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GROWTH OF PLASTIC ZONES DUE TO HEAT LOADING NEAR A CLOSED INTERFACE CRACK

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A bimaterial, consisting of two conjugated half-planes possessing different thermophysical characteristics, with an interface crack closed due to external pressure is considered. The growth of interface plastic zones near the crack tips, caused by a heat flow and thermal resistance of crack faces, is investigated. The complex potentials of the problem that define main characteristics of the bimaterial thermal stress state are expressed in terms of interface jump discontinuities of temperature and tangential displacements, which simulate the crack and plastic zones. A system of singular integro-differential equations in these functions is obtained. The length of plastic zones is derived from the condition for interface shear stress boundedness in the tips of these zones. The closed form expressions for the displacement jumps of the bimaterial components and the plastic zones length are obtained for a certain thermal resistance.

Key words: bimaterial, closed interface crack, heat flow, thermal resistance, plastic zones

Notation

D_1, D_2	–	lower and upper components (half-planes) of the bimaterial
L	–	interface
λ_1, λ_2	–	thermal conductivities, [W/(mK)]
λ	–	averaged thermal conductivity, $\lambda = 2\lambda_1\lambda_2/(\lambda_1 + \lambda_2)$ [W/(mK)]
α_1, α_2	–	coefficients of linear thermal expansion, $[\mathrm{K}^{-1}]$

μ	–	shear modulus for both half-planes, [Pa]
ν	–	Poisson ratio for both half-planes
κ	–	mechanical material constant, $\kappa = 3 - 4\nu$
η	–	thermophysical material constant, $\eta = (\alpha_1/\lambda_1)+(\alpha_2/\lambda_2)$ [m/W]
T	–	temperature, [K]
q_x, q_y	–	components of the heat flow vector, [W/m^2]
$\sigma_x, \sigma_y, \tau_{xy}$	–	components of the stress tensor. [N/m^2]
u, v	–	components of the displacement vector, [m]
p	–	external pressure, [Pa]
q	–	heat flow at infinity, [W/m^2]
Q	–	dimensionless heat flow, $Q = 2a\eta q$
τ_s	–	shear yield value, [N/m^2]
S	–	dimensionless interface shear yield value, $S = \tau_s/\mu$
$\tau_0(x)$	–	step-wise constant function
r_0	–	parameter in the thermal resistance function, [m^{2}K/W]
$r(x)$	–	thermal resistance function, [m^{2}K/W]
R	–	dimensionless thermal resistance, $R = \lambda r_0/(2a)$
a	–	crack half-length, [m]
b	–	distance from the crack center to the end of plastic zone, [m]
d	–	dimensionless plastic zone length, $d = (b - a)/a$
$\gamma(x)$	–	interface jump of crack faces temperature, [K]
$g(x)$	–	interface jump of tangential displacement, [m]
$G(x)$	–	dimensionless jump of tangential displacement, $G(x) = g(x)/a$
$F(z), \Phi_k(z)$	–	complex potentials
$(\cdot)_k$	–	refers a quantity to the kth half-plane, $k = 1, 2$
$(\cdot)^+, (\cdot)^-$	–	denote the boundary values of functions as they approach L from the upper and lower half-planes, respectively
$(\cdot)'$	–	denotes differentiation with respect to x.

1. Introduction

The difficulties, arising when constructing a physically valid solution within a traditional formulation of problems in mechanics and thermomechanics of composites with interface cracks, stimulated a number of works dealing with new approaches to solving such problems. Among them, the papers in which the interface phenomena and processes were taken into account by means of boundary-contact conditions, have received a marked attention. In particular, mechanical contact of crack faces was investigated by Comninou (1977), Atkinson (1982), Simonov (1990), Loboda (1993), Antipov (1995); interface plastic deformations, yield and process zones near crack tips were studied by Yang and Kim (1993), Fujii et al. (1994), Kaminsky et al. (1995), Sheveleva (1997) (see also the references cited there). Barber and Comninou (1983) considered the penny-shaped interface crack making an allowance for zones of perfect and imperfect thermal contact of its faces. Kit and Martynyak (1996) showed the possibility of thermal opening of the initially closed interface crack possessing thermal resistance. Kaczyński and Matysiak (1989) constructed non-oscilating solutions of the thermoelastic problem for the laminate composite with an interface crack using a homogenized model of microperiodic medium.

Formation of thin plastic zones near crack tips in homogeneous solids was studied by Olesiak (1968), Kassir (1969), Pawlowski and Tsai (1997) for bodies subjected to heat fluxes factors and by Olesiak (1988) for bodies subjected to a thermodiffusion process. Kit and Krivtsun (1983), Matczyński and Sokołowski (1989), Gross (1993), Balueva and Matczyński (1997) investigated the contact of crack faces in homogeneous bodies due to a heat load.

A series of problems to be dealt with when considering contact phenomena of bodies interaction making allowance for a non-ideal contact caused by thermal resistance was examined by Shvets and Martynyak (1988), Kryshtafovych and Martynyak (1997), Shvets et al. (1997).

In this paper we are going to study thermal initiation of plastic zones near an interface crack with contacting faces in bimaterial consisting of thermally dissimilar half-planes. The crack is closed due to external pressure, heat flows in the direction perpendicular to the interface and crack faces are in a non-ideal thermal contact characterized by a thermal resistance. Considering the stress and strain state of such a structure within the framework of linear thermoelasticity, Kit and Martynyak (1996) recently showed that the crack thermal resistance gave rise to a considerable concentration of the interface shear stress, which is of root singularity in the vicinity of the crack tips. Thus.

it can yield plastic strain if bimaterial components are elastic-plastic. Here, we consider the structure of elastic-perfectly plastic material and study the interface plastic shear in it. The problem of thermoelastoplasticity is reduced to the corresponding thermoelastic problem in which the plasticity is taken into account by certain interface conditions.

Such a mathematical model is based on the following assumptions:

- The plastic zones form on the interface at the crack direction
- Shear stress in the plastic regions is equal to the interface shear yield value τ_s
- Tangential displacements of bimaterial components have a break in the plastic regions, that is caused by the interface plastic slip of components
- Interface shear stress is limited in the ends of plastic zones.

It is worth pointing out that this approach is also applicable if plastically the deformed material occupies thin regions stretched along the interface. Then τ_s needs to be taken as the least of the yield points of bimaterial components. If a system consists of elastic components joined by a thin elastic-perfectly plastic interlayer, then τ_s is to characterize its yield point. Such a model of plastic behaviour of a glue layer in the bimaterial with an interface crack under a pure force load was applied by Yang and Kim (1993).

2. Formulation of the problem

A bimaterial, i.e. a piecewise-homogeneous body, consisting of two elastic isotropic half-planes D_1 and D_2 is considered. Materials of the components have different thermal characteristics $(\lambda_1 \neq \lambda_2, \alpha_1 \neq \alpha_2)$ but identical the mechanical ones $(\mu_1 = \mu_2 = \mu, \nu_1 = \nu_2 = \nu)$. The $2a$-length crack is located on the dividing line L (Fig.1).

It is assumed that the external load ensures the realization of steady-state of heat conduction and 2D strain state of the body. At infinity the system is subjected to the external uniform pressure $\sigma_y^\infty = -p$ and the uniform heat flow $q_y^\infty = q$ that are perpendicular to the line dividing the components. Besides, at infinity $(x = \pm\infty)$ the bimaterial is attached so that under the prescribed heat and force loading conditions there arises the following stress

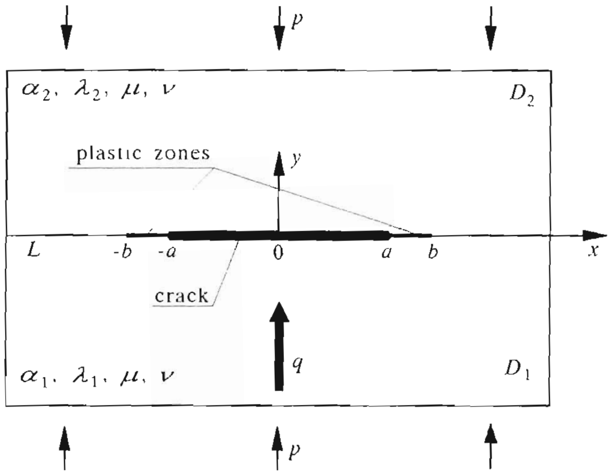

Fig. 1. Mechanical system considered

linearly distributed along the coordinate y

$$\sigma_x^{\infty} = \frac{1}{1-\nu}\Big[\frac{2\alpha_k(1+\nu)\mu q}{\lambda_k} y - \nu p\Big] \qquad k = 1, 2$$

Such a stress will arise in the bimaterial components if they are joined at a considerable distance from the crack $(x = \pm\infty)$ with thermally insulated, rigid flat walls that are parallel to the axis y. Contact of the bimaterial with the walls is such that it allows of free slip $(\tau_{xy}^{\infty} = 0)$ to appear but inhibits displacements along the x-axis $(u^{\infty} = 0)$. The stress σ_x^{∞} compensates global warping and elongation of the half-plane boundaries caused by the applied heat and force loading (see Fig.1).

The crack faces are in contact due to the pressure and their mechanical contact is assumed to be frictionless. The thermal contact of faces is non-ideal and it is characterized by the thermal resistance, given by an even function $r(x)$.

Let us consider formation of symmetric interface plastic zones along the segments $-b < x < -a$ and $a < x < b$ provided the shear stress reaches the shear yield Value τ_s there. The length of these zones is unknown. Let us determine thermal stresses in bimaterial with such interface flaw and establish the dependence of plastic zones length and tangential displacements interface jump on the heat loading.

The boundary conditions of the problem are the following:

— thermal

$$T^{-} - T^{+} = r(x) q_y^{+} \qquad |x| < a \tag{2.1}$$

$$q_y^- = q_y^+ \qquad\qquad\qquad\qquad |x| < a \tag{2.2}$$

$$T^- = T^+ \qquad q_y^- = q_y^+ \qquad |x| \geq a \tag{2.3}$$

$$q_y^\infty = q \tag{2.4}$$

— mechanical

$$\sigma_y^- = \sigma_y^+ \qquad \tau_{xy}^- = \tau_{xy}^+ \qquad v^- = v^+ \qquad x \in L \tag{2.5}$$

$$u^- = u^+ \qquad |x| \geq b \tag{2.6}$$

$$\tau_{xy}^- = 0 \qquad \tau_{xy}^+ = 0 \qquad |x| < a \tag{2.7}$$

$$\tau_{xy}^- = \tau_{xy}^+ = -\tau_s \mathrm{sign}(x) \qquad a \leq |x| < b \tag{2.8}$$

$$\sigma_y^\infty = -p \qquad \tau_{xy}^\infty = 0 \qquad \sigma_x^\infty = \frac{1}{1-\nu}\Big[\frac{2\alpha_k(1+\nu)\mu q}{\lambda_k}y - \nu p\Big] \tag{2.9}$$

3. Solution to the problem

From Eqs (2.1)-(2.3), (2.5)-(2.8) it is clear that the component q_y of the heat flow vector, the displacement vector component v, and the stress tensor components σ_y and τ_{xy} are continuous while passing the boundary line L. The temperature is discontinuous on the crack interval, but the displacement vector component u reveals a jump on the segments of both a crack and plastic zones. Therefore we consider the interface jumps of temperature

$$T^- - T^+ = \gamma(x) \qquad x \in L \tag{3.1}$$

and tangential displacements

$$u^- - u^+ = g(x) \qquad x \in L \tag{3.2}$$

so that

$$\begin{aligned} \gamma(x) &= 0 \qquad \text{if} \quad |x| \geq a \\ g(x) &= 0 \qquad \text{if} \quad |x| \geq b \end{aligned}$$

On the basis of the results of Kit and Martynyak (1996) we write the temperature, stresses and displacements

$$T(x,y) = \mathrm{Re}\Big[F(z) + \mathrm{i}\frac{qz}{\lambda_k}\Big]$$

$$\sigma_x(x,y) + \sigma_y(x,y) = 4\mathrm{Re}\Phi_k(z) + \frac{1}{1-\nu}\Big[\frac{2\alpha_k(1+\nu)\mu}{\lambda_k}y - p\Big]$$

$$\sigma_y(x,y) - \mathrm{i}\tau_{xy}(x,y) = \Phi_k(z) - \Phi_k(\bar{z}) + (z-\bar{z})\Phi_k'(z) - p \tag{3.3}$$

$$2\mu[u'(x,y) + \mathrm{i}v'(x,y)] = \kappa\Phi_k(z) + \Phi_k(\bar{z}) - (z-\bar{z})\overline{\Phi_k'(z)} + 2\mu(1+\nu)\alpha_k F(z)$$

through the piecewise-analytical functions $F(z)$, $\Phi_k(z)$

$$F(z) = -\frac{\lambda}{\lambda_k}\frac{1}{2\pi\mathrm{i}}\int_{-a}^{a}\frac{\gamma(t)}{t-z}\,dt$$

$$\Phi_1(z) = (-1)^k\frac{\mu}{1+\kappa}\Big[2(1+\nu)\alpha_k F(z) + \frac{1}{\pi\mathrm{i}}\int_{-b}^{b}\frac{g'(t)}{t-z}\,dt\Big] \tag{3.4}$$

$$\Phi_2(z) = -\Phi_1(z) \qquad z \in D_k \qquad k = 1,2$$

that are defined by the jumps (3.1), (3.2).

Eqs (3.3) satisfy the conditions (2.2)-(2.6), (2.9). Having satisfied the conditions (2.1), (2.7) and (2.8) by using of Eqs (3.3) and (3.4) we obtain the system of singular integro-differential equations in the functions $\gamma(x)$ and $g(x)$

$$\frac{\gamma(x)}{r(x)} - \frac{\lambda}{2\pi}\int_{-a}^{a}\frac{\gamma'(t)\,dt}{t-x} = q \qquad x \in [-a,a] \tag{3.5}$$

$$\int_{-b}^{b}\frac{g'(t)\,dt}{t-x} - \frac{\lambda(1+\nu)\eta}{2}\int_{-a}^{a}\frac{\gamma(t)\,dt}{t-x} = \tau_0(x) \qquad x \in [-b,b] \tag{3.6}$$

where

$$\tau_0(x) = \begin{cases} 0 & \text{if } \ |x| < a \\ -\tau_s\mathrm{sign}(x) & \text{if } \ a \le |x| < b \end{cases} \tag{3.7}$$

The sought functions should satisfy the conditions

$$\int_{-a}^{a}\gamma'(x)\,dx = 0 \tag{3.8}$$

$$\int_{-b}^{b}g'(x)\,dx = 0 \tag{3.9}$$

$$g'(\pm b) = 0 \tag{3.10}$$

Eqs (3.8) and (3.9) are result from the natural conditions $\gamma(\pm a) = 0$, $g(\pm b) = 0$. Eq (3.10) follows from the requirement that the stresses, defined in the terms of function $g'(x)$ by Eqs (3.3) and (3.4), should be bounded at the ends $x = \pm b$ of the plastic zones.

The temperature jump $\gamma(x)$ is derived from the Prandtl-type integro-differential equation (3.5) and Eq (3.8). Then we define the derivative of the displacement jump $g'(x)$ from Eq (3.6) according to the known formula (see Muskhelishvili, 1953) that expresses the solution to a singular integral equation with the Cauchy kernel. Therefore we seek according to the condition (3.10) the bounded solution to Eq (3.6)

$$g'(x) = \frac{\lambda\eta}{2}\gamma(x) - \frac{\tau_s}{4\pi(1+\kappa)\mu}[H(b,x,a) + H(b,x,-a)] \tag{3.11}$$

where

$$H(b,x,\xi) = \ln\frac{b^2 - \xi x + \sqrt{(b^2 - x^2)(b^2 - \xi^2)}}{b^2 - \xi x - \sqrt{(b^2 - x^2)(b^2 - \xi^2)}}$$

Having substituted for the function $g'(x)$ into Eq (3.9), we define the limits of the plastic zone extension

$$b = \sqrt{a^2 + \Big[\frac{\lambda(1+\kappa)\mu}{2\tau_s}\int\limits_{-a}^{a}\gamma(x)\,dx\Big]^2} \tag{3.12}$$

Hence, Eqs (3.11), (3.12) together with Eqs (3.3), (3.4) give the solution to the problem on the thermal stress state of the bimaterial with an interface closed crack and comming off the interface plastic zones, the appearance of which is caused by a contact thermal resistance of the crack faces. From Eqs (3.5), (3.11), (3.12) we can see that the jumps of temperature and displacements between the crack faces are absent $(\gamma(x) = 0,\ g(x) = 0)$ and the plastic zones do not appear $(b = a)$ if the thermal contact of faces are ideal $(r(x) = 0)$.

4. Example

Let us analyze the formation of plastic zones, when the crack thermal resistance is in the form

$$r(x) = r_0\sqrt{1 - \Big(\frac{x}{a}\Big)^2} \qquad |x| < a \tag{4.1}$$

In this case Eq (3.5) has the following closed-form solution

$$\gamma(x) = \frac{2q\sqrt{a^2 - x^2}}{\lambda + \frac{2a}{r_0}} \qquad |x| < a \tag{4.2}$$

According to these expressions and Eqs (3.11), (3.12) one can write

$$
\begin{aligned}
g(x) &= \frac{-\tau_s}{4\pi(1+\kappa)\mu}\Big[4\sqrt{b^2-a^2}\Big(\pi - \arccos\frac{x}{b}\Big) + \\
&+ (x-a)H(b,x,a) + (x+a)H(b,x,-a)\Big] + \\
&+ \frac{a\eta q}{1+\frac{2a}{r_0\lambda}}\Big\{[s_-(-a) - s_-(a)]\Big[\frac{x}{a}\sqrt{1-\Big(\frac{x}{a}\Big)^2} + \arcsin\frac{x}{a} + \frac{\pi}{2}\Big] + \\
&+ \pi s_-(a)\Big\} \qquad |x| \le b
\end{aligned} \tag{4.3}
$$

$$
b = a\sqrt{1 + \Bigg[\frac{\pi a(1+\kappa)\mu\eta q}{2\tau_s\Big(1+\frac{2a}{r_0\lambda}\Big)}\Bigg]^2} \tag{4.4}
$$

where

$$
s_-(x) = \begin{cases} 1 & \text{if} \quad x > a \\ 0 & \text{if} \quad x \le 0 \end{cases} \tag{4.5}
$$

The results of calculations made for the dimensionless plastic zone length d, thermal resistance R, tangential displacement jump G, interface shear yield point S and abscissa X, are shown in the figures below.

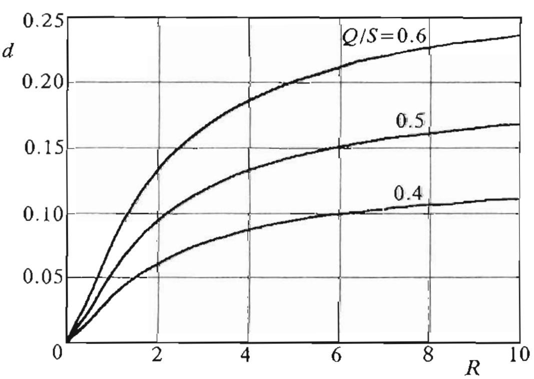

Fig. 2. Effect of the thermal resistance on the plastic zone length

Fig.2 illustrates the dependence of plastic zone length on the crack thermal resistance. As one can see all curves start from the origin of coordinate system. This means that under the prescribed heat loading the interface plastic zones in bimaterial do not arise if the crack faces are in the ideal thermal contact ($r = 0$). The length of these zones enlarges and tends to the maximum value, corresponding to the case of thermally insulated crack ($r = \infty$), if the thermal resistance increases. Besides, it may be easily noticed that the plastic zones length also enlarges with heat flow increasing provided the crack thermal resistance is constant.

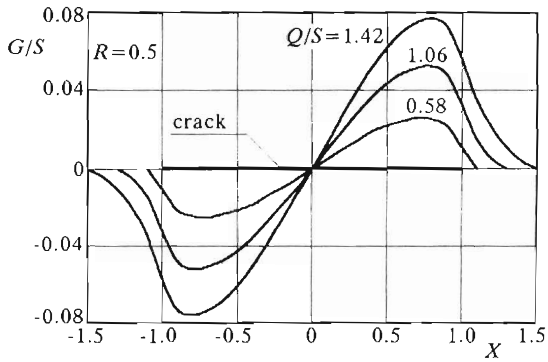

Fig. 3. Dependence of the interface jump of tangential displacements on the heat flow

Fig.3 shows the relative interface shear of bimaterial components for the crack with thermal resistance $R = 0.5$. The jump of tangential displacement is an odd function owing to the problem symmetry. The absolute value of this jump increases with growth of the heat flow intensity. It attains maximum near the crack tips within the intervals $0.7 < |X| < 0.8$ and it is equal to zero at the ends of the plastic zones.

5. Conclusions

The analytical treatment of thermal stresses and displacements in a bimaterial with closed interface imperfections and with an interphase boundary admitting plastic shear of the components makes it possible to draw the following conclusions.

A contact thermal resistance of the crack faces causes formation of the interface plastic zones in a bimaterial subjected to the heat flow perpendicular to the interface. This phenomenon does not occur if the thermal contact of the crack faces is ideal. The length of plastic zones depends on mechanical and thermophysical properties of the bimaterial components in the following way: elongates as the coefficients of linear thermal expansion and the shear moduli increase and the interface shear yield value goes down. Besides, the greater is the heat flow at a constant thermal resistance or the greater is the thermal resistance at a fixed heat flow – the greater is the plastic zones length. An absolute value of the interface jump of tangential displacement enlarges on both the crack interval and the plastic zones if the heat flow increases. Its maximum is not on the plastic segments but on the region of crack.

Referring to the investigation of Kit and Martynyak (1996) the following question may arise: how does the growth of plastic zones effect on the opening of initially closed interface crack possessing a thermal resistance. An answer to this question can be obtained by applying the model proposed here to the bimaterial consisting of thermally and mechanically dissimilar components and by analyzing the influence of plastic zones on the contact pressure of closed crack faces.

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Rozwój stref plastycznych wokół międzyfazowej zamkniętej szczeliny, spowodowany strumieniem cieplnym

Streszczenie

Praca poswięcona jest analizie utworzenia cienkich stref plastycznych wokół końców szczeliny umieszczonej na międzyfazowej granicy bimateriału złożonego z termicznie różnych półpłaszczyzn. Bada się rozwój odkształceń plastycznych spowodowany strumieniem cieplnym i oporem termicznym jej brzegów wokół końców zamkniętej zewnętrznym ciśnieniem szczeliny. Potencjały zespolone, które wyznaczają stan termosprężysty bimateriału, przedstawione są przez międzyfazowe nieciągłości temperatury i przemieszczeń stycznych, którymi modelowana jest szczelina i strefy plastyczne. Dla ich wyznaczenia otrzymany jest układ równań singularnych całkowo-różniczkowych. Długość stref plastycznych oblicza się z warunku ograniczaności naprężeń stycznych międzyfazowych na końcach tych stref. Dla konkretnego oporu termicznego szczegółowo przeanalizowano nieciągłość przemieszczeń brzegów szczeliny i długość stref plastycznych.

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