ANTIPLANE DEFORMATION OF ISOTROPIC BODY WITH A PERIODIC SYSTEM OF THIN RECTILINEAR INCLUSIONS[1]

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A new model of thin elastic rectilinear inclusion has been constructed. An approach to the stress-strain state analysis in an isotropic plane reinforced by a periodic system of thin elastic inclusions has been suggested. The formulae for determination of the effective modulus of composite material and stress intensity factors at the inclusion tip depending on volumetric contents of the reinforcing elements and their elastic characteristics have been obtained. Numerical analysis of the problem for various geometrical and mechanical parameters of the composite has been presented. The effect of the ratio between inclusion and matrix elastic moduli on the values of stress intensity factors has been studied as well.

Key words: elastic inclusion, stress intensity factor, antiplane deformation

1. Introduction

At present, the periodic composites are widely used in various branches of technology. Depending on the kind of reinforcing elements they can be divided into two types; i.e., the discretely and continuously reinforced ones.

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The following two approaches to the analysis of stress state in reinforced composites are most frequently used in mechanics of composite materials: replacement of the real material by a homogeneous one having the effective modulus application of various homogenized models.

The first approach is used if the lateral dimensions of the reinforcing elements are substantial (cf Berezhnitsky, 1987; Christensen, 1980; Vanin, 1982). For very thin reinforcing elements (layers, fibers, inclusions) it is suitable to use the homogenized models (cf Achenbach, 1985; Guz et al., 1982).

Bachvalov and Panasenko (1984) applied a model of the functionally graded material. The principle of energetic smoothing was suggested by Bolotin (Bolotin et al., 1980).

The homogenized model of periodic composites with microlocal parameters was proposed by Woźniak (1986), (1987a,b) and then developed by Matysiak and Woźniak (1987), (1988).

Matysiak (1995) analysed applications of this model to various branches of mechanics of a deformable solid body.

In reinforced fiber composites main part of the load is applied to a matrix while the fibers increase rigidity and strength of the material. On the other hand, as the fiber thickness is small enough, there is the area of stress concentration at its tip, that leads to destruction of a design. Therefore, it is necessary to have the refined models of composites reinforced with discrete fibers at one's disposal.

The stress distribution at the rigid linear inclusion tip was obtained at first by Panasyuk et al. (1972) with the help of conformal mapping method. Later Matysiak and Olesiak (1981) solved this problem by means of the Fourier integral transformation method.

The stress distribution in infinite isotropic plane at the tip of linear rigid inclusion under the cylindrical bending represented within the framework of the Kirchhoff theory was given by Hrushch et al. (1978).

However, such models of discrete fibers are simplified to a substantial extent, since neither the fiber rigidity nor the conditions at its surfaces are taken into account. Therefore, the refined models, which allow for more complete and exact description introducing the influence of thin-walled fiber elastic characteristics on the stress distribution at the inclusion tip are necessary.

The problem of periodic layered plate containing thin elastic inclusions was solved by Yevtushenko et al. (1995) within the framework of the homogenized model (Woźniak. 1986, 1987a,b). The layered composite is replaced with a homogenized material and then the problem is solved for the material with inclusions.

Opanasovich and Dragan (1981) (cf Opanasovich, 1997) using the method of complex potentials constructed the new model of thin rectilinear elastic isotropic inclusion which allowed for taking into account the influence of elastic characteristics of discrete fiber – inclusion and its thickness on the stress state in the composite. The model of a periodic system of line inclusions woth the following assumptions made is proposed in the present paper: the fiber is represented by a thin elastic inclusion; the problem is solved within the framework of fracture mechanics of isotropic body having inclusions.
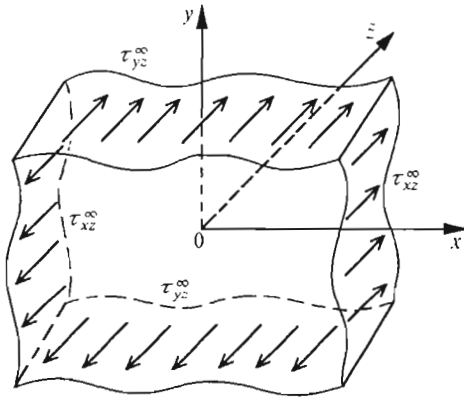
2.   Formulation of the problem

Fig. 1. Scheme of the body

The infinite isotropic body, subject to antiplane deformation (Fig.1) is considered, containing a periodic system of rectilinear inclusions of the height $2h$ and length $2l$. It considered in the Cartesian system $0xy\tilde{z}$; where the axis $0\tilde{z}$ is the deformation axis. Between the matrix and inclusions the conditions for the ideal mechanical contact are satisfied. We take into consideration a complex plane $\mathcal{C}$ $(z = x + iy)$. The centers of inclusions are situated at the points $m\omega_1 + n\omega_2 \in \mathcal{C}$ $(m, n \in \mathcal{Z},\ \mathrm{Im}\omega_1 \geqslant 0,\ \mathrm{Im}(\omega_2/\omega_1) > 0)$, where ω_1 and ω_2 are the periods; $\mathcal{Z}$ is a set of natural numbers (Fig.2).

Let us introduce the local system of coordinates $0x_1y_1\tilde{z}$, in which the axis $0x_1$ coincides with the longer axis of the inclusion symmetry, which make an angle α with the axis $0x$. We assume that the load is applied to the matrix of the body.
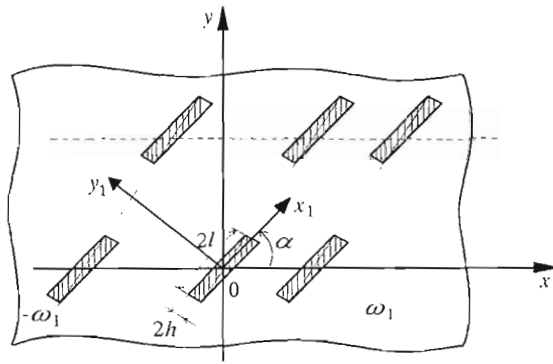
Fig. 2. Body with the periodic system of thin rectilinear inclusions

3.   Solution to the problem

The model of a thin rectilinear inclusion was constructed by Opanasovich
and Dragan (1981). The complex potential and system of integral-differential
equations for a body with a system of heterogeneities included were derived by
Opanasovich and Dragan (1984). If in the obtained formulae we make the limit
transition to a periodic system of inclusions we obtain the complex potential,
that allows for finding the stress-strain state in the matrix

$$F(z) = \frac{h}{2\pi i} \int_{-l}^{l} [\beta_0 P_0'(t) + iQ_0'(t)]\widetilde{V}_j(te^{i\alpha} - z) \, dt + F_0(z) \qquad (3.1)$$

and also the system of integral-differential equations with unknown functions
$P_0(t)$, $Q_0(t)$ which are associated with the stress and displacement jumps at
the inclusion edges $(|x| \leqslant l)$

$$\pi P_0(x) - h\beta_0 \int_{-l}^{l} \frac{P_0'(t)}{t - x} \, dt + hb_0 \int_{-l}^{l} \Big[Q_0'(t)\mathrm{Im}\widetilde{K}_j(t - x) +$$

$$- \beta_0 P_0'(t)\mathrm{Re}\widetilde{K}_j(t - x) \Big] \, dt = -2\pi b_0 \mathrm{Im}\Big[F_*(x) \Big]$$

$$\qquad (3.2)$$

$$\pi \beta_0 Q_0(x) - h \int_{-l}^{l} \frac{Q_0'(t)}{t - x} \, dt - ha_0 \int_{-l}^{l} \Big[\beta_0 P_0'(t)\mathrm{Im}\widetilde{K}_j(t - x) +$$

$$+ Q_0'(t)\mathrm{Re}\widetilde{K}_j(t - x) \Big] \, dt = 2\pi a_0 \mathrm{Re}\Big[F_*(x) \Big]$$

The following notation has been introduced to Eqs (3.1), (3.2)

$$F_0(z) = d_j e^{-i\alpha} + \widetilde{F}_0(z) \qquad\qquad \beta_0 = \frac{\mu}{\mu_0} \qquad\qquad b_0 = 1 - \varepsilon_0$$

$$\varepsilon_0 = \min(1, \beta_0^{-1}) \qquad\qquad a_0 = 1 - \beta_0\varepsilon_0 \qquad d_j = d_j^{(1)} + id_j^{(2)} \tag{3.3}$$

$$\widetilde{K}_j(x) = e^{i\alpha}\widetilde{V}_j\left(xe^{i\alpha}\right) - \frac{1}{x} \qquad\qquad \sum_{n=0}^{\infty}{}' a_n = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n$$

$$\widetilde{V}_j(z) = \frac{2\pi}{\omega_j}\sin\frac{2\pi z}{\omega_j}\sum_{n=0}^{\infty}{}'\left(\cos\frac{2\pi n\omega_{3-j}}{\omega_j} - \cos\frac{2\pi z}{\omega_j}\right)^{-1} \tag{3.4}$$

where

μ, μ_0 — shear moduli of matrix and inclusion, respectively

$F_*(z)$ — complex potential $F_0(z)$ in the system of coordinates $0x_1y_1$

$\widetilde{F}_0(z)$ — known complex potential representing the stress-strain state of a body without inclusions

d_j — unknown constants which are determined from the relations

$$\mathrm{Im}[f(z + \omega_\nu) - f(z)] = R_\nu \qquad\qquad F(z) = f'(z) \qquad\qquad \nu = 1, 2 \tag{3.5}$$

where R_ν are the known force components, acting on the periodic parallelogram sides.

We complete Eqs (3.2) adding the following relations

$$\int_{-l}^{l} Q_0'(t)\, dt = 0 \qquad\qquad \int_{-l}^{l} P_0'(t)\, dt = 0 \tag{3.6}$$

Making the limit transition in Eqs (3.1), (3.2) ($\mu_0 \to 0$ or $\mu_0 \to \infty$) we obtain the formula for complex potential and a singular integral equation for the body with a periodic system of cracks (rigid inclusions.). In the case of homogeneous material ($\mu = \mu_0$) the complex potential $F(z)$ corresponds to $\widetilde{F}_0(z)$.

We assume that the function $\widetilde{F}_0(z)$ reads

$$\widetilde{F}_0(z) = \tau_{xz}^{\infty} - i\tau_{yz}^{\infty} \tag{3.7}$$

Taking into account Eqs (3.1) and (3.7) we can write the function as follows

$$
f(z) = -\frac{h}{2\pi i} \int_{-l}^{l} [\beta_0 P_0'(t) + iQ_0'(t)] \widetilde{W}_j(te^{i\alpha} - z)\, dt +
$$

$$
+ (d_j e^{-i\alpha} + \tau_{xz}^{\infty} - i\tau_{yz}^{\infty})z + C
$$

(3.8)

where

$$
\widetilde{W}_j(z) = \ln\sin\frac{\pi z}{\omega_j} + \sum_{n=1}^{\infty} \ln\left(1 - \frac{\cos\frac{2\pi z}{\omega_j}}{\cos\frac{2\pi n\omega_{3-j}}{\omega_j}}\right)
$$

(3.9)

Here C is an arbitrary constant not affecting the stress state of the composite. Substituting Eq (3.8) into Eq (3.5) we obtain

$$
\mathrm{Im}\left\{\omega_2 d_j e^{-i\alpha} - [1 - (-1)^j]\frac{he^{i\alpha}}{2\omega_j} \int_{-l}^{l} t[\beta_0 P_0'(t) + iQ_0'(t)]\, dt\right\} =
$$

$$
= R_2 - \mathrm{Im}\left[\omega_2(\tau_{xz}^{\infty} - i\tau_{yz}^{\infty})\right]
$$

(3.10)

$$
[1 + (-1)^j]\mathrm{Im}\left\{\frac{he^{i\alpha}}{2\omega_j} \int_{-l}^{l} t[\beta_0 P_0'(t) + iQ_0'(t)]\, dt\right\} +
$$

$$
+\omega_1(d_j^{(2)}\cos\alpha - d_j^{(1)}\sin\alpha) = R_1 + \omega_1\tau_{yz}^{\infty}
$$

We assume that the forces R_j satisfy the following equations

$$
R_1 = -\omega_1\tau_{yz}^{\infty} \qquad\qquad R_2 = \mathrm{Im}\left[\omega_2(\tau_{xz}^{\infty} - i\tau_{yz}^{\infty})\right]
$$

(3.11)

Thus, we have two integral equations for the same problem. One of them corresponds to the kernel for $j = 1$, and the other to $j = 2$. This is caused by the order of summation at the limit transition. The analysis of functions $\widetilde{V}_j(z)$ (see Eq (3.4)) shows that the numerical values of series terms depend on the ratio $\mathrm{Im}(\omega_2/\omega_1) = r$. The equations, corresponding to $j = 1$ are convenient to use, when $r > 1$; otherwise the equations, corresponding meaning $j = 2$ can be used.

4.   Determination of the effective modulus

Let us consider the case of rectangular lattice with the parameters

$$
\omega_1 = \omega \qquad\qquad \omega_2 = id
$$

(4.1)

After Panasjuk et al. (1976) the component W of the displacement vector along the axis $0\tilde{z}$ is determined from the formula

$$w = \frac{1}{\mu} \mathrm{Re} f(z) \qquad (4.2)$$

We determine the effective constants of composite material using the approach formulated by Vanin (1985). Taking into account Eqs (3.8) and (4.2) we can write

$$w(z + \omega_\nu - w(z) = [1 + (-1)^{j+\nu+1}] \cdot$$

$$\cdot \mathrm{Re}\left\{ \frac{he^{i\alpha}}{2\mu\omega_{3-\nu}} \int_{-l}^{l} t[\beta_0 P_0'(t) + iQ_0'(t)]\, dt \right\} + F = \qquad (4.3)$$

$$= \langle \gamma_x - i\gamma_y \rangle \frac{\omega_\nu}{2} + \langle \gamma_x + i\gamma_y \rangle \frac{\overline{\omega}_\nu}{2} \qquad \nu = 1, 2$$

where

$$F = \begin{cases} \frac{\omega}{\mu}(\tau_{xz}^\infty + d_j^{(1)} \cos\alpha + d_j^{(2)} \sin\alpha) & \nu = 1 \\ \frac{d}{\mu}(\tau_{yz}^\infty - d_j^{(2)} \cos\alpha + d_j^{(1)} \sin\alpha) & \nu = 2 \end{cases}$$

and $\langle \gamma_x \rangle$, $\langle \gamma_y \rangle$ are the average strains related to the average stresses τ_{xz}^∞, τ_{yz}^∞ (3.11) by Hooke's law

$$\langle \gamma_x \rangle = \frac{\tau_{xz}^\infty}{G_x} + \frac{\mu_x}{G_y}\tau_{yz}^\infty \qquad \langle \gamma_y \rangle = \frac{\tau_{yz}^\infty}{G_y} + \frac{\mu_y}{G_x}\tau_{xz}^\infty \qquad (4.4)$$

By virtue of Eq (4.3), (4.3) we have

$$G_y = \mu\left(1 - d_j^{(2)} \cos\alpha + d_j^{(1)} \sin\alpha - Q_{1j1}\right)^{-1}$$

$$\mu_x = G_y\mu^{-1}\left(d_j^{(1)} \cos\alpha + d_j^{(2)} \sin\alpha + Q_{2j1}\right)$$

$$(4.5)$$

$$G_x = \mu\left(1 + d_j^{(1)} \cos\alpha + d_j^{(2)} \sin\alpha + Q_{2j2}\right)^{-1}$$

$$\mu_y = G_x\mu^{-1}\left(d_j^{(1)} \sin\alpha - d_j^{(2)} \cos\alpha - Q_{1j2}\right)$$

The following notation has been are introduced to Eq (4.5)

$$Q_{kjn} = a[1 + (-1)^{j+k}] \begin{cases} \mathrm{Re}Q_{3n} & k = 1 \\ \mathrm{Im}Q_{3n} & k = 2 \end{cases}$$

$$(4.6)$$

$$Q_{3n} = e^{i\alpha} \int_{-l}^{l} t[\beta_0 P'_{0n}(t) + iQ'_{0n}(t)]\, dt \qquad \alpha = \frac{h}{2d\omega_1}$$

Here, $P'_{01}(t)$, $Q'_{01}(t)$ is the solution of the set of equations (3.2), (3.6), (3.10) at $\tau_{xz}^\infty = 0$, $\tau_{yz}^\infty = 1$ while $P'_{02}(t)$, $Q'_{02}(t)$ is the solution of the same system at $\tau_{xz}^\infty = 1$, $\tau_{yz}^\infty = 0$.

Solving Eqs (3.10) for d_j and substituting the obtained solutions into Eq (4.5) we obtain

$$G_y = \mu(1 - 2a\mathrm{Re}Q_{31}) \qquad \mu_x = 2G_y\mu^{-1}\mathrm{Im}Q_{31}a$$

$$(4.7)$$

$$G_x = \mu(1 + 2a\mathrm{Im}Q_{31}) \qquad \mu_y = -2G_x\mu^{-1}\mathrm{Re}Q_{32}a$$

5. Determination of the stress intensity factors

The set of Eqs (3.2), (3.6), (3.10) we solve numerically by means of the method of mechanical quadratures (cf Panasjuk et al., 1976)

$$\left[P_0(xl)\right]'_x = \frac{v_0(x)}{\sqrt{1-x^2}} \qquad \left[Q_0(xl)\right]'_x = \frac{u_0(x)}{\sqrt{1-x^2}} \qquad (5.1)$$

For determination of the nodal values of functions $v_0(x)$ and $u_0(x)$ and the constants $d_j^{(1)}$ and $d_j^{(2)}$ we have the system of linear algebraic equations

$$\sum_{m=1}^{M} v_0(t_m)\left[\eta(x_r - t_m)\pi - \frac{h^*\beta_0}{t_m - x_r}\right] + b_0 h^* \sum_{m=1}^{M}\left[u_0(t_m)\mathrm{Im}K_j(t_m - x_r) + \right.$$

$$\left. -\beta_0 v_0(t_m)\mathrm{Re}K_j(t_m - x_r)\right] + 2b_0 M d_j^{(2)} = -2b_0 M[\tau_{xz}^\infty \sin\alpha - \tau_{yz}^\infty \cos\alpha]$$

$$(5.2)$$

$$\sum_{m=1}^{M} u_0(t_m)\left[\pi\beta_0\eta(x_r - t_m) - \frac{h^*}{t_m - x_r}\right] - a_0 h^* \sum_{m=1}^{M}\left[\beta_0 v_0(t_m)\mathrm{Im}K_j(t_m - x_r) + \right.$$

$$\left. +u_0(t_m)\mathrm{Re}K_j(t_m - x_r)\right] - 2a_0 M d_j^{(1)} = -2a_0 M[\tau_{xz}^\infty \cos\alpha + \tau_{yz}^\infty \sin\alpha]$$

where $r = \overline{1, M-1}$ and

$$\sum_{m=1}^{M} u_0(t_m) = 0 \qquad \sum_{m=1}^{M} v_0(t_m) = 0$$

$$\mathrm{Im}\left\{\frac{2}{\lambda_2}d_j e^{-i\alpha} - [1-(-1)^j]\frac{\pi h^* \lambda_j e^{i\alpha}}{4M}\sum_{m=1}^{M}t_m[\beta_0 v_0(t_m) + iu_0(t_m)]\right\} = 0 \tag{5.3}$$

$$[1+(-1)^j]\mathrm{Im}\left\{\frac{\pi h^* \lambda_i e^{i\alpha}}{4M}\sum_{m=1}^{M}t_m[\beta_0 v_0(t_m) + iu_0(t_m)]\right\} +$$

$$+\frac{2}{\lambda}(d_j^{(2)}\cos\alpha - d_j^{(1)}\sin\alpha) = 0$$

where

$$\lambda_1 = \frac{2l}{\omega} \qquad \lambda_2 = \frac{2l}{id} = -i\tilde{\lambda}_2 \qquad t_m = \cos\frac{2\pi(2m-1)}{2M}$$

$$x_r = \cos\frac{r\pi}{M} \qquad K_j(x) = e^{i\alpha}V_j\left(xe^{i\alpha}\right) - \frac{1}{x} \qquad h^* = \frac{h}{l}$$

$$V_1(x) = \pi\lambda_1\sin(\pi\lambda_1 x)\sum_{n=0}^{\infty}{}'\frac{1}{\cosh\gamma_{1n} - \cos\pi\lambda_1 x} \tag{5.4}$$

$$V_2(x) = -\pi\lambda_2\sinh(\pi\tilde{\lambda}_2 x)\sum_{n=0}^{\infty}{}'\frac{1}{\cosh\gamma_{2n} - \cosh\pi\tilde{\lambda}_2 x}$$

$$\gamma_{1n} = 2\pi n\frac{\lambda_1}{\tilde{\lambda}_2} \qquad \gamma_{2n} = 2\pi n\frac{\tilde{\lambda}_2}{\lambda_1}$$
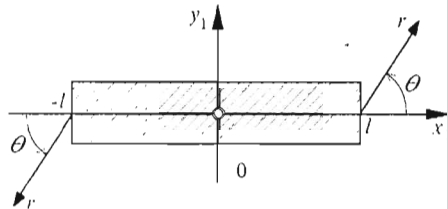

Fig. 3. Polar coordinate system at the inclusion tip

The stress distribution at the thin inclusion tip (Fig.3) was given by Opanasovich and Dragan (1981) in the Cartesian coordinate system

$$\tau_{xz} = \frac{1}{\sqrt{2r}}(K_2 \cos \theta_1 - K_1 \sin \theta_1) + O(1)$$

(5.5)

$$\tau_{yz} = \frac{1}{\sqrt{2r}}(K_2 \sin \theta_1 + K_1 \cos \theta_1) + O(1)$$

and in the polar coordinate system

$$\tau_{rz} = \frac{1}{\sqrt{2r}}(K_2 \cos \theta_1 + K_1 \sin \theta_1) + O(1)$$

(5.6)

$$\tau_{\theta z} = \frac{1}{\sqrt{2r}}(-K_2 \sin \theta_1 + K_1 \cos \theta_1) + O(1) \qquad \theta_1 = \frac{\theta}{2}$$

It is expressed in terms of the generalized stress intensity factors K_j, $j = \overline{1,2}$, which are determined from the formulas

$$K_1^{\pm} = \mp \frac{h^* \sqrt{l}}{2} u_0(\pm 1) \qquad\qquad K_2^{\pm} = \mp \frac{h^* \sqrt{l}}{2} \beta_0 v_0(\pm 1)$$

(5.7)

Here

$$u_0(\pm 1) = \frac{1}{M} \sum_{m=1}^{M} u_0(t_m)(-1)^r \tan^{\mp 1} \frac{(2m-1)\pi}{4M}$$

$$v_0(\pm 1) = \frac{1}{M} \sum_{m=1}^{M} v_0(t_m)(-1)^r \tan^{\mp 1} \frac{(2m-1)\pi}{4M}$$

(5.8)

$$r = m + 1 + \frac{(M+1)(1 \mp 1)}{2}$$

We determine the effective constants of a composite material from Eqs (4.3), taking into account the relations

$$a = \frac{h^* \lambda_1 \tilde{\lambda}_2}{8}$$

(5.9)

$$Q_{3n} = \frac{\pi}{M} e^{i\alpha} \sum_{m=1}^{M} t_m [\beta_0 v_{0n}(t_m) + i u_{0n}(t_m)]$$

6.   Analysis of the results

The results of numerical analysis carried out for various values of geometrical and mechanical parameters shown in Fig.4 $\div$ Fig.7.
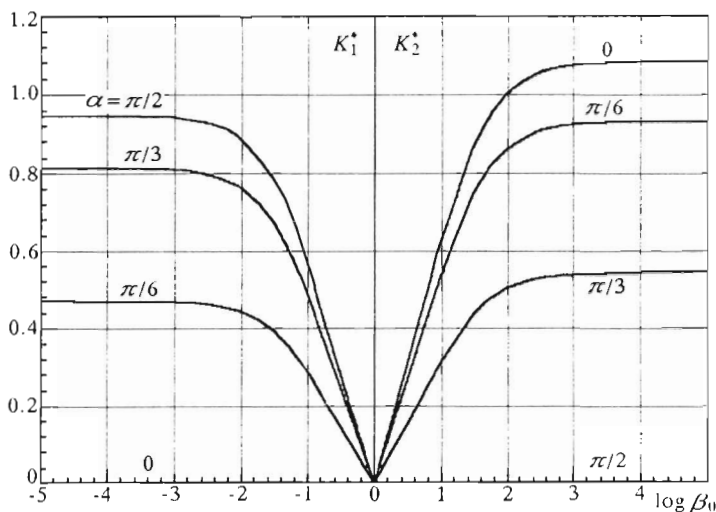
Fig. 4. SIF $K_j^* = K_j/(\tau_{yz}^\infty \sqrt{l})$ versus the ratio between the matrix and inclusion rigidities $\beta_0 = \mu/\mu_0$ at $\tau_{zz}^\infty = 0$, $h/l = 0.1$ for a square lattice with periods $\lambda_1 = \tilde{\lambda}_2 = 0.4$ and for various angles α
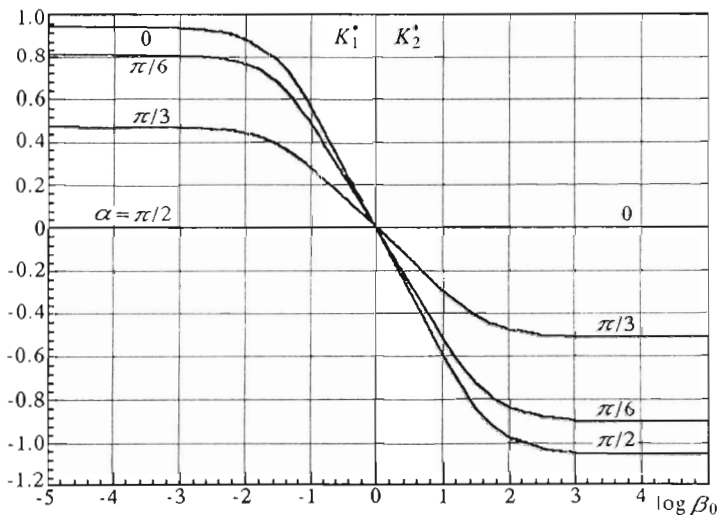

Fig. 5. SIF $K_j^* = K_j/(\tau_{xz}^\infty \sqrt{l})$ versus the ratio between the matrix and inclusion rigidities $\beta_0 = \mu/\mu_0$ at $\tau_{yz}^\infty = 0$, $h/l = 0.1$ for a square lattice with periods $\lambda_1 = \tilde{\lambda}_2 = 0.4$ and for various angles α

The calculations were made for 30 nodal values of the system (5.2) and (5.3).

Basing on the results the following conclusions can be drawn:

- In the case of $\alpha = 0°$, small values of the parameter $\tilde{\lambda}_2$ and for the ratio $\beta_0 = \mu/\mu_0 = 10^5$ the value of stress intensity factor K_2 coincides with the corresponding one, obtained for a periodic system of collinear cracks by Panasjuk et al. (1976)

- In the case of $\alpha = 0°$, small values of the parameter λ_1 and for the ratio of $\beta_0 = \mu/\mu_0 = 10^5$ the value of stress intensity factor K_2 coincides with the corresponding one, obtained for a periodic system of coplanar cracks by Panasjuk et. al. (1976)

- In the case of small values of the parameters λ_1 and $\tilde{\lambda}_2$ and for the ratio $\beta_0 = \mu/\mu_0 = 10^5$ the value K_2 coincides with the corresponding one, obtained for a single crack by Panasjuk et al. (1976)

- In the case of $\lambda_1 = \tilde{\lambda}_2 = \lambda$ and $\beta_0 = \mu/\mu_0 = 10^5$ the value K_2 coincides with the stress intensity factor obtained for periodic systems of cracks by Panasjuk et al. (1976).

It was supposed, that $\tau_{xz}^\infty = 0$, and $K_1 = 0$ for the cracks, and $K_2 = 0$ for the rigid inclusions. From the obtained results it follows that in this case, the approximate formula presented by Panasjuk et al. (1976) can be used not only for small values of the parameter λ since even for $\lambda = 0.8$ the discrepancy between the exact and approximate values does not exceed 7%. We have $K_1 = 0$ for the cracks and $K_2 = 0$ for rigid inclusions.

The relations $K_j^* = K_j/(\tau_{yz}^\infty \sqrt{l})$ and $K_j^* = K_j/(\tau_{xz}^\infty \sqrt{l})$, $j = \overline{1,2}$ versus the ratio between matrix and inclusion rigidities $\beta_0 = \mu/\mu_0$ are shown in Fig.4 and Fig.5 for a square lattice with $\lambda_1 = \tilde{\lambda}_2 = 0.4$ at various angles α and $h/l = 0.1$. The curve in Fig.4 is constructed for the case $\tau_{xz}^\infty = 0$, while Fig.5 corresponds to $\tau_{yz}^\infty = 0$. In these figures $K_2^* = 0$ for the inclusions more rigid than the matrix, while $K_1^* = 0$ if the material of inclusion is more soft than that of the matrix. The influence of the ratio $\beta_0 = \mu/\mu_0$ on the elastic constants is shown in Fig.6 and Fig.7 for a rectangular lattice with the periods $\lambda_1 = 2/3$, $\tilde{\lambda}_2 = 5/3$ and $h/l = 0.1$. The curves in these figures are constructed for $\alpha = 0°$ and for $\alpha = 30°$, respectively. The performed computations have shown that

$$\frac{\mu_x}{G_y} = \frac{\mu_y}{G_x} = \frac{\mu}{G} \tag{6.1}$$

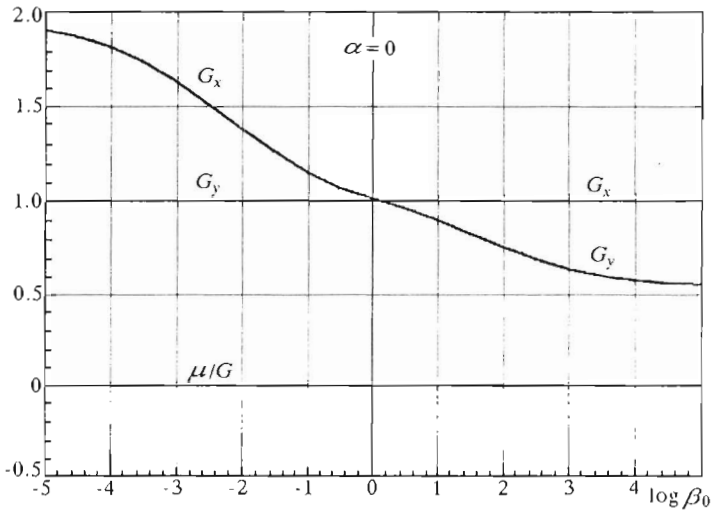Moreover, $\mu_x = \mu_y = 0$ if $\alpha = 0°$.

Fig. 6. Elastic moduli of composites versus the ratio between the matrix and inclusion rigidities $\beta_0 = \mu/\mu_0$ for a rectangular lattice with the periods $\lambda_1 = 2/3$, $\widetilde{\lambda}_2 = 5/3$ at $\alpha = 0°$ and $h/l = 0.1$
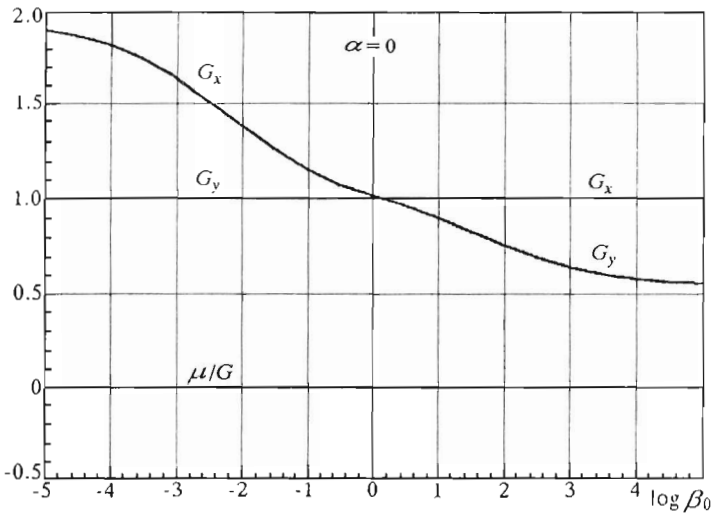

Fig. 7. Elastic moduli of composites versus the ratio between the matrix and inclusion rigidities $\beta_0 = \mu/\mu_0$ for a rectangular lattice with the periods $\lambda_1 = 2/3$, $\widetilde{\lambda}_2 = 5/3$ at $\alpha = 30°$ and $h/l = 0.1$

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Antypłaska deformacja ciał izotropowych z periodycznym układem cienkich liniowych inkluzji

Streszczenie

W pracy przedstawiono metodę modelowania stanu naprężeń i odkształceń w ciele izotropowym wzmocnionym periodycznym układem cienkich sprężystych inkluzji. Otrzymano równania określające moduły efektywne kompozytu oraz współczynniki intensywności naprężeń w wierzchołkach inkluzji. Przeprowadzono analizę współczynników intensywności naprężeń w zależności od efektywnych modułów.