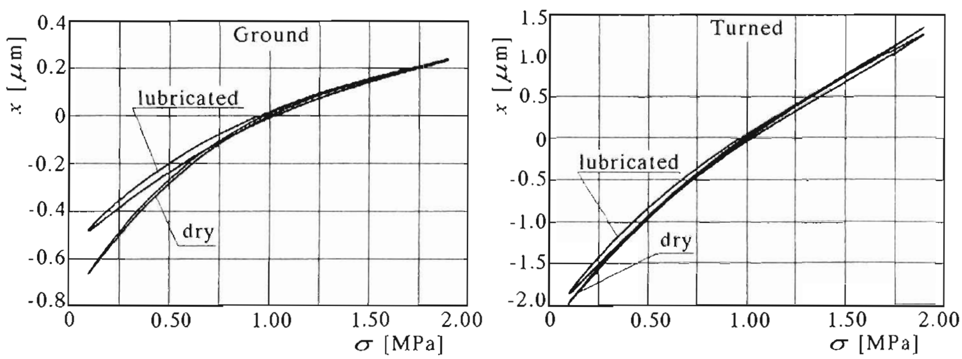


Fig. 3. Deformability characteristics of the tested contact joints

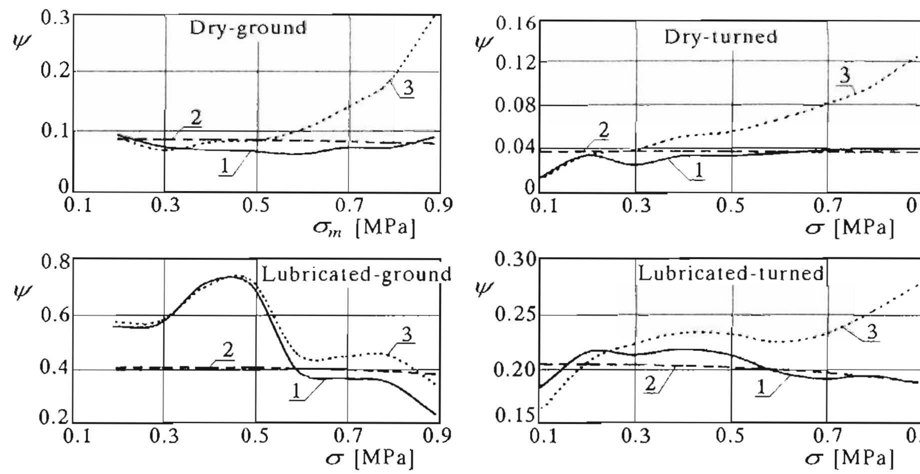


Fig. 4. Dependence of the dissipation energy coefficient $\psi$ on the amplitude of contact load $\sigma_m$

In Fig.4 the characteristics illustrating the dependence of dissipation energy coefficient $\psi$ on the load amplitude are given.

In this figure, the following types of particular curves are assumed:

1 – object characteristics – calculated according to (2.11)
2 – nonlinear model characteristics – calculated according to (2.11)
3 – linear model characteristics – calculated according to the following formula

$$\psi_1(\sigma_m) = 2\pi \tan[\delta_1(\sigma_m)] \tag{3.1}$$

## 4. Conclusions

It seems that the main advantage of the proposed mathematical model is that two essential parameters which characterize each contact joint, i.e. deformability of contact and dissipation energy of contact, are taken into account in the set of parametric equations.

The presented method of estimation of the nonlinear mathematical model can be useful not only for the description of the metal-metal contact joint properties, but for the description of dissipative properties of various materials, e.g. plastics, rubber and also the metal-plastic joints.

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**Nieliniowe modele matematyczne połączeń stykowych słabo obciążonych**

Streszczenie

W pracy przedstawiono nieliniowe modele matematyczne opisujące właściwości połączeń stykowych suchych i smarowanych obciążonych w kierunku normalnym. Podano także zależności współczynnika rozproszenia energii $\psi$ w funkcji amplitudy obciążenia. Strukturę modeli określono na podstawie eksperymentalnie wyznaczonych charakterystyk widmowych sygnałów przemieszczeń względnych nominalnych powierzchni styku i obciążenia styku w kierunku normalnym. Postaci czynników funkcyjnych modelu wyestymowano stosując metody regresji liniowej.

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