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PRACTICAL APPLICABILITY OF IDENT PACKAGE TO IDENTIFICATION OF PARAMETERS OF DYNAMIC MODELS OF MACHINE TOOL SUPPORTING SYSTEMS

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In the paper a short description of the identification method of parameters of machine tool supporting system dynamic models, constructed according to the rigid finite element method convention is given. The identification is carried out on the basis of selected points of the experimentally determined amplitude-frequency characteristics of tested machine tool. It is possible to identify the parameters of spring-damping elements which model slideway joints, lead screws, turntables and rolling blocs as well as the parameters of the object foundation on spring-damping vibration isolators. Principal mathematical relationships of the identification algorithm are given. An example of practical use of this method for identifying selected parameters of FWD-32J type milling machine model is presented and discussed.

Key words: identification, machine tool, modelling, dynamics, stiffness

1. Introduction

Theoretical analysis of as complex mechanical structures as those formed by machine tool supporting systems is a very laborious and complicated process with respect to both their static properties and the dynamic ones (Marchelek, 1991). Experiments are, in turn, very expensive, labour- and time-consuming, and require sophisticated and expensive apparatus and adequate software, and, what is most important, require a very experienced the research team in order to assure a their proper realization, and adequate processing of the signals obtained from measurements and, finally, a consistent interpretation of the results. Hence, in view of minimization of the research costs, it is most advisable to develop adequate procedures and methodology of physical models construction and, basing on them mathematical models of real objects which enable researchers to carry out simulation tests. The need for necessary experiments, carried out on real object, can be then reduced to an indispensable minimum.

At the first stage of creating such a model there appears an obvious necessity for the real object idealization, which consists in taking into account only the most important properties, i.e. those decisive of a given phenomenon, neglecting, at the same time, the properties of secondary significance. The real object idealization cannot, however, go too far since an oversimplified model will inadequately reflect the properties of the structure tested and it will be impossible to effectively shape or analyze these properties based on the simulation model tests.

Therefore, in practice, it is often necessary to reach a reasonable compromise in the form of relatively simple models which would enable the researchers to carry out easily (basing on limited information data from experiments) the simulation tests with some reduction in the accuracy of the results obtained. In the case of the machine tool supporting systems especially great capabilities of creating such models are offered by the rigid finite element (RFE) method (Kruszewski et al., 1975), supplemented by the slideway joint modelling option (Szwengier, 1991).

As early as at the initial stage of the model development it is necessary to assume definite values of the parameters (those of stiffness and damping), which describe the spring-damping properties of slideway joints, rolling bearings, lead screws, clutches, etc., since a proper selection of them is highly decisive on the degree of consistence between the model and the real object. Should such a consistence be lacking the cause of this discrepancy is to be sought, first of all, in the model structure errors, too radical model simplifications, or just in the "mistuning" of the model by taking wrong values of the parameters. Taking these values on the basis of the literature recommendations can, therefore, be justified only at the initial stage of the model development, and that within a limited range, since it does not guarantee a sufficient conformity to be attained between the model and object. In the next steps it is necessary to identify these parameters based on the results of experiments carried out on the real object. If it only can be done, the greatest possible number of factors should be determined from independent experiments, preferably carried out under loading conditions similar to those actual. In particular, such a possibility exists in relation to vibration isolators, the parameters of which can be determined by means of individual tests done on each vibration isolator on a special setup (Witek, 1992).

At the Mechanical Engineering Faculty of Technical University of Szczecin a special package of the IDENT programs was developed in order to identify the parameters of the spring-damping elements (the SDEs) of the machine tool supporting system models, constructed according to the RFE method. based on limited amount of information from the tests done on the real object. This package allows for a relatively quick and effective identification of selected coefficients of stiffness and damping of the slideway joints, lead screws, and rolling turntables and blocks. Also identified in the same way can be parameters of the machine tool foundation on spring-damping vibration isolators. However, while the model simulation tests do not involve, in general, any computational problems the situation is quite different when doing the identification based on experimentally determined data. In this case, there is a problem of various sensitivity of amplitude-frequency (A-F) characteristics, on the basis of which the identification is to be conducted, to the changes in values of parameters taken as the decision variables. These characteristics are. as a rule, most sensitive to the changes of the slideway joints stiffness and less (and within a lower frequency band) sensitive to the changes of stiffness of those elements which model the vibration isolators. A clearly weakest effect on the machine tool A-F characteristics is displayed by the damping of particular joints.

And this is why the selection of proper initial values of these coefficients has an unusually radical effect on the modelling accuracy. Identification of the damping of particular slideway joints in the form of independent, individual decision variables is very difficult or simply impossible whereas the global effect of damping, produced by all the joints, on the A-F characteristics may be considerably greater than that of the identified stiffness coefficients characterized by a high, even predominant, individual sensitivity. In testing the real structures it was therefore necessary either to give up evaluating these parameters by means of identification and take the damping of slideway joint modellling elements according to the literature data (this, however, questioned the use of identifying other parameters) or develop a methodology which would allow for identification of damping even on the basis of those A-F characteristics which display a low sensitivity to damping.

This work presents an example of identification, done on an FWD-32J type milling machine model, and some results obtained for stiffness and damping of the SDEs of this model. The identification was carried out based on experimentally determined A-F characteristics of the machine table, which were transposed to the principal central axes of inertia of the table. The machine tool model was formed according to the RFEs convention, making use of the DOUNO program elaborated by Witek et al., (1983).

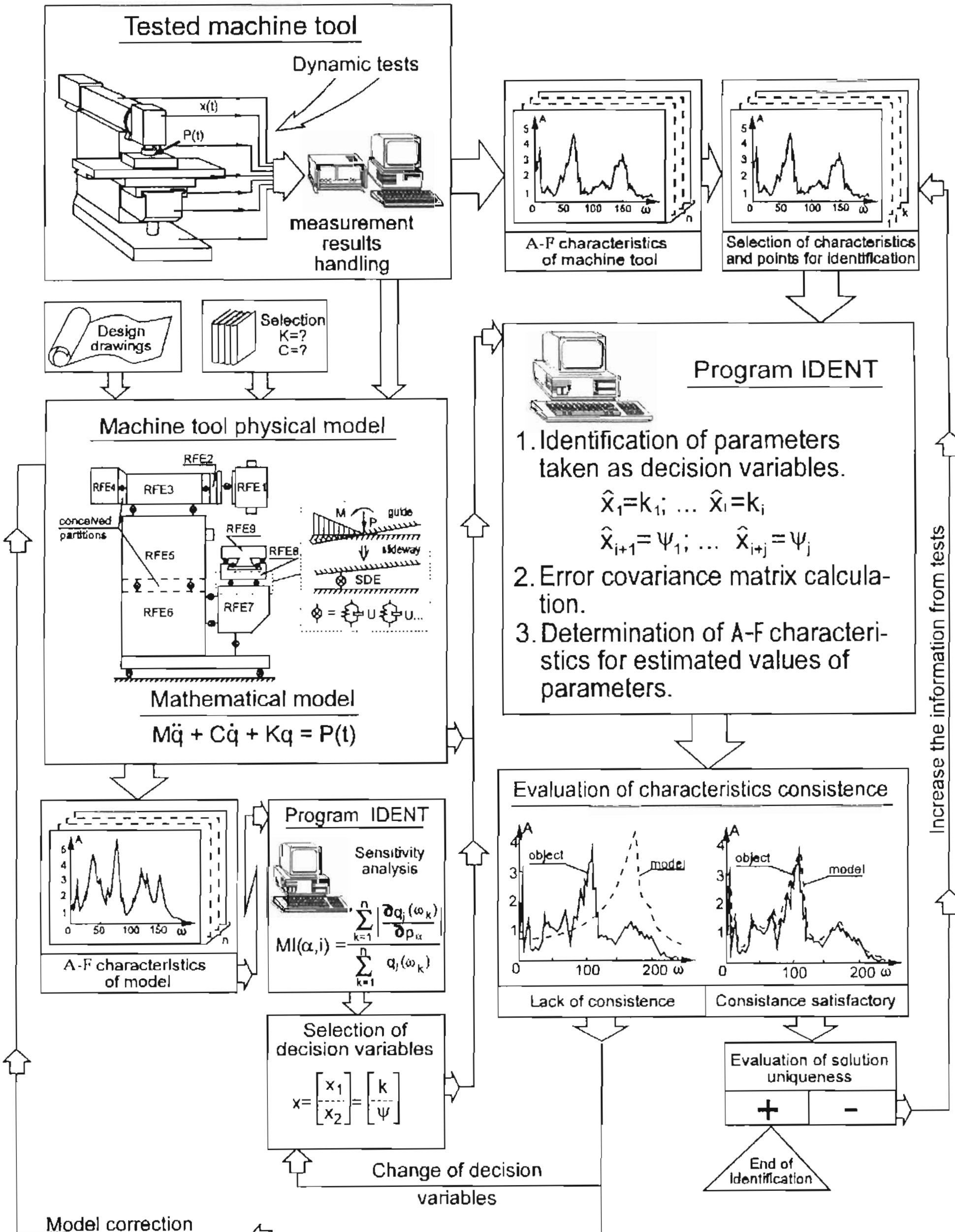


Fig. 1. The general scheme of modelling the machine tool supporting systems and identifying parameters of SDEs of their models

2. Description of the method

A general block diagram of the identification process is presented in Fig.1. Initially, the process runs in a two-path way. The first path represents the experiments done on the entire machine tool or on its particular subassemblies. The basic purpose of these tests is to determine the A-F characteristics of some selected solids of the machine. These characteristics. after transforming them to the systems of principal central axas of inertia of the solids selected, form a basis for the identification. The other path represents the physical model development, carried out on the basis of the design drawings supplemented with necessary weightings and geometric measurements of the real machine tool. Through solving differential equations, which describe small vibrations of the physical model, the A-F characteristics of the selected solids of this model are determined. These characteristics are compared, using a specified criterion, with the respective experimental characteristics. If there is lack of consistence between them the parameters of the model SDEs are identified according to the algorithm developed. If the discrepancy between the model characteristics and the experimental ones persists, the model structure is corrected and the identification process is repeated.

3. Principles of developing a physical model of machine tool supporting system – basic mathematical relationships

The machine tool is considered as a set of spatially arranged rigid solids (RFEs) interconnected by means of the spring-damping elements (the SDEs) of linear characteristics. Dividing the machine tool into the rigid solids is usually performed in a natural way, that is following the planes of contact between its respective elements. If, however, the accuracy of the obtained model is too poor it is possible to carry out imaginary divisions. Each RFE has six degrees of freedom. The SDEs which connect them and which model the slideway joints and rolling turntables and blocks (Fig.2a), are composed of six weightless springs which represent three translational and three rotational stiffnesses and dampings. SDEs which model the lead screws (Fig.2b) are composed of single weightless spring situated along the lead screw axis. SDEs which model vibration isolators (Fig.2c) are composed of three such springs. Modelling is always preceded by an analysis of the loading conditions of particular slideway joints, in order to determine which slideways are operative under given load, what is

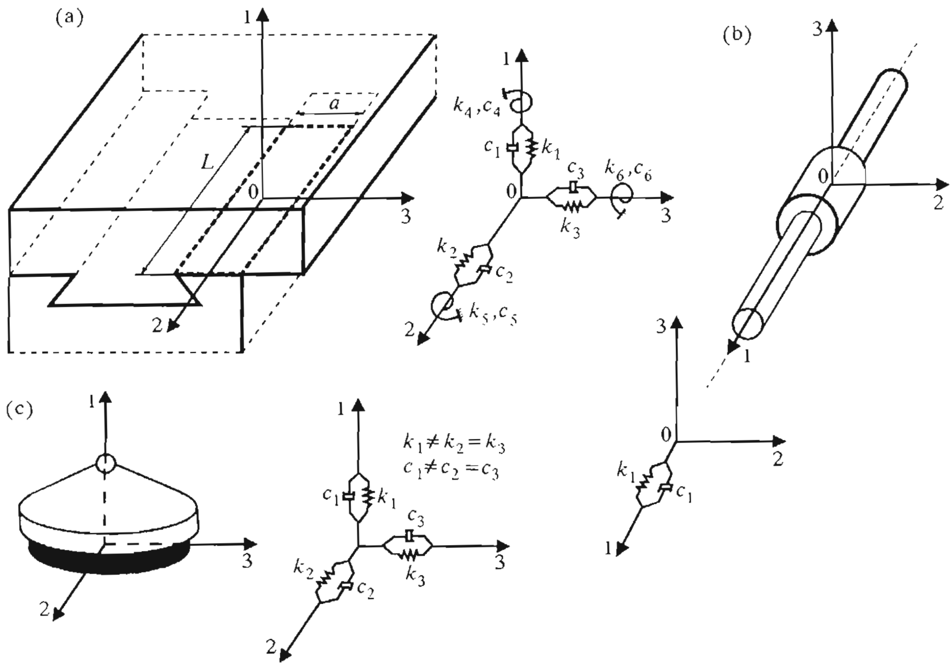


Fig. 2. Spring-damping elements that model: (a) slideway joints, (b) lead screws, (c) vibration isolators

the shape of the pressure distributions over the contacting surfaces and what are the values of this pressure. Just on this basis one can define the model structure and determine its parameters.

Differential equation of the model motion assumes the following form

$$\mathbf{M}\ddot{q} + \mathbf{C}\dot{q} + \mathbf{K}q = P(t) \tag{3.1}$$

where

$\mathbf{M}$	–	diag$\{m_i, m_i, m_i, J_{i1}, J_{i2}, J_{i3}\}$, inertia matrix, $i = 1, 2, ..., u$
$\mathbf{C}$	–	damping matrix
$\mathbf{K}$	–	stiffness matrix
q	–	generalized coordinate vector
$P(t)$	–	generalized force vector
u	–	number of RFEs.

The matrices $\mathbf{K}$ and $\mathbf{C}$ are symmetrical, positive definite real matrices

$$\mathbf{K} = \{\mathbf{K}_{ij}\} \qquad \mathbf{C} = \{\mathbf{C}_{ij}\} \qquad i = 1, 2, ..., u \qquad j = 1, 2, ..., u \tag{3.2}$$

According to Kruszewski et al. (1975), the blocks $\mathbf{K}_{pp}$ of the stiffness matrix, which lie on the diagonal, are determined from the following relationship

$$\mathbf{K}_{pp} = \sum_{\kappa=1}^{i_p} \mathbf{K}_{pp\kappa} \qquad \text{where} \qquad \mathbf{K}_{pp\kappa} = \mathbf{S}_{p\kappa}^{\mathsf{T}} \mathbf{\Theta}_{p\kappa}^{\mathsf{T}} \mathbf{K}_{\kappa} \mathbf{\Theta}_{p\kappa} \mathbf{S}_{p\kappa} \quad . \tag{3.3}$$

where

- i_p – number of the SDEs attached to the RFE No. p
- $\mathbf{S}_{p\kappa}$ – matrix of coordinates of the point, at which the SDE is attached to the RFE No. p
- $\mathbf{\Theta}_{p\kappa}$ – matrix of directional factors between the main axes of the SDE and the principal axes of inertia of the RFE No. p
- $\mathbf{K}_{\kappa}$ – stiffness coefficient block of the SDE No. κ attached to the RFE No. p.

The blocks $\mathbf{K}_{pr}$, which lie out of the main diagonal can be determined from the following relationship

$$\mathbf{K}_{pr} = \sum_{\kappa=1}^{i_{pr}} \mathbf{K}_{pr\kappa} \qquad \text{where} \qquad \mathbf{K}_{pr\kappa} = -\mathbf{S}_{p\kappa}^{\mathsf{T}} \mathbf{\Theta}_{p\kappa}^{\mathsf{T}} \mathbf{K}_{\kappa} \mathbf{\Theta}_{r\kappa} \mathbf{S}_{r\kappa} \tag{3.4}$$

where

- i_{pr} – number of SDE which interconnect the RFEs No. p and r
- $\mathbf{S}_{p\kappa}$ – matrix of coordinates of the point, at which the SDE is attached to the RFE No. r
- $\mathbf{\Theta}_{p\kappa}$ – matrix of directional factors between the main axes of the SDE and the principal axes of inertia of the RFE No. r
- $\mathbf{K}_{\kappa}$ – stiffness coefficient block of the SDE which interconnects the RFEs No. p and r, and at the same time

$$\mathbf{K}_{\kappa} = \operatorname{diag}\{k_{\kappa i}\} \qquad i = 1, 2, ..., 6 \tag{3.5}$$

The first three terms of this block present the translational while the remaining three the rotational stiffness coefficients. The method of constructing the matrices $\mathbf{S}_{p\kappa}$ and $\mathbf{\Theta}_{p\kappa}$ is described by Kruszewski et al. (1975). In the case of slideway joints the SDEs are attached at the geometric centres of working parts of particular surfaces of a joint. their stiffness coefficients are determined from the following relationships

$$k_{\kappa 1} = \frac{A_\kappa}{e_{n\kappa}} \qquad k_{\kappa 2} = k_{\kappa 3} = \frac{A_\kappa}{e_{t\kappa}}$$
$$k_{\kappa 4} = \frac{J_{\kappa 1}}{e_{t\kappa}} \qquad k_{\kappa 5} = \frac{J_{\kappa 2}}{e_{n\kappa}} \qquad k_{\kappa 6} = \frac{J_{\kappa 3}}{e_{n\kappa}} \tag{3.6}$$

where

A_κ	–	working part of the slideway surface
$J_{\kappa 1}, J_{\kappa 2}, J_{\kappa 3}$	–	principal moments of inertia of the working part of the slideway surface
$e_{n\kappa}, e_{t\kappa}$	–	surface contact deformability factors in the normal and tangential directions, respectively.

The structure of the damping matrix $\mathbf{C}$ is analogous to that of the stiffness matrix $\mathbf{K}$. At the same time, the damping coefficient block $\mathbf{C}_\kappa$ of the SDE κ, which interconnects the RFEs No. r and p, assumes the following form

$$\mathbf{C}_\kappa = \frac{1}{2\pi\omega}\text{diag}\Big\{\psi_{n\kappa}, \psi_{t\kappa}, \psi_{t\kappa}, \psi_{t\kappa}, \psi_{n\kappa}, \psi_{n\kappa}\Big\}\mathbf{K}_\kappa \tag{3.7}$$

where

$\psi_{n\kappa}, \psi_{t\kappa}$ – coefficients of relative dissipation of vibration energy in the normal and tangential directions, respectively.

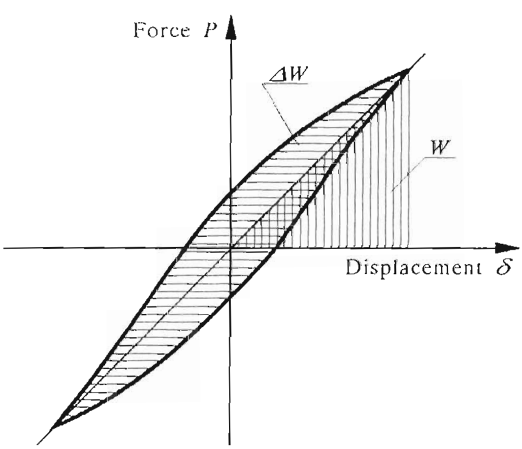


Fig. 3. Dissipated energy ΔW and maximum potential energy W of the system

These coefficients are generally defined as the rations of energy dissipated during a single period of vibration ΔW, to maximum potential energy W during this period (Fig.3), i.e.

$$\psi = \frac{\Delta W}{W} \tag{3.8}$$

The stiffness and damping matrices of the SDEs which model the lead screws are diagonal and assume the following forms

$$
\begin{aligned}
\mathbf{K}_\kappa &= \operatorname{diag}\{k_\kappa, 0, 0, 0, 0, 0\} \\
\mathbf{C}_\kappa &= \frac{1}{\omega}\operatorname{diag}\{c_\kappa, 0, 0, 0, 0, 0\}
\end{aligned}
\tag{3.9}
$$

where

$$
c_\kappa = \frac{\psi_\kappa k_\kappa}{2\pi}
\tag{3.10}
$$

Next, for the SDEs that model vibration isolators, on which the machine tool is placed, the stiffness and the damping matrices have the following forms

$$
\begin{aligned}
\mathbf{K}_\kappa &= \operatorname{diag}\{k_{\kappa n}, k_{\kappa t}, k_{\kappa t}, 0, 0, 0\} \\
\mathbf{C}_\kappa &= \frac{1}{\omega}\operatorname{diag}\{c_{\kappa n}, c_{\kappa t}, c_{\kappa t}, 0, 0, 0\}
\end{aligned}
\tag{3.11}
$$

and, in the same time

$$
c_{\kappa n} = \frac{\psi_\kappa k_{\kappa n}}{2\pi} \qquad\qquad c_{\kappa t} = \frac{\psi_\kappa k_{\kappa t}}{2\pi}
\tag{3.12}
$$

4. Identification algorithm – basic mathematical relationships

The IDENT program algorithm was already discussed in detail by Berczyński and Gutowski (1993), (1994). In this section given are only basic mathematical relationships that form its basis.

In order to determine the model frequency characteristics $\boldsymbol{q}(j\omega)$ the equation of motion, which is expressed by Eq (3.1), is subject to the Fourier transformation, after which assumes the following form

$$
[\mathbf{K} - \omega^2\mathbf{M} + j\omega\mathbf{C}]\boldsymbol{q}(j\omega) = \boldsymbol{P}(j\omega)
\tag{4.1}
$$

The characteristics sought are obtained through successive solving this equation at a fixed excitation amplitude, and at succesively altered frequency ω.

The experimental characteristics $\boldsymbol{z}$, determined basing on the dynamic tests on the real object, are distorted in some measure by systematic errors and random errors. Assuming that the systematic errors will be eliminated when processing the results, these characteristics can then be expressed as follows

$$
\boldsymbol{z} = \boldsymbol{q}(\boldsymbol{x}) + \boldsymbol{\nu}
\tag{4.2}
$$

where

- z – vector of experimentally determined characteristics of an object
- $\boldsymbol{q}(\boldsymbol{x})$ – vector of model frequency characteristics
- $\boldsymbol{x}$ – vector of those parameters, the components of which are to be estimated based on $\boldsymbol{z}$
- $\boldsymbol{\nu}$ – vector of observation random errors.

It is obvious that, in case of discrepancy between the model and the object, also their frequency characteristics will not agree with each other. It is therefore necessary to identify those parameters $\boldsymbol{x}$ whose function is expressed by these characteristics. The necessity for expressing the consistence between the model characteristics and the experimental ones requires adopting an adequate criterion of identification. Its form depends upon the assumed model of uncertainty as to the observation and to the model parameters $\boldsymbol{x}$, and upon the form of an estimator that will be used for transforming the information contained in the observations z into the estimates $\widehat{\boldsymbol{x}}$. In the IDENT program algorithm Fisher's uncertainty model is used (Schweppe, 1978), thereby it is assumed that the vector $\boldsymbol{x}$ is fully unknown whereas the observation error vector $\boldsymbol{\nu}$ is a random vector with the known of probability density function $f(\boldsymbol{\nu})$. Moreover, it has been assumed that $\boldsymbol{\nu}$ has the $\mathbf{N}(0,\mathbf{R})$ distribution. For such a case the cumulative distribution function z with given $\boldsymbol{x}$, i.e. the so called likelihood function $f(z:\boldsymbol{x})$ assumes the following form

$$f(z:\boldsymbol{x}) = [(2\pi)^n|\mathsf{R}|]^{-\frac{1}{2}} \exp\Big\{-\frac{1}{2}[z-\boldsymbol{q}(\boldsymbol{x})]^\top \mathsf{R}^{-1}[z-\boldsymbol{q}(\boldsymbol{x})]\Big\} \tag{4.3}$$

The strongest (in terms of statistic properties) estimator for the Fisher models is the maximum likelihood estimator, that is a vector which maximizes the likelihood function. For a function given by the relationship (4.3) this is equivalent to minimization of the following expression

$$J(\boldsymbol{x}) = \frac{1}{2}[z-\boldsymbol{q}(\boldsymbol{x})]^\top \mathsf{R}^{-1}[z-\boldsymbol{q}(\boldsymbol{x})] \tag{4.4}$$

In this way this expression becomes an identification criterion for the uncertainty model assumed.

The frequency characteristics $\boldsymbol{q}(\boldsymbol{x})$ versus model parameters $\boldsymbol{x}$ relationship is nonlinear and its form is unknown. Therefore, the identification criterion (4.4) minimization can only be carried out by means of iterative methods of optimization. From among them the highest computational effectiveness is offered by the gradient methods of variable metric (Szymanowski, 1984). Within these methods the iterating process is realized according to the following recurrence relationship

$$\boldsymbol{x}_{k+1} = \boldsymbol{x}_k + \lambda_k \boldsymbol{d}_k \tag{4.5}$$

where

- $\boldsymbol{x}_k$ – vector of decision variables determined in the kth iteration
- $\boldsymbol{d}_k$ – search direction vector
- λ_k – step factor which minimizes the criterion $J(\boldsymbol{x})$ in the search direction $\boldsymbol{d}_k$

$$\boldsymbol{d}_k = \mathbf{v}_k \nabla J(\boldsymbol{x}_k) \tag{4.6}$$

where

- $\nabla J(\boldsymbol{x}_k)$ – gradient of the criterion $J(\boldsymbol{x})$ at the point $\boldsymbol{x}_k$
- $\mathbf{v}_k$ – search direction modification matrix, the form of which is dependent upon the type of the variable metric method used.

The iterative process is conducted until the convergence is obtained, i.e. to a point at which the difference between two succesive solutions, and the difference between two succesive gradients is less then the assumed value. The identification criterion gradient is determined from the following relationship

$$\nabla J(\boldsymbol{x}_k) = -\mathsf{H}^\top(\boldsymbol{x})\mathsf{R}^{-1}[z - \boldsymbol{q}(\boldsymbol{x})] \tag{4.7}$$

where $\mathsf{H}(\boldsymbol{x})$ is the Jacobian matrix of the vector function $\boldsymbol{q}(\boldsymbol{x})$. The values of the decision variables $\boldsymbol{x}$, for which the identification criterion (4.4) reaches the minimum, are the sought estimates $\widehat{\boldsymbol{x}}$ of the identifiable parameters.

The basic optimization method used in the IDENT program algorithm is the Wolf-Broyden-Davidon (W-B-D) method of variable metric. Since, in some cases, this method is not sufficiently effective, which was proved by authors (Gutowski and Berczyński, 1995), the Gauss-Newton (G-N) linearization method is optionally used. In the latter method minimization of the identification criterion (4.4) is achieved in another way. The components of the vector $\boldsymbol{q}$ of the model frequency characteristics are nonlinear functions of the identifiable parameters $\boldsymbol{x}$

$$\boldsymbol{q}_k = \boldsymbol{q}_k(\boldsymbol{x}) \tag{4.8}$$

Eq (4.8) can therefore be expanded into the Taylor series with respect to $k-1$ iteration. Taking only the first two terms of this expansion it is obtained

$$\boldsymbol{q}(\boldsymbol{x}_k) = \boldsymbol{q}(\boldsymbol{x}_{k-1}) + \mathsf{H}_{k-1}(\boldsymbol{x}_k - \boldsymbol{x}_{k-1}) \tag{4.9}$$

The vector $\boldsymbol{x}_k$, which satisfies the minimum of the criterion (4.4) is here sought by setting the derivative $J_k(\boldsymbol{x})$, with respect to $\boldsymbol{x}_k$, equal to zero, that is

$$\frac{dJ_k(\boldsymbol{x})}{d\boldsymbol{x}_k} = \mathbf{0} \tag{4.10}$$

hence

$$\mathsf{H}_{k-1}^\top \mathsf{R}^{-1} \boldsymbol{q}_{k-1} + \mathsf{H}_{k-1}^\top \mathsf{R}^{-1} \mathsf{H}_{k-1}(\boldsymbol{x}_k - \boldsymbol{x}_{k-1}) - \mathsf{H}_{k-1}^\top \mathsf{R}^{-1} \boldsymbol{z} = \mathbf{0} \tag{4.11}$$

From this equation, after some transformations, one can obtain

$$\boldsymbol{x}_k = \boldsymbol{x}_{k-1} + (\mathsf{H}_{k-1}^\top \mathsf{R}^{-1} \mathsf{H}_{k-1})^{-1} \mathsf{H}_{k-1}^\top \mathsf{R}^{-1} [\boldsymbol{z} - \boldsymbol{q}(\boldsymbol{x}_{k-1})] \tag{4.12}$$

The iterative process is conducted until a sufficiently small difference between two succesive solutions is obtained. Conducting this process in accordance with the relationship (4.12) may, however, lead to numerical instability. This is why Eq (4.4) is subject to minimization in each iterative step in the search direction given by the following relationship

$$\boldsymbol{d}_k^* = (\mathsf{H}_{k-1}^\top \mathsf{R}^{-1} \mathsf{H}_{k-1})^{-1} \mathsf{H}_{k-1}^\top \mathsf{R}^{-1} [\boldsymbol{z} - \boldsymbol{q}(\boldsymbol{x}_{k-1})] \tag{4.13}$$

The quality of identification computations performed is evaluated based on the differences $\boldsymbol{z} - \boldsymbol{q}(\widehat{\boldsymbol{x}})$ between the object characteristics $\boldsymbol{z}$ and the model characteristics $\boldsymbol{q}(\widehat{\boldsymbol{x}})$ determined for the estimated values $\widehat{\boldsymbol{x}}$ of the parameters identified. However, in the case of nonlinear estimation the solution may be ambiguous, i.e., the same consistence between the characteristics may be obtained for different sets of values of the parameters estimated. The compatibility of the characteristics, therefore, need not mean that the calculated estimates $\widehat{\boldsymbol{x}}$ are equal to or close to the actual values $\boldsymbol{x}_r$ of the object parameters. It is, therefore, essential that the errors of the determined estimates be evaluated. Indices which characterize the error of any estimate are the error of estimator bias and the variance of error. However, in case of nonlinear Fisher's estimators the basic problem consists in that these estimators are usually biased and, as yet, there is no general method which would allow us to determine the error of estimator bias. And this error can be predominant, i.e. can be greater than the variance of error (Shweppe, 1978). In the IDENT program the lower constraints of the variance of estimate error are calculated upon assumption that the estimators are unbiased. Computations are made basing on the so called Cramer-Rao informational inequality

$$D^2(\widehat{x}_i) \geq \Big[-\frac{\partial \boldsymbol{q}^\top(\widehat{x}_i)}{\partial x_i} \mathsf{R}^{-1} \frac{\partial \boldsymbol{q}(\widehat{x}_i)}{\partial x_i}\Big]^{-1} \tag{4.14}$$

which, after generalization over a case of a vector $\boldsymbol{x}$, assumes the following form

$$\mathbf{CV}(\widehat{\boldsymbol{x}}) \geq \left[\mathbf{H}^{\top}(\widehat{\boldsymbol{x}})\mathbf{R}^{-1}\mathbf{H}(\widehat{\boldsymbol{x}})\right]^{-1} \tag{4.15}$$

where $D^2(\widehat{x}_i) = E\{(x_{ri} - \widehat{x}_i)^2\}$ is the variance of the error of ith estimate, and $\mathbf{CV}(\widehat{\boldsymbol{x}}) = E\{(\boldsymbol{x}_r - \widehat{\boldsymbol{x}})(\boldsymbol{x}_r - \widehat{\boldsymbol{x}})^{\top}\}$ is the covariance matrix of the errors of parameters identified.

Computation of the **CV** matrix is of great practical significance since the analysis of its element values allows one not only to gain an idea of the estimate error values but also to evaluate the solution uniqueness, which is the particularly essential when the identified model is to be used in the process of shaping dynamic properties of machine tool.

When the variances of same parameter estimates are great the cause usually does not consist in great observation errors but rather in that the amount of information from the experiments is too small for the values of these parameters to be explicitly determined. One should then either increase the amount of information from the experiments (increase the number of frequency characteristics, or broaden the frequency band) or reduce the number of degrees of freedom of the model so that the parameters, whose estimates are characterized by great errors, be considered in the model by a single equivalent parameter.

The effectiveness of the identification computations depends, in great measure, on a skilful selection of parameters which are to be identified. Theoretically, any elements of the stiffness matrix **K** or damping matrix **C** of Eq (3.1) can be subject to identification, i.e., can be selected as the decision variables. However, on account of dimensions of the problem solved as well as making allowance for limited capability of the computers used and, also, the necessity for achieving an unique solution, the number of the decision variables is to be restricted. One should, therefore, always properly select these variables. This selection is carried out based on the results of sensitivity analysis of the characteristics to changes in parameters values. This sensitivity is different for various characteristics and, within a given characteristic, is different at various points of this characteristic. A measure of sensitivity, which is used in the IDENT program algorithm, is the value of mean coefficient of band sensitivity MI, calculated according to the following relationship

$$MI(\alpha, i) = \sum_{k=1}^{n} \left|\frac{\partial q_i(\omega_k)}{\partial p_\alpha}\right| / \sum_{k=1}^{n} q_i(\omega_k) \tag{4.16}$$

This coefficient evaluates the effect of the parameter p_α $(\alpha = 1, ..., m)$ on the characteristic q_i $(i = 1, ..., \beta)$ within the frequency band from ω_1 to ω_n.

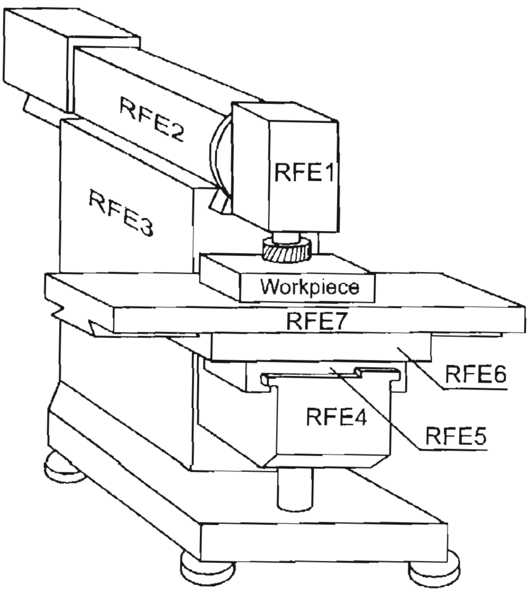
In principle, those parameters should be selected as the decision variables which have the strongest effect on the A-F characteristics taken as the identification basis. However, one should realize that such a procedure may, in extreme cases, cause even gross errors of the identification, since, at a great number of degrees of freedom, the cumulative effect of those model parameters which are not subject to identification (taken a priori as those correct on the basis of e.g. the literature data) may be considerably greater than that of the parameters identified. This will result in an erroneus evaluation of the values of the parameters estimated.

5. Example of practical use of the method

The presented method was used in practice for conducting an experimental identification of parameters of SDEs of the FWD-32J milling machine model. The original model of this machine tool was constructed by Witek (1990). It was however, considerably corrected and modified on the basis of detailed geometric measurements done on the real machine tool. In this model the milling machine is divided into seven rigid solids (RFEs) which are interconnected by four systems of slideway joints (overarm – column, column – knee, knee – lower saddle, upper saddle – table), and by two rotary joints (spindle head – overarm and lower saddle – upper saddle) as well as by three lead screws. The slideway designs have forced acceptance of minimum number of the SDEs which model them. In the presented model (Fig.4) assumed are 29 SDEs, that model the slideway joints, lead screws and rotary joints, and four SDEs that model the spring-dissipative properties of vibration isolators, on which the machine was being founded during the tests. It is to be clearly stressed, however, that various types of loading of the milling machine with the cutting forces as well as various reciprocal positions of particular sub-assemblies involve operation of various slideway joints (some of them may be passive under given load). In this connexion the number of SDEs that are active within the model will also vary, depending on the load conditions taken.

Each slideway joint, rotary joint and vibration isolator is characterized by the definite stiffnesses in normal k_n and tangential k_t directions, and by the coefficients of vibration energy dissipation also in normal ψ_n and tangential ψ_t directions. The lead screws are chracterized by two parameters only, i.e., the stiffness k_n and the coefficient of energy dissipation ψ. The knowledge of these parameter values is necessary to calculate the elements of the stiffness $\mathbf{K}$ and damping $\mathbf{C}$ matrices of the model motion equation (3.1).

(a)



(b)

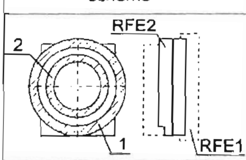
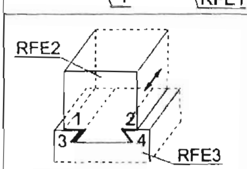
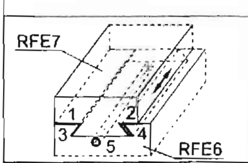
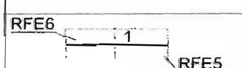
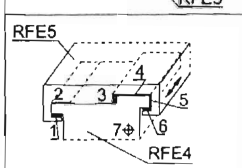
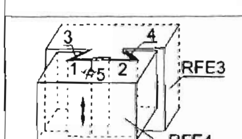
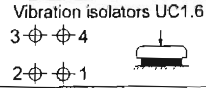
Slideway joint scheme	Way No.	SDE No.
	1 2	1
	1 2 3 4	2 3 4, 5 6, 7
	1 2 3 4 5	8 9 10 11 12
	1	13
	1 2 3 4 5 6 7	14,15,16 17 18 19 20 21 22
	1 2 3 4 5	23, 24 25, 26 27 28 29
Vibration isolators UC1.6 	1 2 3 4	30 31 32 33

Fig. 4. FWD-32J milling machine physical model: (a) division into RFEs, (b) designation of SDEs that model particular joints and vibration isolators

According to the above presented algorithm, just these parameters can be subject to identification. From a simple calculation it can be stated that the amplitude-frequency characteristics of the model formed will depend on as many as 126 parameters. Taking so great a number of parameters as the decision variables, which are to be subject to identification based on three or, at most, six (the IDENT program constraints) experimentally determined A-F characteristics of selected milling machine solids would be irrational. This number had to be restricted, and that radically. The first of the constraints taken was the assumption of constant relationship between the normal stiffness and the tangential one in the slideway joints. This was taken so because from the literature data (Back, Burdekin and Cowley, 1973) one can conclude that a constant value of $k_n/k_t \approx 2.5$ can be assumed for these joints. In this way the number of variables has been limited to 74. This, however, was too great number. Then, the next constraint was introduced, consisting in assuming identical values of the normal stiffness k_n, identical values of the tangential stiffness k_t and identical values of the vibration energy dissipation coefficients in normal ψ_n and in tangential ψ_t directions for all those SDEs which modelled the vibration isolators. In this way the number of possible decision variables become more reduced, down to 62. Further selection was to be carried out based on the results from the analysis of the sensitivity of the A-F characteristics to changes in the parameters.

The basis for conducting the identification was taken in the form of the milling machine table A-F characteristics, experimentally determined for one of four machine tool structures, that could be distinguished as those representative when milling with the face milling cutter (Witek, 1992). The selected structure corresponds to maximum foreward shift of the overarm, and to the cutting force orientation angles $\alpha = 60°$ and $\beta = 81°$ (Fig.5). The mean value of the exciting force P under test conditions was $P = 4$ kN. The experimentally determined characteristics were transformed into the system of principal central axes of inertia of the table, and 38 points were selected from each of three translational characteristics in this system, on the basis of which the identification was carried out. The table characteristics are, as it has been proved by the sensitivity analysis, very sensitive to changes in the slideway joint stiffness and are hardly sensitive to the changes in damping of individual joints. On the contrary, the cumulative effect of the entire structure damping – of all the joints on the A-F chracteristics is considerable. It was not, therefore, possible to identify the slideway joint stiffness without a simultaneous correction of damping, i.e. with a constant values of damping being assumed on the literature data, the more so because these values are strongly dependent upon the slideway load and the lubricant used. According to data given by Levina and Resetov (1971), within the load range (0.1 up to 1.2) MPa, these values may vary within a very wide range ($\psi = 2$ down to 0.4, for a strictly defined lubricating oil). It was therefore impossible to carry out correct identification of the model without conducting the damping identification. This parameter has to be identified. The coefficients ψ_n and ψ_t of the slideway joints were therefore grouped and were subject to identification as the so called group decision variables. Assumed was a single, the same for all slideway joints, value of the coefficient of vibration energy dissipation in the normal direction $\psi_n = 0.8$ and a single, common to all slideway joints, value of the coefficient of vibration energy dissipation in the tangential direction $\psi_t = 1.6$. These values roughly corresponded to the data taken by Witek (1990) for this milling machine.

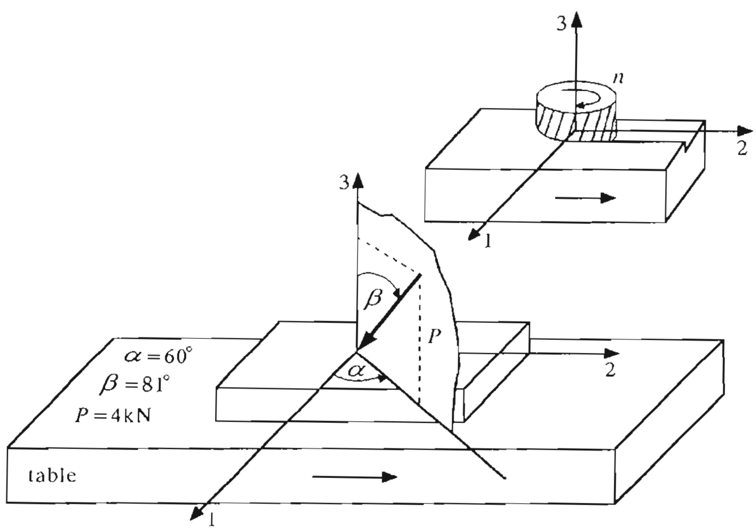


Fig. 5. Taken in dynamic tests on milling machine variant of reciprocal motion of cutting tool and workpiece

The milling machine physical model was supplemented with an additional SDE (the global number 34) that modelled the spring-damping properties of a hydraulic actuator, by means of which the force excitation was effected. The axis of this SDE coincided with the actuator axis while the point of its attachment lay in the place where the actuator plunger directly acted on the special ball fixed, during tests, to the machine spindle head.

Such a way to carry out the identification has resulted in a significant improvement in consistence of the model characteristics with those experimental and, at the same time, the lower constraints of the estimate errors were small.

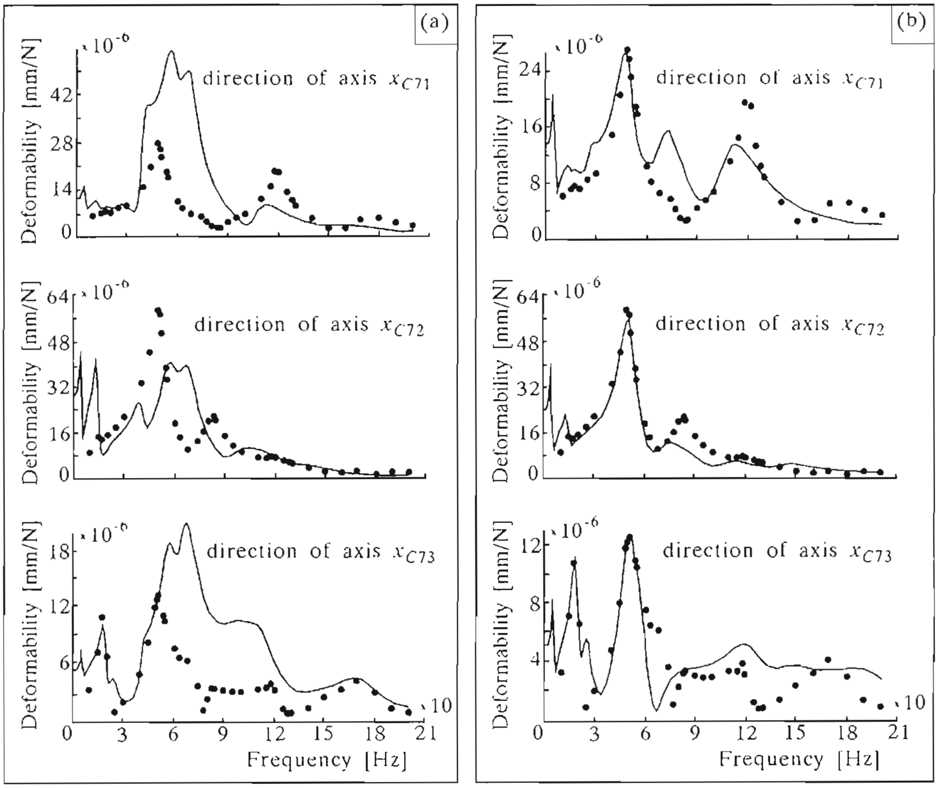


Fig. 6. A-F characteristics of the table of FWD-32J milling machine model in the system of principal central axes of inertia: (a) for initial values of decision variables, (b) for identified values, against the points chosen from A-F characteristics of the real machine, determined from dynamic tests

The results of the computations are presented tabularly in Table 1, and illustrated in Fig.6. This figure shows the model A-F characteristics in three directions, in the system of principal central axes of inertia of the machine tool table against a background of the points chosen from experimental characteristics as a basis for identification. The model characteristics, presented in this figure, were determined for the initial values of the parameters chosen as decision variables (Fig.6a), and for the values estimated after one computational cycle (Fig.6b). A radical improvement can be seen in the consistence of the tested machine tool and its model characteristics for the estimated values of the parameters.

Table 1. Results of identification selected parameters of SDEs of FWD-32J type milling machine supporting system model based on A-F characteristics of machine tool table

Global SDE No.	Working SDE No.	Parameter value, k_n [MPa/μm] initial	Parameter value, k_n [MPa/μm] identified	Standard error
2	2	0.05	0.0199	0.00061
3	3	0.05	0.0073	0.00022
6	6	0.1	0	0.00161
7	7	0.1	0.0128	0.00256
8	8	0.05	0.0795	0.07596
11	10	0.025	0.3242	0.03206
17	14	0.05	0.0294	0.00141
20	16	0.05	0.1401	0.02526
21	17	0.025	0.0607	0.00889
23	19	0.036	0.0397	0.03027
24	20	0.036	0.0133	0.00682
27	21	0.071	0.0976	0.06725
28	22	0.048	0.1093	0.00740
Global SDE No.	**Working SDE No.**	**Parameter value, ψ [-] initial**	**Parameter value, ψ [-] identified**	**Standard error**
Group	*)	0.8	1.093	0.05305
Group	**)	1.6	1.608	0.091
12,22,29,34	11,18,23,28	0.6	0.840	0.1906
Global SDE No.	**Working SDE No.**	**Parameter value, k_n [N/μm] initial**	**Parameter value, k_n [N/μm] identified**	**Standard error**
22	18	10	9.706	6.217
29	23	16.7	8.942	5.949
34	28	0.5	0.5639	0.02689
30,31,32,33	24,25,26,27	0.459	0.4941	0.00754

Global parameter for SDEs No.: $2 \div 10$, $13 \div 17$, $19 \div 22$
*) – in normal direction; **) – in tangential direction

Analyzing the arranged in Table 1 values of estimated parameters of normal stiffness of SDEs, which model the slideway joints in the real tested machine tool, it is seen, that these values are contained in the wide range $(0.007 \div 0.1401\,\mathrm{MPa}/\mu\mathrm{m}$. The identified zero value of SDE No. 6 stiffness indicates that most probably under given loading conditions of the whole structure this SDE is either subject a very low load or it does not work at all. Here, it should be, however, strongly stressed, that estimated values of these parameters relate to the dynamic load of joints, so they represent so called dynamic stiffness which strongly depends on loading conditions (on the constant component of load as well as on load amplitude – the altered component). One can not mistake it for static contact stiffness of joints else, determined under the static load, given e.g. by Levina and Resetov (1971).

The estimated values of the coefficients of the vibration energy dissipation of those SDEs which model the slideway joints are: $\psi_n = 1.093$ – in normal direction, and $\psi_t = 1.608$ – in tangential direction. These are relatively great values, however, as it was mentioned above, at low surface pressures occuring in the slideway joint contacts (in an order of 0 to 1.0 MPa), the coefficients of vibration energy dissipation of just this order were to be expected. Very small variability factors and law values of lower constraints of the estimate errors of these parameters allow the authors to belive that the estimates have been properly determined. It is to be stressed here, that these are mean values for all the slideway joints of the entire structure. The values of the vibration energy dissipation coefficients of each individual joint may, therefore, differ from them. This is demonstrated by computations made after taking the coefficients of vibration energy dissipation of particular joints as the individual decision variables. A better consistence of the model characteristics with those experimental is then achieved. However, the errors of so determined estimates of the parameters (taken as decision variables) are too great for these estimates to be accepted as those determined correctly.

It is also to be clearly stressed that the obtained estimate values are neither the stiffness coefficients nor coefficients of vibration energy dissipation of the real machine tool slideway joints or lead screws, but are the values of parameters of those SDEs which model these joints in the machine tool model constructed according to the RFE method. The stiffness coefficient estimates contain not only the stiffness of the joint but also the stiffness of the interconnected elements which are assumed to be undeformable in the model. Similarly, with regard to the coefficients of vibration energy dissipation, the estimate values obtained contain not only damping of the joints but also the material damping of the connected elements as well as the damping of other joints, neglected in the model. The obtained estimates should, therefore, be considered as so called equivalent parameters.

6. Conclusions

The results presented in this paper make only a partial illustration of the IDENT package capabilities, since the final results of the identification depend not only on the effectiveness of this package but also, in a great measure, on the correctness of realizing experiments, the results of which are next used as the basis for the identification, and on the accuracy of previously done calculations associated with the machine tool physical model development (the modelling accuracy). Making errors at any of these steps either makes the identification realization generally impossible or leads to great errors in the estimates obtained.

So far, the authors have carried out mainly the simulation investigations – the model ones (Berczyński and Gutowski, 1993, 1994, 1995), with the aim to develop the package capabilities so as to make it useful for identifying the models of mass-spring-damping (MSD) systems with many degrees of freedom, that is ones which are formed by e.g. the supporting structures of metal-cutting machine tools. The results presented in this work confirm the fact that the main goal – practical usefulness of these package – has been achieved.

The IDENT program makes a versatile tool that enables the researchers to effectively identify, in principle, any number of parameters of those SDEs which model spring-damping properties of machine tool. Its todays constraints result mainly from limited capabilities of computers, on which this program is used. By means of this program it was possible to go beyond the range of two, three or four-mass models with several degrees of freedom, which have been used so far in the attempts to model complex structures.

Identified in the presented solution are parameters of those SDEs which model strictly defined joints in a machine tool. Having an adequate (both in terms of structure and in terms of parameters evaluation correctness) model it is possible to simulate and investigate the machine tool behaviour under various, even extreme loads without the need to carry out expensive labour- and time-consuming tests. One can trace thoroughly the effect of changes in each parameter on the behaviour of the machine tool during operation and, consequently, one can recommend the designers definite modifications, which allow them to upgrade the parameters of the machine tool operation.

One should also realize that certain parameters, such as stiffness and damping of a given joint are strongly dependent upon the load amplitude, even if the mean values of the loads remain unaltered. This variability should, therefore, be taken into account in conducting the simulation investigations.

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Praktyczne możliwości wykorzystania pakietu IDENT do identyfikacji parametrów modeli dynamicznych układów nośnych obrabiarek

Streszczenie

W pracy przedstawion krótki opis metody identyfikacji parametrów modeli dynamicznych układów nośnych obrabiarek budowanych w konwencji metody sztywnych elementów skończonych. Identyfikację prowadzi się na podstawie wybranych punktów z wyznaczonych doświadczalnie charakterystyk amplitudowo-częstotliwościowych badanej obrabiarki Możliwe jest prowadzenie identyfikacji parametrów elementów sprężysto-tlumiących modelujących połączenia prowadnicowe, śruby pociągowe, obrotnice i bloki toczne, a także parametrów elastycznego posadowienia obrabiarki na wibroizolatorach. Podano podstawowe zależności matematyczne algorytmu identyfikacji oraz podano i omówiono przykład praktycznego wykorzystania metody do identyfikacji wybranych parametrów modelu ukladu nośnego frezarki FWD-32J.

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