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AUTOMATIC CONTROL OF AN AIRCRAFT FLYING A TURN

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Co-ordination of a turn is a complex case. Correct turn is done with such a bank that the resultant force of centrifugal and gravity forces will be in the symmetry plane of the aeroplane.
The paper deals with a general mathematical model of an aeroplane in spatial flight. The following problems have been analysed: characteristic parameters of a correct turn, general physical and mathematical control model, approved steering principles and block schema of the equipment. The most important thing is a correct choice of the control system and suitable amplification factors.
The autopilot contains four channels, which can control the flight thanks to deflection of the height control surface, direction control surface, ailerons and the engine control lever.
Results of the research were obtained by means of numerical simulation of complete model of dynamics of an aeroplane with automatic control system, on the basis of a programme written in the calculation environment of MATLAB package, and were presented in graphic forms.

Key words: automatic control system, autopilot, co-ordinated turn, flight simulation

1. Introduction

In order to make a turn it is necessary to bank the plane by moving the ailerons. The vector of aerodynamic lift follows the motion of the plane that gives the necessary horizontal component of the lift force equal to $P_Z \sin\Phi$ that changes plane's horizontal movements. Banking of the aerodynamic lift P_Z decreases its vertical component force $P_Z \cos\Phi$. As it does not compensate

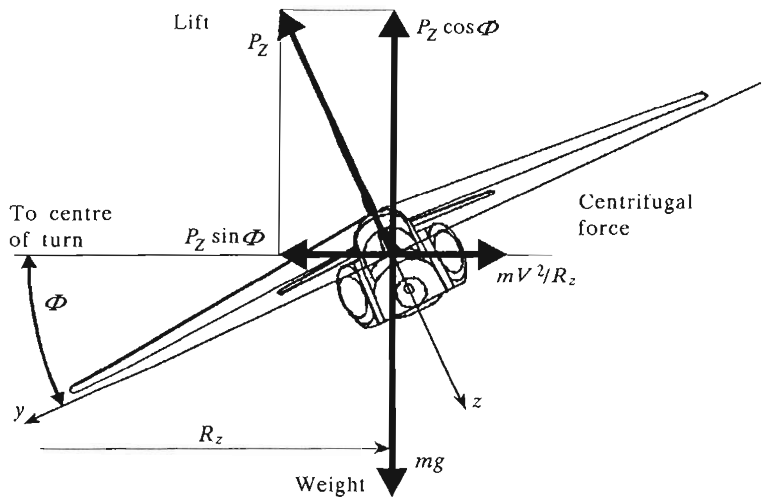

Fig. 1. Forces acting in a turn

the gravitation forces, the height is lost. To counterbalance this tendency, the autopilot must tend to increase the aerodynamic lift by increasing the angle of attack, i.e. by displacing the elevator δ_H. Simultaneous displacing of the ailerons and the elevator, and mainly the increase of the angle of attack α, result in the aerodynamic drag increase. Certain position of the engine control lever causes losing the speed, however, the propulsive thrust is stable. To compensate the increasing drag, the pilot must tend to increase the engine control lever. In addition to all these phenomena, the course change is followed by an undesired angle of sideslip. To avoid it, the automatic pilot uses the rudder. Therefore, to make a co-ordinated turn, the autopilot uses all channels of control. Banking is the main control channel and yawing is an auxiliary channel.

It is necessary to use automatic devices to control the plane. These devices take over some pilot's functions. Their task is to stabilise the flight parameters or to control and navigate the plane. Since they make the job of the pilot easier, the pilot can pay less attention to direct flight control actions. These devices are also used to improve some technical characteristics of the plane, e.g. its longitudinal stability or transverse stability.

2. Physical model of the plane

Physical model of the controlled plane has been created and based on the following assumptions:

- The plane is an infrasound training-and-combat aircraft with a conventional symmetric and compact body, equipped with more than one jet propulsion engine.
- The plane is considered to be a rigid body with movable but undeformable steering devices of six degrees of freedom (the plane moves in a three-dimensional space).
- Control surfaces have no weight and their movements influence only aerial forces and moments.
- Flow around the body is considered as quasi-stationary (quasi-stationary aerodynamics has been used).
- The movements of the plane are supposed to occur in the plane and in quiet air (the weather is windless).
- Any changes of the external parameters are considered to be in conformity with International Weather Standards; the flights are carried out at the altitude of 11 km.
- The symmetry plane of the aircraft is xOz (geometric, mass, aerodynamic).
- Origin of the co-ordinates $Oxyz$ is fixed up invariably with the moving aircraft, the centre of the aerodynamic forces influencing the plane and the centre of mass are located on the symmetry plane xOz of the aircraft.
- Thrust of the engines and rotation of the turbo-compressor are variable for each engine.
- The mass of the aircraft is constant and invariable during the flight.

3. General mathematical model of the aircraft during the flight in three-dimensional space

The dynamics equations of aircraft motion are derived in a plane-fixed coordinate system, on the basis of the Boltzmann-Hamel formalism for material

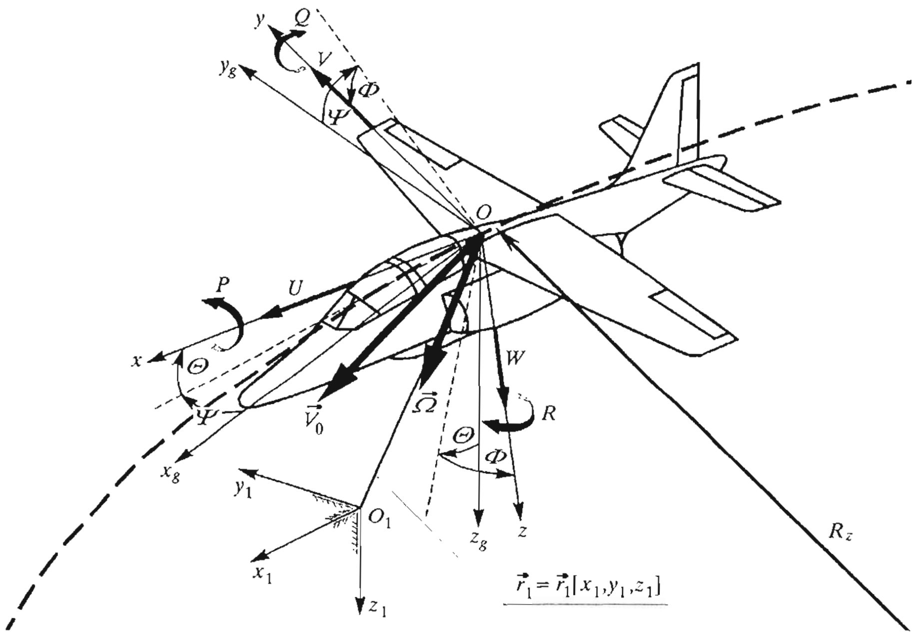

Fig. 2. Reference systems

systems with holonomic constraints; below they are presented in a general form

$$
\begin{aligned}
&m(\dot{U} + QW - RV) - S_x(Q^2 + R^2) - S_y(\dot{R} - PQ) + S_z(\dot{Q} + PR) = X \\
&m(\dot{V} + RU - PW) + S_x(\dot{R} + QP) - S_y(P^2 + R^2) - S_z(\dot{P} - QR) = Y \\
&m(\dot{W} + PV - QU) - S_x(\dot{Q} - PR) + S_y(\dot{P} + QR) - S_z(Q^2 + P^2) = Z \\
&I_x\dot{P} - (I_y - I_z)QR - I_{xy}(\dot{Q} - PR) - I_{xz}(\dot{R} + PQ) - I_{yz}(Q^2 - R^2) + \\
&+S_y(\dot{W} + VP - QU) + S_z(PW - UR - \dot{V}) = L \\
&I_y\dot{Q} - (I_z - I_x)RP - I_{xy}(\dot{P} + QR) - I_{yz}(\dot{R} - PQ) - I_{xz}(R^2 - P^2) + \\
&-S_x(\dot{W} + VP - UQ) + S_z(\dot{U} - VR + QW) = M \\
&I_z\dot{R} - (I_x - I_y)PQ - I_{yz}(\dot{Q} + PR) - I_{xz}(\dot{P} - RQ) - I_{xy}(P^2 - Q^2) + \\
&+S_x(\dot{V} - WP + RU) - S_y(\dot{U} - RV + WQ) = N
\end{aligned}
\tag{3.1}
$$

where

U, V, R – forward, side and yawing velocities
L, M, N – roll, pitch and yaw moments
P, Q, R – angular velocities, roll, pitch and yaw
Φ, Θ, Ψ – roll, pitch and yaw angles.

Aerodynamic angles are expressed as:

— angle of attack

$$\alpha = \arctan \frac{W}{U} \tag{3.2}$$

— angle of sideslip

$$\beta = \arcsin \frac{V}{V_0} \tag{3.3}$$

General equation of dynamics of an object being controlled, i.e. the vector equation of state for a mathematical non-linear model is as follows

$$\widetilde{\mathbf{M}}\dot{\boldsymbol{V}} + \mathbf{KM}\boldsymbol{V} = \boldsymbol{Q} + \mathbf{U}\boldsymbol{\delta} \tag{3.4}$$

where

$\widetilde{\mathbf{M}}$ – modified inertia matrix, $\widetilde{\mathbf{M}} = \mathbf{M} + \mathbf{M}_{\dot{\mathbf{U}}\dot{\mathbf{W}}}$
$\mathbf{B}$ – matrix of rigidity, $\mathbf{B} = \mathbf{KM}$
$\dot{\boldsymbol{V}}$ – vector of accelerations, $\dot{\boldsymbol{V}} = [\dot{U}, \dot{V}, \dot{W}, \dot{P}, \dot{Q}, \dot{R}]^\top$
$\boldsymbol{V}$ – vector of velocities, $\boldsymbol{V} = [U, V, W, P, Q, R]^\top$
$\boldsymbol{Q}$ – vector of external forces, $\boldsymbol{Q} = [X, Y, Z, L, M, N]^\top$
$\mathbf{U}$ – matrix of steering
$\boldsymbol{\delta}$ – steering vector, $\boldsymbol{\delta} = [\alpha_{zH}, \delta_H, \delta_L, \delta_V, \delta_T]^\top$.

Non-linear equation of motion and parallel transforming processes can be displayed as a matrix function (that can be proceeded with a numeric simulation)

$$\dot{\boldsymbol{V}} = \widetilde{\mathbf{M}}^{-1}(-\mathbf{B}\boldsymbol{V} + \boldsymbol{Q} + \mathbf{U}\boldsymbol{\delta}) \tag{3.5}$$

Formula (3.5) that can be proceeded with integral equation in simulating programs.

There are four channels of control: pitching Θ – through displacing elevator δ_H, rolling Φ – through aileron displacement δ_L, yawing Ψ – through rudder displacement δ_V, and speeding V_0 – through changing the engine propulsive thrust T through positioning of the engine control lever δ_T.

4. General physical and mathematical model of steering

Automatic pilot structure. Two parts of the automatic pilot can be described: the logical one and the executive one, Fig.3 (Szczepański, 1986).

Logical part – responsible for generating the signals of errors and due to the accepted principles of guidance, signals of control;

Executive part – responsible for displacing the control devices according to the signals received from the logical part.

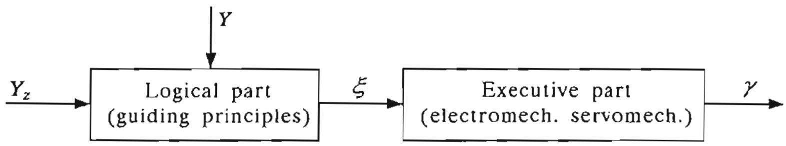

Fig. 3. Automatic pilot structural scheme

Errors: deviations of the controlled real variables (measured) of an aircraft from the variables required

$$\boldsymbol{\varepsilon} = (\boldsymbol{Y} - \boldsymbol{Y_z}) \tag{4.1}$$

Measuring system: distorts the signals describing an object due to errors of measurement and the activity inertia. However, we disregard signal distortion and assume that the variables under measurement $\boldsymbol{Y_p}$ are equal to real variables of the aircraft $\boldsymbol{Y}$. Therefore $\boldsymbol{Y_p} \equiv \boldsymbol{Y}$.

Guiding executive part of automatic pilot: any delays of the autopilot system should not affect general characteristics of the aircraft. Taking into consideration the forces influencing the aircraft, very well powered servomechanisms are built in automatic pilots. Any influence of servo-mechanism dynamics on the aircraft dynamics has in fact no meaning and can be described as the following inertial first order lag

$$T_I \frac{d\gamma_i}{dt} + \gamma_i = K_{i\gamma}\xi_i \tag{4.2}$$

where

T_i, K_i – respectively the time-constant and gain factor of the electromechanical servo-mechanism

γ_i – position of the control device

ξ_i – steering signal, duly determined in each separate channel according to the accepted principles of steering.

Servo-mechanisms in each channel have different values of factors T_i and K_i, due to their construction differences. Status of each of those servo-mechanisms is determined separately for each of the four channels.

Disregarding the time-constant (4.2)

$$\gamma_i = K_{i\gamma}\xi_i \tag{4.3}$$

Executive steering system: to process the steering signals obtained form the pilot or autopilot (displacement of control devices) and to transform them into displacing of the control surfaces (Fig.4)

$$T_i \frac{d\delta_i}{dt} + \delta_i = K_{i\delta}\gamma_i + \delta_{i0} \tag{4.4}$$

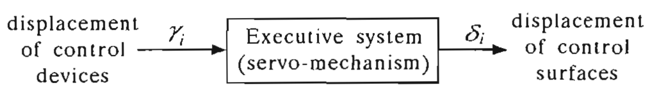

Fig. 4. Structure of executive steering syste

Disregarding the time-constant

$$\delta_i = K_{i\delta}\gamma_i + \delta_{i0} \tag{4.5}$$

A system of equations can be determined from Eqs (4.3) and (4.5)

$$\delta_i = K_{i\delta}\gamma_i + \delta_{i0} \qquad\qquad \gamma_i = K_{i\gamma}\xi_i$$

Therefore

$$\delta_i = K_{i\delta}K_{i\gamma}\xi_i + \delta_{i0}$$

and finally

$$\delta_i = K_i\xi_i + \delta_{i0} \tag{4.6}$$

and if

$$\begin{aligned} \delta_i < \delta_{i\,\min} \quad &\Rightarrow \quad \delta_i = \delta_{i\,\min} \\ \delta_i > \delta_{i\,\max} \quad &\Rightarrow \quad \delta_i = \delta_{i\,\max} \end{aligned}$$

<u>*Conclusion:*</u> displacement of control surfaces δ_i, results from multiplying the gain factor K_i by error ξ_i, increased by the position of control surfaces in equilibrium δ_{i0}.

Any procedure of automatic steering consists of components which measure particular parameters of the aircraft movements, processing components and steering devices.

The general block diagram of a steering system (automatic pilot) and the controlled object is presented in Fig.5.

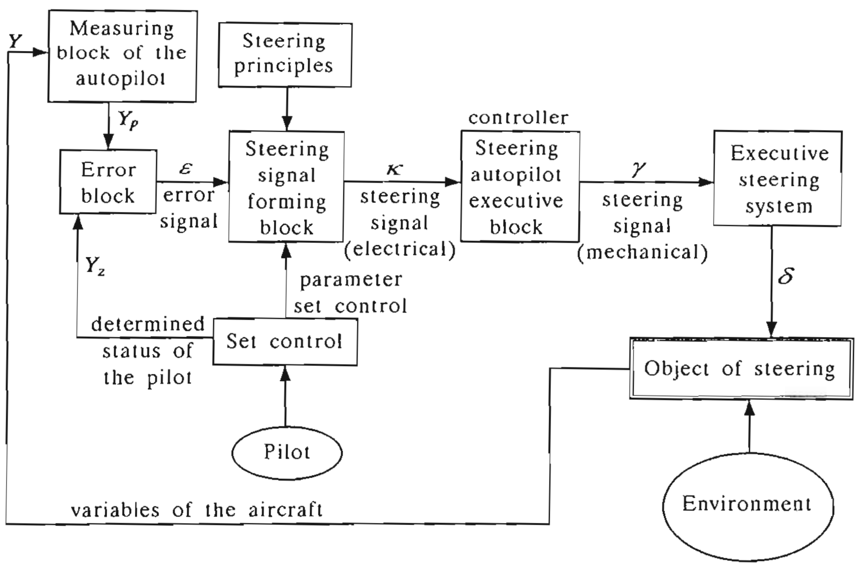

Fig. 5. General block scheme of the steering system with the object of steering

5. Accepted principles of automatic pilot steering

Steering factors are assumed as constants. However, while constructing an adaptive automatic pilot (working in different conditions), they might be required to be assumed as functions.

Values of the gain factors are different in different channels due to different construction of the servo-mechanism concerned. Incorrect evaluation of some factors, and sometimes wrong evaluation of even a single one, might affect the other factors and might cause unreliable results of the system.

It is very well known that a proper choice of the gain factor values for an automatic pilot is not very simple and might cause many problems (there exists no adequate method to determine the gain factors). The best way is to minimise their quantity to a very small number. Sometimes a compromise and purely "brain work" is needed to get simple and satisfactory results concerning the formerly determined conditions.

Looking for the gain factors for an automatic pilot, one can consider integral and square criteria of steering quality and supplement it with evaluation of transitional processes in all channels of the autopilot.

Steering principles have been determined as follows:

— rolling channel Φ

$$\delta_L = K^L_\Phi(\Phi - \Phi_z) + K^L_\Psi(\Psi - \Psi_z) + \delta_{L0} \tag{5.1}$$

— pitching channel Θ

$$\delta_H = K^H_{z_1}(z_1 - z_{1z}) + K^H_\Theta(\Theta - \Theta_z) + \delta_{H0} \tag{5.2}$$

— yawing channel Ψ

$$\delta_V = K^V_V(V - V_z) + \delta_{V0} \tag{5.3}$$

— speeding channel V_0

$$\delta_T = K^T_U(U - U_z) + K^T_\Theta(\Theta - \Theta_z) + \delta_{T0} \tag{5.4}$$

6. Device block diagram

The block diagram based on Eqs (5.1) $\div$ (5.4) is presented in Fig.6.

7. Flight in a regular turn

PZL I-22 "IRYDA" has been chosen as a test plane since it presents the best possibility to obtain the most important data necessary to proceed with the analysis of the problems concerned.

An analysis of the flight of an aircraft performing a regular turn has been taken into consideration. Dynamic characteristics of the PZL I-22 "IRYDA" have been analysed in order to get proper values for the steering strategies concerned.

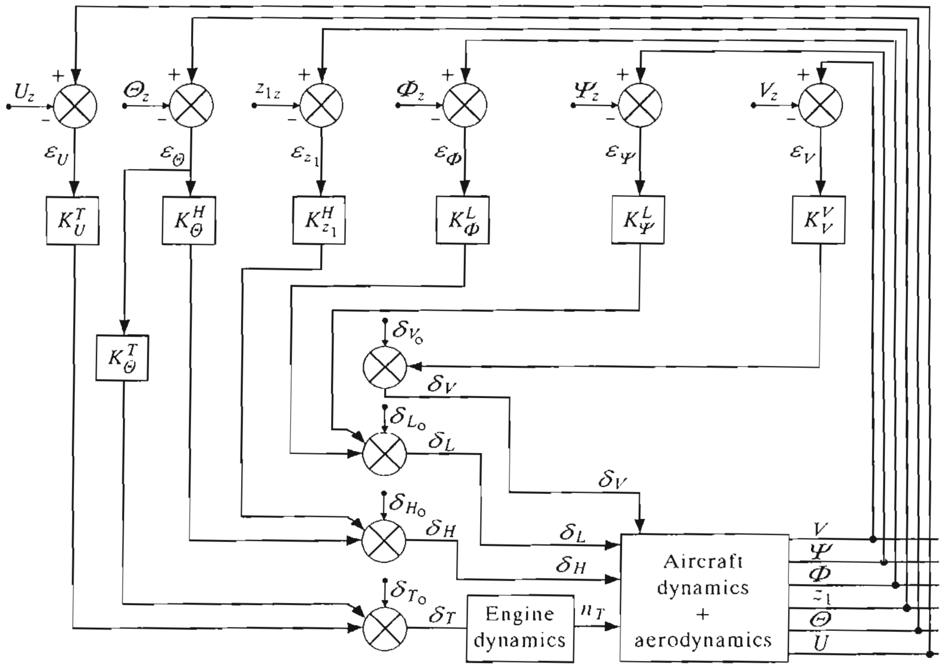

Fig. 6. Block diagram of the device

Basing on curves of the required thrust $T_n = f(V_s, \Phi)$ and curves of the available thrust $T_r = f(V_s,)$, it might be solved for different speeds $\Phi_{\max}$, $R_{\min}$, $t_{\min}$ as well as $n_{z\,\max}$.

The turn speed is proportionally linked with the horizontal component of the aerial force $P_Z \sin\Phi$ which makes the plane to turn. Therefore it is proportional to the banking angle Φ.

Typical parameters for a turn performed at altitude of 3000 m are:

— for the rotation speed $n_T = 15025$ rev/min, at the best and constant conditions

— for the rotation speed $n_T = 14000$ rev/min, at regular flight conditions.

To perform a proper control of a turn manoeuvres, the following instruments are helpful: a compass, an artificial horizon and, most of all, a turn indicator with a slide indicator, also called bank indicator, that allows to control properly the turns performed. A turn indicator is not a quantitative instrument but a qualitative device. A turn is properly performed if the ball inside the device does not leave the central area of the indicator.

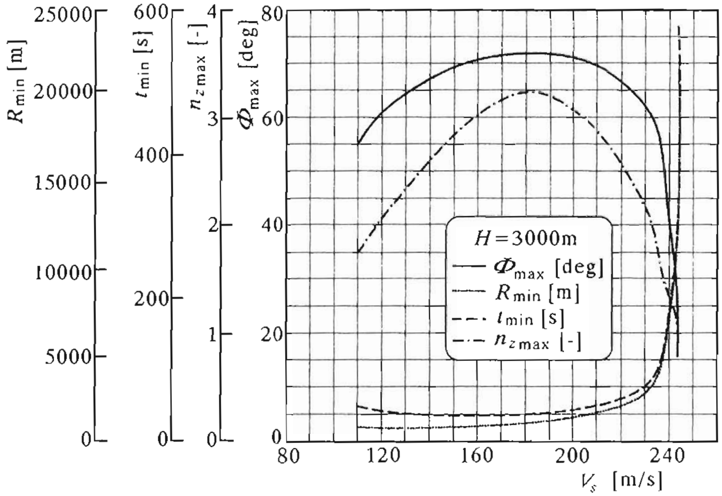

Fig. 7. Typical parameters for a turn with $n_T = 15025\,\text{rev/min}$

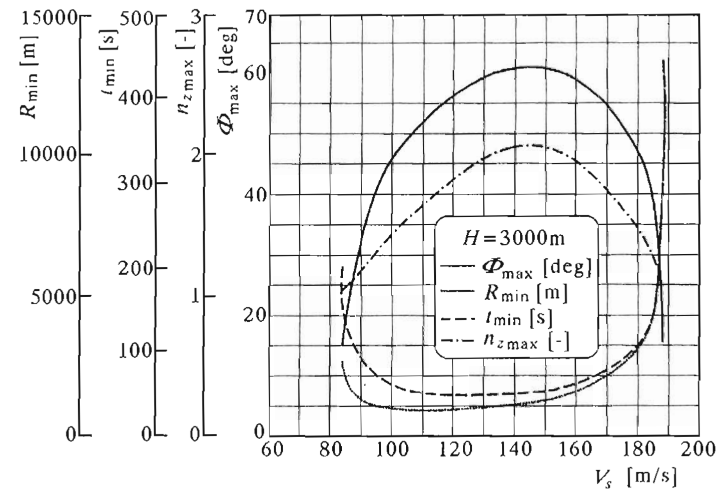

Fig. 8. Typical parameters for a turn with $n_T = 14000\,\text{rev/min}$

8. Results of the analysis

A numerical simulation of a turn has been performed for a flight at altitude of 3000 m. Automatic pilot system starts its functioning after 10 s of stable flight. The whole flight has been carried out in the following conditions: angle of tail plane setting $\alpha_{zH} = -2° = \text{const}$, mass centre position equals 22% (of mean aerodynamic chord).

Example I: Turn followed by a 90 deg course.

Automatic pilot parameters in this case are as follows

$$
\begin{array}{lll}
\delta_H : & K^H_{z_1} = -0.04 & K^H_\Theta = 0.82 \\
\delta_L : & K^L_\Phi = 0.012 & K^L_\Psi = 0.0055 \\
\delta_V : & K^V_V = -0.05 & \\
\delta_T : & K^T_U = -5.0 & K^T_\Theta = 4.0
\end{array}
$$

Example II: Turn followed by a 180 deg course

Automatic pilot parameters in this case are as follows

$$
\begin{array}{lll}
\delta_H : & K^H_{z_1} = -0.04 & K^H_\Theta = 0.85 \\
\delta_L : & K^L_\Phi = 0.01 & K^L_\Psi = 0.003 \\
\delta_V : & K^V_V = -0.05 & \\
\delta_T : & K^T_U = -5.0 & K^T_\Theta = 4.0
\end{array}
$$

Example III: Turn followed by a 360 deg course

Automatic pilot parameters in this case are as follows

$$
\begin{array}{lll}
\delta_H : & K^H_{z_1} = -0.045 & K^H_\Theta = 1.2 \\
\delta_L : & K^L_\Phi = 0.008 & K^L_\Psi = 0.00168 \\
\delta_V : & K^V_V = -0.05 & \\
\delta_T : & K^T_U = -5.0 & K^T_\Theta = 4.0
\end{array}
$$

Results of numerical calculation are shown in a graphic chart basing on functions of the MATLAB program.

Chosen parameters of flight and steering are displayed in Fig.9 ÷ Fig.16.

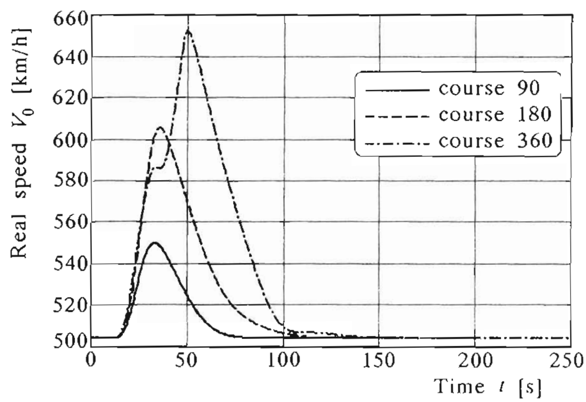

Fig. 9. Real speed as function of time

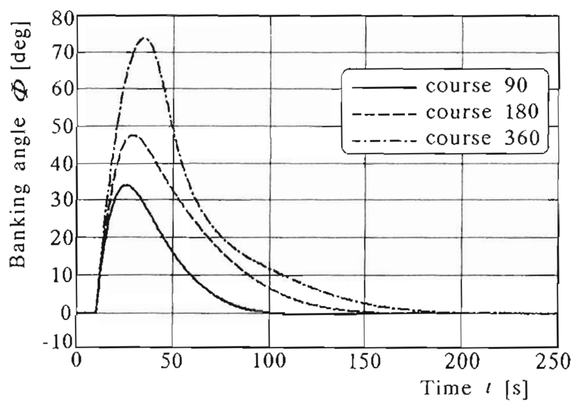

Fig. 10. Banking angle as function of time

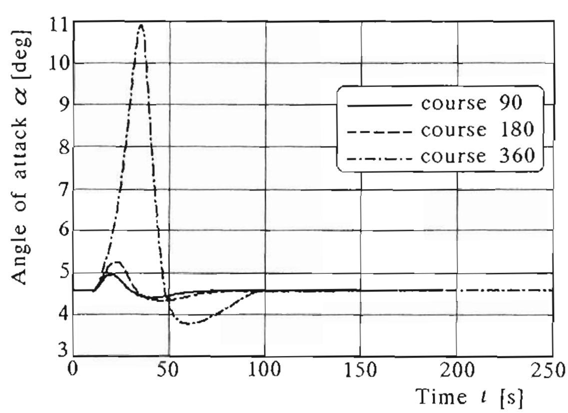

Fig. 11. Angle of attack as function of time

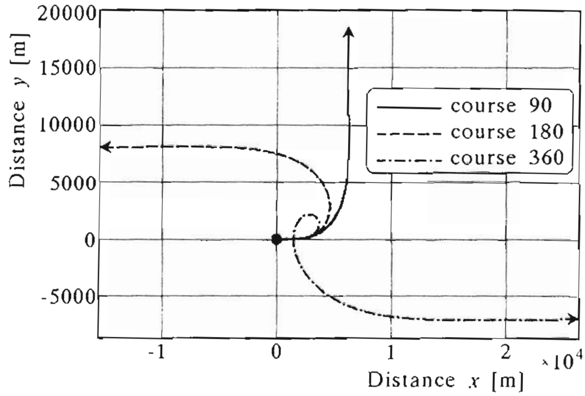

Fig. 12. Distance $y(x)$

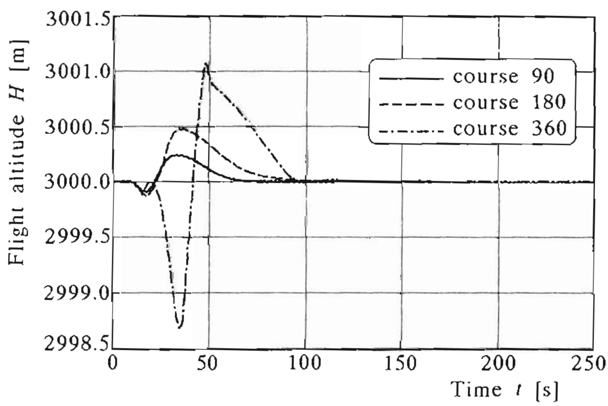

Fig. 13. Flight altitude as function of time

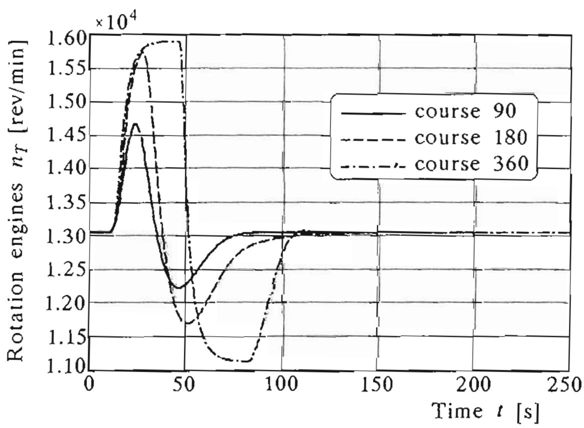

Fig. 14. Rotation of right and left engines as function of time

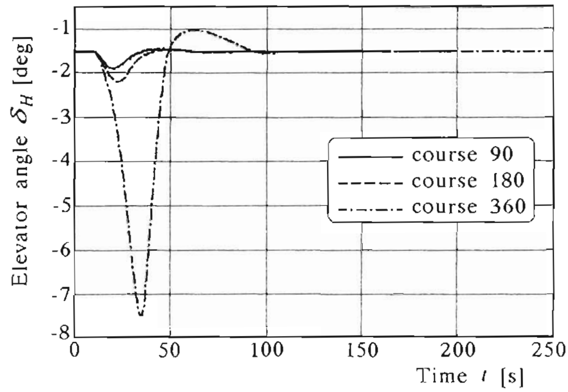

Fig. 15. Elevator angle as function of time

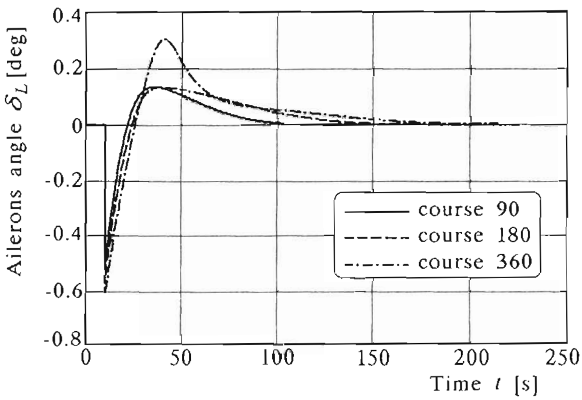

Fig. 16. Ailerons angle as function of time

9. Conclusions

Considering the results obtained it can be stated that the simulation program based on the assumed aircraft dynamics model represents a proper instrument to calculate the real physical phenomena. Physical model of an aircraft in space is necessary to display properly this particular case of flight.

Automatic pilot system with determined principles of steering guarantees preservation of proper flight altitude, guiding the aircraft to a proper course Ψ with a proper banking angle Φ. It can be stated that the aircraft gains and maintains the required parameters. The existing gravity force does not exceed the acceptable values based on physiological capacities of the pilot.

The obtained results show correct application of the steering principles, excellent plane guidance and remarkable steering results.

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Automatyczne sterowanie lotem samolotu w zakręcie

Streszczenie

Koordynacja zakrętu jest sprawą złożoną. Prawidłowy zakret powinien być wykonany przy takim przechyleniu, aby wypadkowa siły ciężkości i siły odśrodkowej leżała w płaszczyźnie symetrii samolotu.

W pracy pokazano ogólny model matematyczny samolotu sterowanego w locie przestrzennym. Przedstawiono charakterystyczne parametry zakrętu prawidłowego, ogolny model fizyczny i matematyczny sterowania, postać przyjetych praw sterowania oraz schemat blokowy urządzenia. Najważniejszy jest prawidłowy dobór układu sterowania i odpowiednich współczynników wzmocnienia. Przyjęto, że badany autopilot jest czterokanałowy, mogący sterować lotem za pomocą wychylenia steru wysokości, steru kierunku, lotek i dźwigni sterowania silnikami.

Wyniki badań uzyskano w wyniku symulacji numerycznej pełnego modelu dynamiki samolotu z układem automatycznego sterowania w oparciu o program napisany w środowisku obliczeniowym pakietu MATLAB. Zostały one przedstawione w postaci graficznej.

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