

THE PHENOMENON OF COLMATAGE AS A GENERALIZATION OF FILTRATION

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In this paper various models of the process of colmatage are presented. Three kinds of kinetics are discussed, describing the course of this phenomenon, and the relevant systems of partial differential equations. Examples of the solutions of these equations (2.2), (2.3); (2.2), (2.4); (2.2), (2.5) are given and the function of porosity of a medium during the colmatage process (2.7), (2.11), (2.15) is determined from these equations. Besides, by introducing the adequate equation of motion (2.17), the way of determining the pressure distribution in a medium when the flow proceeds at the assumed discharge (2.19) and at the constant difference of pressure, Eq (3.3), is presented. In the latter case the discharge of flow as a decreasing function of time Eq (3.2) is additionally determined.

Calculations are made for each kinetics. The diagram enclosed illustrates the distribution of pressure versus time t when the flow proceeds at a constant pressure difference. It should be noticed that the distribution obtained from the theory of colmatage at the moment $t = 0$ agrees with that obtained when Darcy's law is applied.

Key words: colmatage, filtration, porous media, suspension

1. Introduction

In the 1850s Darcy initiated a series of investigations into filtration meant as a way to purify water in sand filters for municipal and industrial purposes. However, he did not complete that task and abandon it. He focused on examining the flow of a pure liquid through porous media. This work resulted in what we know now as Darcy's law or the equation of filtration. The term *filtration* has nothing in common with the term *purification*. It means the flow

of pure liquids through porous media which does not cause any physical changes in those media. Contrasted with the flow of pure liquids, the transport of suspension through porous media may result in sedimentation of the particles in the media space. In consequence, this leads to a decrease in the medium porosity, variation of pressure distribution and permability reduction. Such a phenomenon is known as the colmatage, and is the subject matter of this paper. It was described theoretically and experimentally in a series of papers both by the authors of this work and by other investigators (see Reference).

2. Three models of the colmatage process

The phenomenon of colmatage is described by a system of partial differential equations, i.e. the equations of balance-transport and suitable kinetics of the process.

Examining the effect of colmatage on the pressure distribution in the medium the equation of motion is additionally used.

The equation of balance-transport for a one-dimensional flow obtained by using the mass conservation law has the form

$$\frac{\partial[N(x, t)\varepsilon(x, t)]}{\partial t} + q(t)\frac{\partial N(x, t)}{\partial x} - \frac{\partial\varepsilon(x, t)}{\partial t} = 0 \quad (2.1)$$

where

- $\varepsilon(x, t)$ – medium porosity
- $N(x, t)$ – concentration of solid particles suspended in a liquid medium
- $q(t)$ – unit flow discharge

or

$$q(t)\frac{\partial N(x, t)}{\partial x} - \frac{\partial\varepsilon(x, t)}{\partial t} = 0 \quad (2.2)$$

Eq (2.2) is deprived of a local derivative of function $N(x, t)$, which is equivalent to neglecting the time in which the wave front of a colmatant is transmitted along the porous medium.

The process of colmatage starts at a given point x at the moment when the wave front reaches this point. When the local derivative mentioned is neglected, and shifting of the wave front is left out, the time of the process run is thought to be the same for each point x . Thus, it is possible to leave out the above only when the time needed for the wave front to travel from the

point $x = 0$ to the point $x = L$ in a given porous medium is negligibly small when compared to colmatage process duration.

The use of a proper equation of kinetics depends on the way of arresting solid particles, on the mechanism of this action, or on the kind of the process run.

Three kinetics of the process can be distinguished. They are represented by the following equations

$$\frac{\partial \varepsilon(x, t)}{\partial t} = -\alpha(x)q(t)N(x, t) \quad (2.3)$$

$$\frac{\partial \varepsilon(x, t)}{\partial t} = -\alpha(x)q(t)\varepsilon(x, t)N(x, t) \quad (2.4)$$

$$\frac{\partial \varepsilon(x, t)}{\partial t} = \alpha(x)q(t)\varepsilon(x, t)[\varepsilon(x, t) - \beta^*]N(x, t) \quad (2.5)$$

where

- $\varepsilon(x)$ - colmatage coefficient determining the relationships between suspension particles and a solid medium
- β^* - certain constant, $\beta^* > \varepsilon_0$.

The first kinetics (Eq (2.3)) is supposed to present the processes proceeding at a constant rate of mass exchange.

In the case of second kinetics (Eq (2.4)) the highest rate of mass exchange appears at $t = 0$, and with the progressing saturation of the medium it drops to zero.

The third kinetics (Eq (2.5)) describes the processes in which the rate of mass exchange increases at the beginning and then, reaching a certain maximum value, decreases to zero.

The processes described by the first kinetics appear, for example, in electrical filters, while the second kinetics represents, e.g., the adsorption processes. In sand filters during the flow of clay suspensions, and not only, the processes were observed which were characterised by non-monotonic rates of saturation. It is the third kinetics (Eq (2.5)), which describes the flows accompanied by the colmatage in most general way. The two other kinds of kinetics can be treated as its specific cases.

As seen from the above, the flow of suspensions through porous media accompanied by colmatage leads to the change of the medium porosity, its permeability followed by the change of the pressure head and, quite obviously, to the concentration of the particles flowing through. The description of the phenomenon of colmatage is reduced to the determination and presentation of these magnitudes, these functions.

The system of the equations of balance-transport combined with the proper kinetics of the process of colmatage as well as the equations of motion allow for determination of these functions. Basing on the system of the two first equations the functions $\varepsilon(x, t)$ and $N(x, t)$ will be determined.

In this paper, solutions of the systems of equations (2.2), (2.3); (2.2), (2.4) and (2.2), (2.5) with the initial-boundary conditions are presented

$$\varepsilon(x, 0) = \varepsilon_0 \qquad N(0, t) = n \qquad (2.6)$$

where n denotes concentration of solid particles in a suspension flowing into a medium.

Thus, the equation of balance (2.2) is combined with one of the formulated kinetics.

For the first kinetics, the system (2.2), (2.3) has a solution in the form

$$N(x, t) = ne^{-\int_0^x \alpha(x) dx} \qquad \varepsilon(x, t) = \varepsilon_0 - nQ(t)A_1(x) \qquad (2.7)$$

where

$$Q(t) = \int_0^t q(t) dt \qquad A_1(t) = \alpha(x)e^{-\int_0^x \alpha(x) dx} \qquad (2.8)$$

For the second kinetics, the function $N(x, t)$ is eliminated from Eq (2.2), (2.4). The following equation is obtained

$$\frac{\partial^2 \ln \varepsilon(x, t)}{\partial t \partial x} - \frac{\alpha'(x)}{\alpha(x)} \frac{\partial \ln \varepsilon(x, t)}{\partial t} + \alpha(x) \frac{\partial \varepsilon(x, t)}{\partial t} = 0$$

The above equation is linearized

$$\ln \frac{\varepsilon(x, y)}{\varepsilon_0} \approx \frac{1}{\varepsilon_0} [\varepsilon(x, t) - \varepsilon_0]$$

Thus, we have

$$\frac{\partial \varepsilon(x, t)}{\partial x} = \left[\frac{1}{\varepsilon_0} \frac{\alpha'(x)}{\alpha(x)} - \alpha(x) \right] \varepsilon(x, t) [\varepsilon(x, t) - \varepsilon_0] \qquad (2.9)$$

The boundary condition for the above equation is found applying Eq (2.4) at the point $x = 0$. We obtain

$$\varepsilon(0, t) = \varepsilon_0 e^{-\alpha(0)nQ(t)} \qquad (2.10)$$

The following function is a solution of Eqs (2.9) and (2.10)

$$\varepsilon(x, t) = \frac{\varepsilon_0}{1 + [e^{\alpha(0)nQ(t)} - 1]A_2(x)} \tag{2.11}$$

where

$$A_2(x) = \frac{\alpha(x)}{\alpha(0)} e^{-\varepsilon_0 \int_0^x \alpha(x) dx} \tag{2.12}$$

Basing on the third kinetics (2.5) and on the equation of balance (2.2) after eliminating function $N(x, t)$ we have

$$\frac{\partial^2 \ln \frac{\varepsilon(x,t)}{\beta^* - \varepsilon(x,t)}}{\partial t \partial x} - \frac{\alpha'(x)}{\alpha(x)} \frac{\partial \ln \frac{\varepsilon(x,t)}{\beta^* - \varepsilon(x,t)}}{\partial t} + \alpha(x)\beta^* \frac{\partial \varepsilon(x,t)}{\partial t} = 0 \tag{2.13}$$

An auxiliary function is introduced

$$F(x, t) = \ln \frac{\varepsilon(x, t)}{\beta^* - \varepsilon(x, t)}$$

Thus

$$\varepsilon(x, t) = \frac{\beta^* e^{F(x,t)}}{1 + e^{F(x,t)}} \tag{2.14}$$

Eq (2.13) is integrated over t using is integrated in relation to t using the above substitution. We have

$$\frac{\partial F(x, t)}{\partial x} - \frac{\alpha'(x)}{\alpha(x)} [F(x, t) - F_0] + \alpha(x)\beta^* \left[\frac{\beta^* e^{F(x,t)}}{1 + e^{F(x,t)}} - \varepsilon_0 \right] = 0$$

Using the linearization

$$\frac{\beta^* e^{F(x,t)}}{1 + e^{F(x,t)}} \approx \frac{\varepsilon_0}{\beta^*} (\beta^* - \varepsilon_0) [F(x, t) - F_0] + \varepsilon_0$$

where $F_0 = F(x, 0) = \ln \frac{\varepsilon_0}{\beta^* - \varepsilon_0}$, the following equation is obtained

$$\frac{\partial F(x, t)}{\partial x} = \left[\frac{\alpha'(x)}{\alpha(x)} - \alpha(x)\varepsilon_0(\beta^* - \varepsilon_0) \right] [F(x, t) - F_0]$$

The function $F(x, t) = F_0 - n\beta^*Q(t)\alpha(x) \exp\left[-\varepsilon_0(\beta^* - \varepsilon_0) \int_0^x \alpha(x) dx\right]$ is its solution.

By virtue of Eq (2.14) the function of porosity is expressed by the formula

$$\varepsilon(x, t) = \frac{\varepsilon_0 \beta^*}{\varepsilon_0 + (\beta^* - \varepsilon_0) e^{n \beta^* Q(t) A_3(x)}} \quad (2.15)$$

where

$$A_3(x) = \alpha(x) e^{-\varepsilon_0(\beta^* - \varepsilon_0) \int_0^x \alpha(x) dx} \quad (2.16)$$

Now, we will determine the pressure distribution in a porous medium during the process of colmatage. To this end the following of motion will be used

$$\frac{\partial h(x, t)}{\partial x} = - \frac{aq(t)}{[\varepsilon(x, t)]^3} \quad (2.17)$$

The function $h(x, t)$ determines the sought pressure distribution in a porous medium, and a is a certain constant.

If the flow proceeds at the assumed discharge $q(t)$, and the pressure h_L under which the suspension flows out of the medium at the point $x = L$ is given, then the pressure distribution can be found by integrating Eq (2.17) over to x and taking the boundary condition (2.18) into account

$$H(L, T) = h_L \quad (2.18)$$

We have

$$h(x, t) = aq(t) \int_x^L \frac{dx}{[\varepsilon(x, t)]^3} + h_L \quad (2.19)$$

The function $\varepsilon(x, t)$ appearing in the above equation is expressed by: Eq (2.7) for the first kinetics; Eq (2.11) for the second kinetics, and Eq (2.15) for third kinetics, respectively.

Now, the case when the flow proceeds at a constant pressure difference will be examined. Let h_0 denote the pressure at which the suspension flows into the medium at the point $x = 0$. We have

$$h(0, t) = h_0 \quad (2.20)$$

The pressure distribution in the medium $h(x, t)$ and the discharge of flow $q(t)$ will be determined below.

3. Sample solutions

Integrating the equation of motion (2.17) with the conditions (2.18), (2.20) the following relation is obtained

$$h_0 - h_L = \int_0^L \frac{aq(t)}{[\varepsilon(x, t)]^3} dx \quad (3.1)$$

Hence

$$q(t) = \frac{h_0 - h_L}{a \int_0^L \frac{dx}{[\varepsilon(x, t)]^3}} \quad (3.2)$$

Combining Eq (3.2) with (2.19) we have

$$h(x, t) = h_L + (h_0 - h_L) \frac{\int_0^x \frac{dx}{[\varepsilon(x, t)]^3}}{\int_0^L \frac{dx}{[\varepsilon(x, t)]^3}} \quad (3.3)$$

Let us notice that if the function $Q(t)$ were known, it would be possible to determine the function of porosity $\varepsilon(x, t)$ for different kinds of kinetics using Eqs (2.7), (2.11) or (2.15), and introducing them into Eqs (3.2) and (3.3). These formulae could make it possible to determine the discharge of flow $q(t)$ and the pressure distribution in the deposit $h(x, t)$.

Now the function $Q(t)$ for different kinds of kinetics should be found.

Let us analyse the first kinetics. Eq (2.7) will be introduced into Eq (3.1). We obtain the relation

$$h_0 - h_L = \int_0^L \frac{aq(t) dx}{[\varepsilon_0 - nQ(t)A_1(x)]^3}$$

Integrating the above equation over t we obtain

$$(h_0 - h_L)t = \int_0^L \frac{a dx}{2nA_1(x)[\varepsilon_0 - nQ(t)A_1(x)]^2} - \int_0^L \frac{a dx}{2nA_1(x)\varepsilon_0^2} \quad (3.4)$$

where $A_1(x)$ is expressed by Eq (2.8)₂.

Eq (3.4) can be treated as an implicit equation of function $Q(t)$.

When the second kinetics is considered, Eq (2.11) should be introduced into Eq (3.1). The following is obtained

$$\frac{\partial h(x, t)}{\partial x} = -\frac{aq(t)}{\varepsilon_0^3} \left[1 + 3A_2(x) \left(e^{\alpha(0)nQ(t)} - 1 \right) + 3[A_2(x)]^2 \left(e^{\alpha(0)nQ(t)} - 1 \right)^2 + [A_2(x)]^3 \left(e^{\alpha(0)nQ(t)} - 1 \right)^3 \right]$$

where $A_2(x)$ is given by Eq (2.12).

After integrating the above over x with the conditions (2.18) and (2.20) we obtain

$$h_0 - h_L = \frac{aq(t)}{\varepsilon_0^3} \left[L + 3B_1 \left(e^{\alpha(0)nQ(t)} - 1 \right) + 3B_2 \left(e^{\alpha(0)nQ(t)} - 1 \right)^2 + B_3 \left(e^{\alpha(0)nQ(t)} - 1 \right)^3 \right]$$

where

$$B_k = \int_0^L [A_2(x)]^k dx \quad \text{for } k = 1, 2, 3$$

The above relation is integrated in over time. An implicit equation of function $Q(t)$ for the second kinetics is obtained

$$\begin{aligned} (h_0 - h_L)t &= \frac{a}{\varepsilon_0^3} \left[Q(t)(L - 3B_1 + 3B_2 - B_3) + \right. \\ &+ \frac{3}{n\alpha(0)} \left(e^{n\alpha(0)Q(t)} - 1 \right) (B_1 - 2B_2 + B_3) + \\ &\left. + \frac{3}{2n\alpha(0)} \left(e^{2n\alpha(0)Q(t)} - 1 \right) (B_2 - B_3) + \frac{1}{3n\alpha(0)} \left(e^{3n\alpha(0)Q(t)} - 1 \right) B_3 \right] \end{aligned} \tag{3.5}$$

Now the third kinetics is under consideration. Eq (2.15) is introduced into Eq (3.1), thus, we obtain

$$\begin{aligned} h_0 - h_L &= \frac{aq(t)}{\varepsilon_0^3(\beta^*)^3} \int_0^L \left[\varepsilon_0^3 + 3\varepsilon_0^2(\beta^* - \varepsilon_0)e^{n\beta^*Q(t)A_3(x)} + \right. \\ &\left. + 3\varepsilon_0(\beta^* - \varepsilon_0)^2 e^{2n\beta^*Q(t)A_3(x)} + (\beta^* - \varepsilon_0)^3 e^{3n\beta^*Q(t)A_3(x)} \right] dx \end{aligned}$$

$A_3(x)$ is expressed by Eq (2.15).

The above equation is integrated over t

$$(h_0 - h_L)t = \frac{a}{\varepsilon_0^3(\beta^*)^3} \int_0^L \left[\varepsilon_0^3 Q(t) + 3\varepsilon_0^2(\beta^* - \varepsilon_0) \frac{e^{n\beta^* Q(t)A_3(x)} - 1}{n\beta^* A_3(x)} + 3\varepsilon_0(\beta^* - \varepsilon_0)^2 \frac{e^{2n\beta^* Q(t)A_3(x)} - 1}{2n\beta^* A_3(x)} + (\beta^* - \varepsilon_0)^3 \frac{e^{3n\beta^* Q(t)A_3(x)} - 1}{3n\beta^* A_3(x)} \right] dx$$

In this way an implicit equation of function $Q(t)$ for the third kinetics is obtained.

The authors want to stress that the equation given by Darcy and known as the filtration law is a specific case of the problem discussed, which, for the sake of distinction, is called colmatage.

The information on pressure distribution during the flow without the exchange of mass with the neighbourhood is obtained from the equation of filtration. The theory of colmatage, to the development of which the authors of this paper have contributed, makes it possible to determine these distributions during the flow with the mass exchange. We have directions (functions) at our disposal how to obtain those distributions on the path x and at time t of the colmatage process duration under the conditions of constant discharge of flow both in homo- and heterogeneous media and in reference to homogeneous and heterogeneous suspensions.

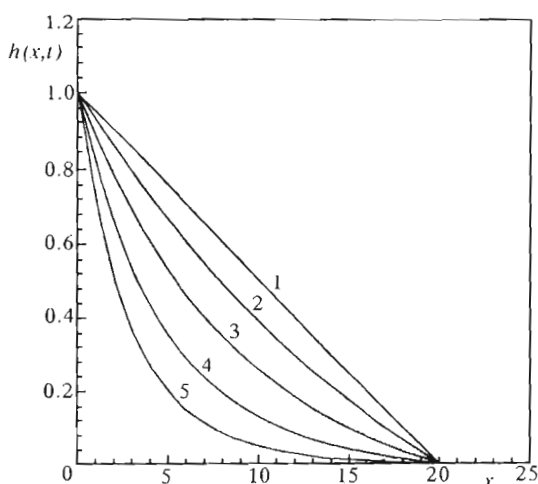


Fig. 1.

Fig.1 shows the pressure distribution in the flow without and with the exchange of mass with neighbourhood, respectively. Curve 1 presents the pres-

sure distribution in the flow without the exchange of mass with neighbourhood. Such a process can be obtained on the strength of Darcy's law. The other curves illustrate the distributions when the flow proceeds with the exchange of mass with neighbourhood, i.e. when the colmatage occurs. The curves present these processes at the chosen moments t . At the beginning of the process at the moment $t = 0$, the pressure distribution obtained on the basis of the theories of colmatage agrees with curve 1 resulting from the theory of filtration presented by Darcy. Therefore the statement that the theory of colmatage is the generalisation of filtration is quite justified.

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Zjawisko kolmatacji jako uogólnienie filtracji i jego wpływ na zmianę porowatości i ciśnienia

Streszczenie

W pracy zaprezentowano różne modele przebiegu zjawiska kolmatacji. Omówiono mianowicie trzy kinetyki opisujące przebieg zjawiska oraz odpowiadające im układy równań różniczkowych cząstkowych. Przedstawiono przykłady rozwiązań tych układów (2.2), (2.3); (2.2), (2.4) oraz (2.2), (2.5) i wyznaczono w ich wyniku funkcję porowatości ośrodka w trakcie trwania procesu kolmatacji (2.7), (2.11), (2.15). Ponadto wprowadzając odpowiednie równanie ruchu (2.17) przedstawiono sposób określenia rozkładu ciśnienia w ośrodku w przypadku gdy przepływ następuje przy zadanym wydatku (2.19) oraz przy stałej różnicy ciśnień (3.3). W tym drugim przypadku wyznaczono dodatkowo wydatek przepływu jako funkcję malejącą czasu (3.2).

Obliczenia podano dla wszystkich trzech kinetyk.

Zamieszczony wykres ilustruje rozkłady ciśnienia dla różnych chwil t , przy przepływie realizowanym przy stałej różnicy ciśnień. Należy zwrócić uwagę, że w chwili $t = 0$ uzyskany z teorii kolmatacji rozkład pokrywa się z rozkładem uzyskanym z prawa Darcy'ego.