MECHANIKA TEORETYCZNA
I STOSOWANA
Journal of Theoretical
and Applied Mechanics
1, 35, 1997

THE EFFECT OF SHALLOW WATER ON THE MANOEVERABILITY OF SHIP

ZYGMUNT KITOWSKI

Naval Academy, Gdynia

e-mail: zkit@amw.gdynia.pl

The paper presents an analysis of the effect of shallow water on the dynamic characteristics of a ship. Basing on the equation of ship's motion in horizontal plane, the effect of shallow water and ship's speed influence, respectively, on the values of hydrodynamic coefficients of the hull and rudder of ship have been considered. It also shows the nature of changes in the values of differential equations coefficients, which describe the ship motion and the results of computer simulation of the course changes.

Key words: shallow water, steering of ship

1. Introduction

The depth of a water region is a significant factor exerting influence on steering of a ship. Ship parameters in the case when the depth of water region H is comparable with the draught value T_z show, that in many cases deterioration of manoeverability of a ship is observed. The ship begins to yaw, so to keep it on a straight course, more frequent lays of the rudder (cf Zemlyanovskiĭ (1976)) are required. The concept of shallow water is not well-defined. As its measure is often used the ratio of water region depth H to ship's draught T_z. Another important parameter which characterises the sailing conditions in shallow water is Froude's shallow-water number $F_{nH} = \vartheta/\sqrt{gH}$, being a ratio of the ship's speed to the speed of shallow water wave. The value of ship's speed equal to the maximal speed of shallow water wave $\vartheta_{cr} \approx \sqrt{gH}$, is called the critical speed. An especially acute effect of the critical speed appears at the speed of the ship $\vartheta > 0.4\vartheta_{cr}$ (cf Dudziak (1988)).

When formulating dynamical models of the ship, the functions representing forces and hydrodynamic moments acting on the ship hull are usually extended in a Taylor series, omitting the components, contribution of which. At present, two main methods of formulation of equations of ship dynamics are used to investigate its manoeverability. So arose many similar mathematical models which were verified in terms of ship models in towing tanks or on real ships. For example, we have the models elaborated by Eda and Crane (1965), Abkowitz (1964), Norrbin (1971), Clark (1971), Nomoto (1969), etc. The components of higher order in the Taylor series have no physical interpretation in such a approach. Another procedure of determination the ship dynamical model was accepted by Basin (1977), Sobolev (1976) and Voĭtkunskiĭ et al. (1985), in which the formulae for forces and hydrodynamic moments were defined as empirical relationships.

According to the above aspects, in this paper, the dynamical model of a ship is considered, where the coefficients of the forces and hydrodynamic moments are given explicity (cf Voĭtkunskiĭ et al. (1985)).

2. Equations of ship's motion in horizontal plane

When ship steering is studied, the considerations are most often confined to an analysis of its motion in horizontal plane. Then differential equations of motion, on the assumption that the ship is propelled by screw propellers and controlled by a steering gear, have the following forms (cf Voĭtkunskiĭ et al. (1985))

(a) force equation

$$\frac{d\beta}{dt} + \frac{(C_y^\beta + C_2|\beta|)\vartheta F_g + \mu_k \chi_p \vartheta S_z}{2V(1+k_{22})}\beta + \frac{\mu_k \chi_p \varepsilon L S_z - 2V(1+k_{11})}{2V(1+k_{22})}\omega + \tag{2.1}$$
$$- \frac{\mu_k S_z \vartheta}{2V(1+k_{22})}\alpha - \frac{1}{\gamma V(1+k_{22})\vartheta}F_w = 0$$

(b) moment equation

$$\frac{d\omega}{dt} + \frac{C_m^\omega \gamma\vartheta F_g L^2 + \mu_k l_s \varepsilon L \gamma \vartheta S_z \chi_p}{2I(1+k_{66})}\omega + \frac{-C_m^\beta \gamma \vartheta^2 F_g L + \mu_k l_s \chi_p \gamma \vartheta^2 S_z}{2I(1+k_{66})}\beta + \tag{2.2}$$
$$- \frac{\mu_k l_s \gamma \vartheta^2 S_z}{2I(1+k_{66})}\alpha - \frac{1}{I(1+k_{66})}M_w = 0$$

Introducing the following notation

$$
\begin{aligned}
q_{21} &= \frac{(C_y^\beta + C_2|\beta|)\vartheta F_g + \mu_k \chi_p \vartheta S_z}{2V(1+k_{22})} \\
r_{21} &= \frac{\mu_k \chi_p \varepsilon L S_z - 2V(1+k_{11})}{2V(1+k_{22})} \\
s_{21} &= \frac{\mu_k S_z \vartheta}{2V(1+k_{22})} \\
t_{21} &= \frac{1}{\gamma V(1+k_{22})\vartheta} \\
q_{31} &= \frac{-C_m^\beta \gamma \vartheta^2 F_g L + \mu_k l_s \chi_p \gamma \vartheta^2 S_z}{2I(1+k_{66})} \\
r_{31} &= \frac{C_m^\omega \gamma \vartheta F_g L^2 + \mu_k l_s \varepsilon L \gamma \vartheta S_z \chi_p}{2I(1+k_{66})} \\
s_{31} &= \frac{\mu_k l_s \gamma \vartheta^2 S_z}{2I(1+k_{66})} \\
t_{31} &= \frac{1}{I(1+k_{66})}
\end{aligned}
$$

where

β	–	drift angle at the center of gravity
ω	–	angular velocity of the ship
V	–	volume displacement
ϑ	–	ship's speed
L	–	length on the waterline
γ	–	sea water specific gravity
I	–	mass moment of inertia of the ship
k_{11}	–	added mass coefficient at longitudinal ship's motion
k_{22}	–	added mass coefficient at transverse ship's motion
k_{66}	–	added mass moment of inertia coefficient
S_z	–	reduced rudder area
C_m^β	–	positioning moment coefficient (for aplane motion)
C_m^ω	–	damping moment coefficient (for aplane motion)
χ_p	–	reduced coefficient of the hull and screw propeller influence on direction of the rudder inflow
μ_k	–	rudder lateral force coefficient
l_s	–	distance between the rudder stock axis and midship section

ε	–	relative distance from the rudder to the midship section
C_2, C_y^β	–	transverse positioning force coefficient (for a plane motion)
C_y^ω	–	transverse damping force coefficient (for a plane motion)
F_g	–	area of middle buttock of the ship
α	–	rudder angle
F_w	–	resultant of exciting forces acting on ship's hull
M_w	–	moment of external forces.

Eqs (2.1) and (2.2), with the external excitations neglected can be rewriten as

$$\frac{d\beta}{dt} + q_{21}\beta + r_{21}\omega - s_{21}\alpha = 0 \tag{2.3}$$

$$\frac{d\omega}{dt} + q_{31}\beta + r_{31}\omega - s_{31}\alpha = 0 \tag{2.4}$$

These equations constitute the basis for analysis of a ship's motion in the water plane.

3. The effect of water region depth on values of hydrodynamic coefficient

The phenomenon of deterioration of ship steering in shallow water is appears one to factiss: the first of them is the slowing down in shallow water as a result of fast increase of hull resistance, the other is a change in the character of hull flow around (cf Anosov and Licyk (1976)). The effect of the depth of the water region can be taken into account by making use of the results published by Antonenko et al. (1978), where the experimental formulae for coefficients of forces and hydrodynamic moments, established on the basis of model testing were presented. From the calculations it followed that coefficients C_y^ω, C_m^ω, C_y^β, C_m^β depended on the ratios T_z/H and $\lambda = 2T_z/L$. They are not, however, functions of the drift angle β and angular velocity of the ship ω. These dependencies are shown in Fig.1.

Representing the diagrams shown in Fig.1 in terms of simple functions, the following formulae for force and moment coefficients acting on the ship's hull when sailing in shallow water are obtained (cf Basin et al. (1976))

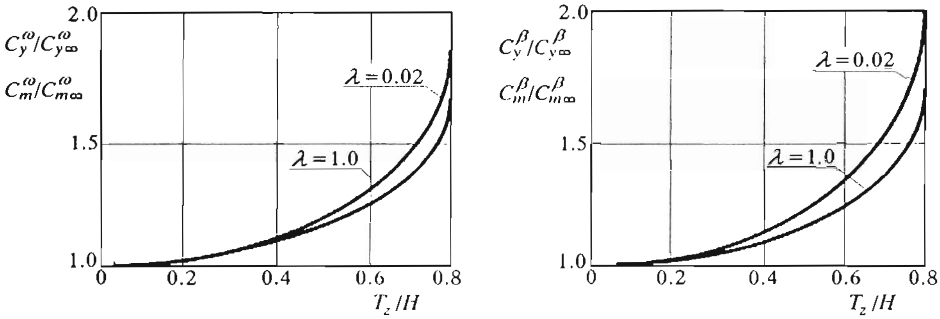

Fig. 1. Courses of force and moment coefficients acting on ship's hull versus the depth of water below keel H

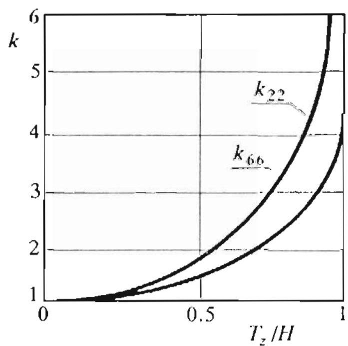

Fig. 2. Diagrams of coefficients k_{22} and k_{66} versus the depth of water below keel H

$$C_y^\omega = C_{y\infty}^\omega + 1.6C_{y\infty}^\omega\left(\frac{T_z}{H}\right)^2 \qquad C_m^\omega = C_{m\infty}^\omega + 1.6C_{m\infty}^\omega\left(\frac{T_z}{H}\right)^2$$
$$C_y^\beta = C_{y\infty}^\beta + 2.0C_{y\infty}^\beta\left(\frac{T_z}{H}\right)^2 \qquad C_m^\beta = C_{m\infty}^\beta + 2.0C_{m\infty}^\beta\left(\frac{T_z}{H}\right)^2 \tag{3.1}$$

where $C_{y\infty}^\omega$, $C_{m\infty}^\omega$, $C_{y\infty}^\beta$, $C_{m\infty}^\beta$ – values of ther coefficients for deep water.

The change in added mass of water, caused by a limited depth of the water region, can be taken into account by making use of the data published by Basin et al. (1976). Approximation of the dependence between the added mass coefficient for ship's transverse motion k_{22} and the added mass moment of inertia coefficient k_{66}, which takes into account the effect of limited depth of the water region shown in Fig.2, leads to the following formulas

$$k_{22} = 1.8k_{22\infty}\left(\frac{T_z}{H}\right)^2 \qquad k_{66} = 2.2k_{66\infty}\left(\frac{T_z}{H}\right)^2 \tag{3.2}$$

where k_{22}, k_{66} – values of the coefficients for deep water.

The analysis of equations describing ship's motion in horizontal plane and Eqs (3.1) and (3.2) allows us to discover the cause of deterioration of ship's manoeverability when sailing in shallow water. The increase in magnitudes of forces depending on drift angle β and angular velocity of a ship ω, which act on the ship's hull, is to a certain degree compensated by a adequate change in the added mass, which results in diminishing of the rudder effectiveness and consequently in deterioration of ship's manoeverability. Thus in the case, when the depth of water region H is comparable with the ship's draught T_z, it is necessary, for the process of steering to be performed properly, to take into account the changes in the ship dynamic characteristics, expressed by the changes in values of hydrodynamic coefficient of ship's hull and rudder, respectively. In Fig.3 the change in values of ΔC_y^ω, ΔC_m^ω, ΔC_y^β, ΔC_m^β, Δk_{22}, Δk_{66} versus the changes in the depth of water below keel, for one of the ships[1], has been shown as an example.

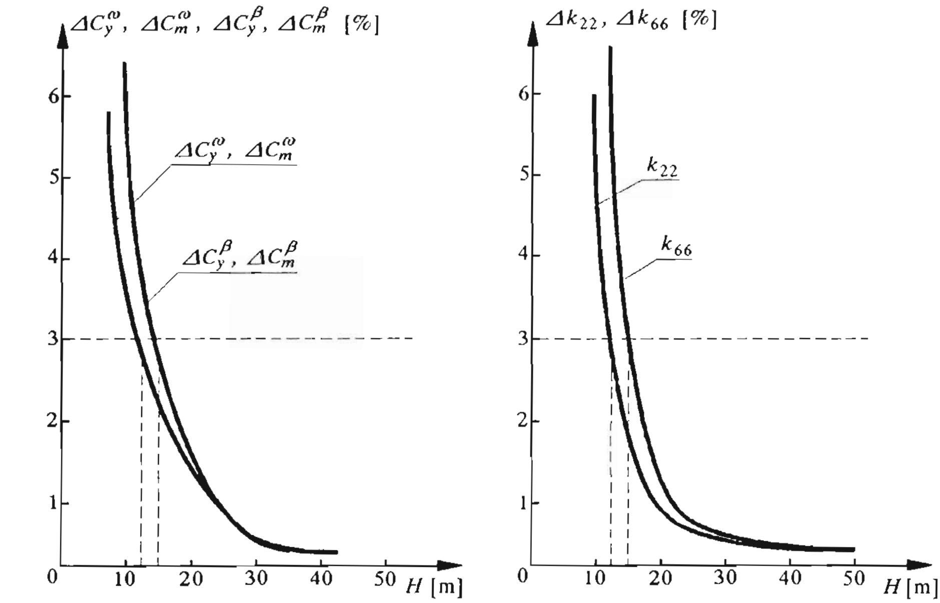

Fig. 3. Percentage change of coefficients C_y^ω, C_m^ω, C_y^β, C_m^β, k_{22}, k_{66} as a function of the depth of water region H

From the presented diagrams it follows that, when 3%-change in the value

[1]Ship's exploitation data: $V = 213.76\,\mathrm{m}^3$, $L = 36.3\,\mathrm{m}$, $B = 7\,\mathrm{m}$, $T_z = 1.742\,\mathrm{m}$. The vessel is equipped with two screw propellers and two rudders

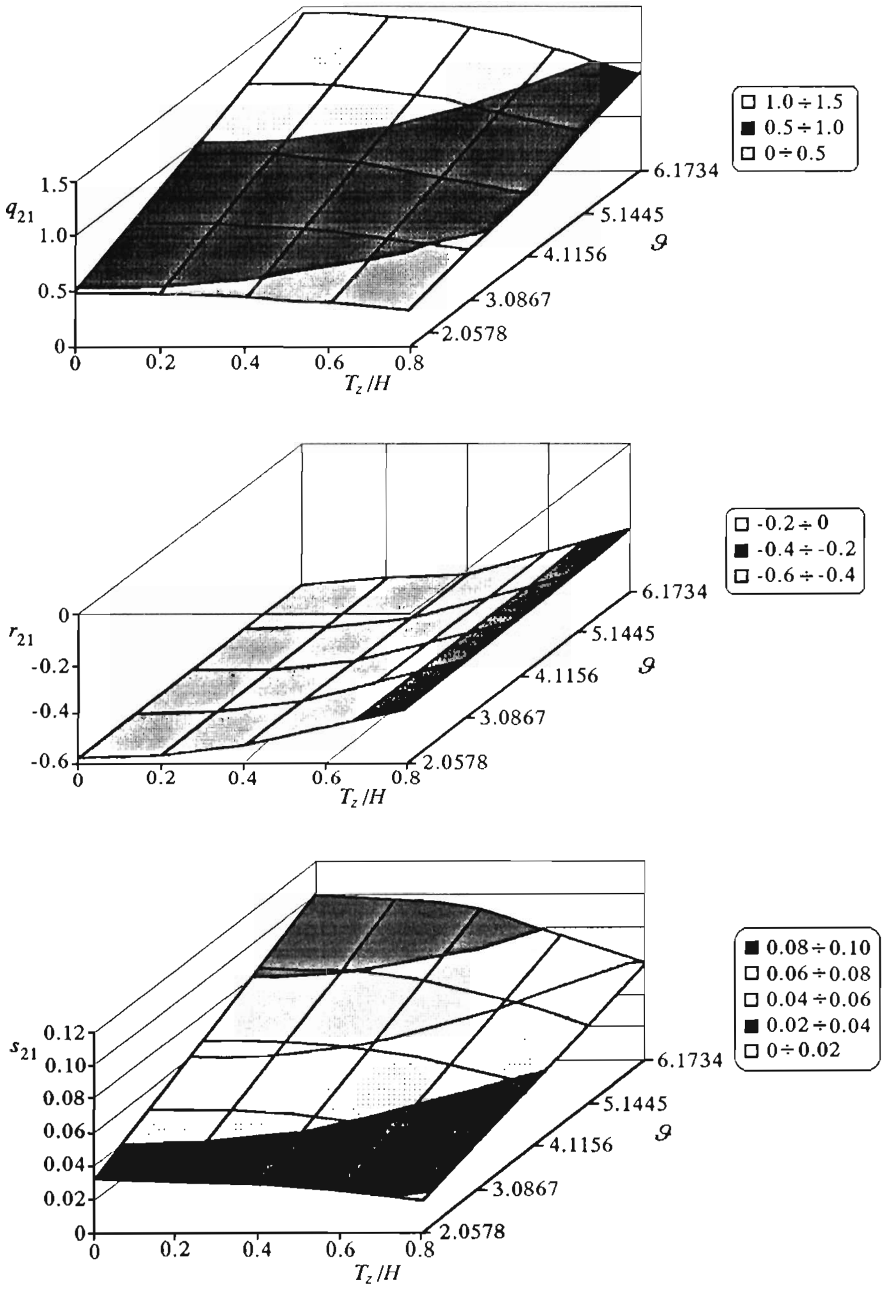

Fig. 4. Values of the hydrodynamic coefficients $q_{21} = f_1(\vartheta, T_z/H)$, $r_{21} = f_2(\vartheta, T_z/H)$, $s_{21} = f_3(\vartheta, T_z/H)$

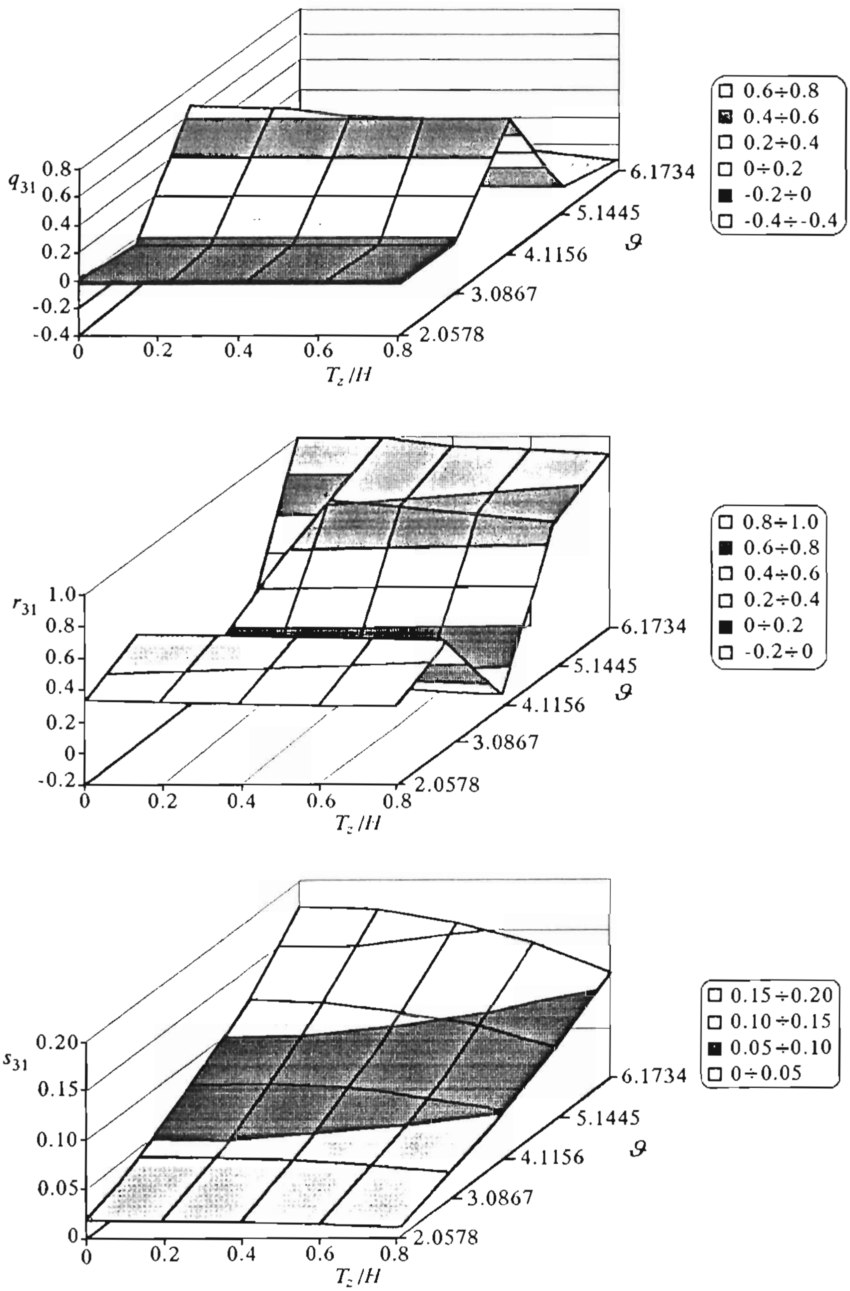

Fig. 5. Values of the hydrodynamic coefficients $q_{31} = g_1(\vartheta, T_z/H)$, $r_{31} = g_2(\vartheta, T_z/H)$, $s_{31} = g_3(\vartheta, T_z/H)$

of coefficient is assumed (this accuracy lies within the accuracy of their determination), it is necessary to take into account the effect of the depth of water region on the dynamic characteristics of the ship for $H < 15$. For depths over 15 m the effect of H has to be taken into account when corrections of the values of C_y^ω, C_m^ω, C_y^β, C_m^β, k_{22}, k_{66} are being introduced in to calculations of hydrodynamic coefficients of ship's hull and rudder. The influence of the hydrodynamic coefficient values on the coefficients of differential equations describing ship's motion is shown in Fig.4 and Fig.5.

4. The influence of shallow water on ship steering

In order to investigate the influence of shallow water on the ship steering, general equations of ship's motion, Eqs (2.3) and (2.4), have been used. The influence of the depth of water region on ship's performance at a course change by 20° is shown in Fig.6 and Fig.7.

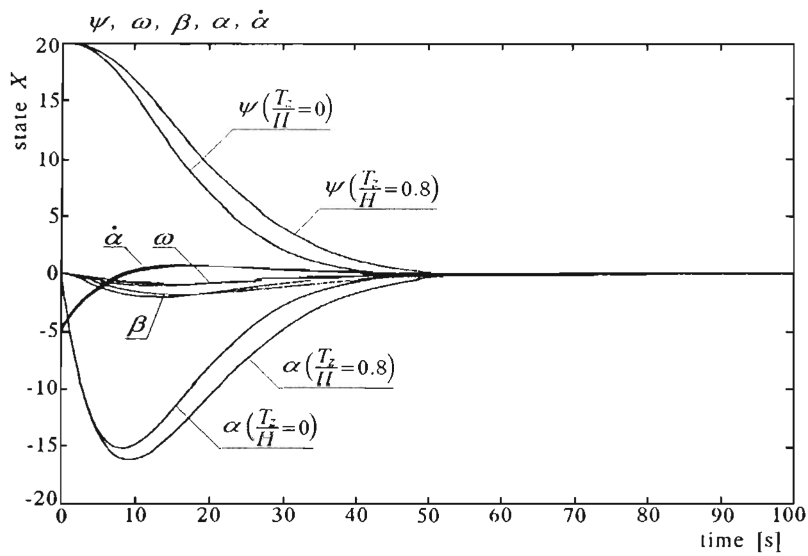

Fig. 6. Change of ship's course by $\Delta\psi = 20°$ at the speed $\vartheta = 2.0578$ m/s in shallow and deep water

In both cases under consideration, the change in course was executed at the same setting of giropilot but at different speeds. As it can be seen from the presented diagrams, the influence is visible for a speed close to the critical

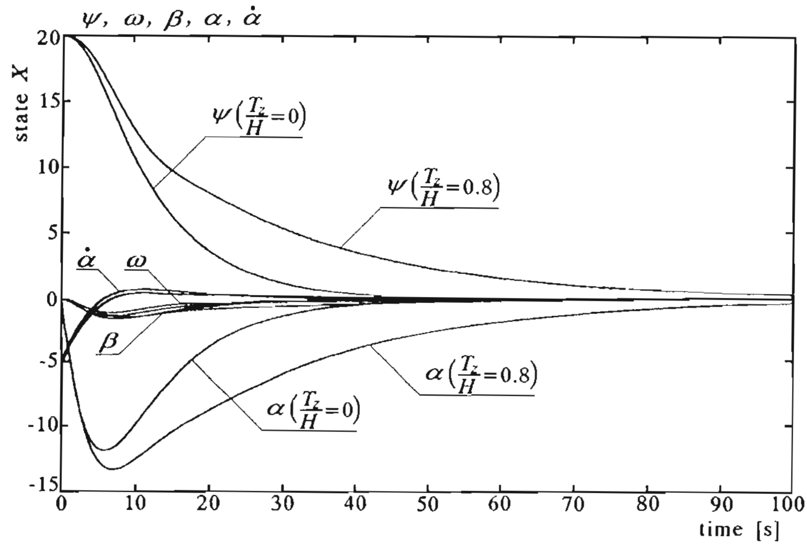

Fig. 7. Change of ship's course by $\Delta\psi = 20°$ at the speed $\vartheta = 4.1156\,\mathrm{m/s}$ in shallow and deep water

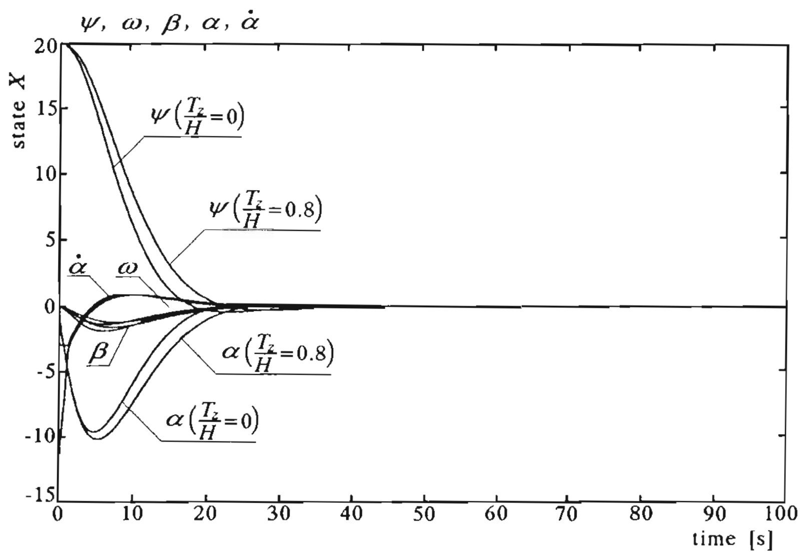

Fig. 8. Change of ship's course by $\Delta\psi = 20°$ at the speed $\vartheta = 6.1734\,\mathrm{m/s}$ in shallow and deep water

speed $\vartheta_{cr} = \sqrt{g(T_z/0.8)} = 4.62\,\mathrm{m/s}$. From the diagrams it can be seen that, the ship's turning ability in shallow water impairs, i.e. an increase in the ship's turning radius is observed. The interesting fact is, however, that with further increase in speed, the steering improves (Fig.8). The fact stated above proves complexity of the processes when the ship is moving in shallow water. The step-wise change of parameters q_{31} and r_{31} has not been justified in publications so far. Therefore it is necessary to conduct further studies.

5. Conclusions

When sailing in shallow water, the pressure decreases, and rotational motion of water is appears, especially under aft part of the ship, which changes the normal distribution of pressure acting on the rudder blade. This leads to the yaw increase and brings about deterioration of ship steering. The shallow water does not show the same effect for all draught values. Quite often the loss of steerability in shallow water occurs suddenly. The causes of similar phenomena are little known. Clearly, it seems that the ship tends to turn towards deep water.

The depth of water region at which the ship begins to yaw and poorly answers the helm, depends strongly on ship's speed. Commonly the ship starts to answer the helm poorly, when the ship floats over an area where the depth of water is 1.5 larger than its draught. With the derease of depth or increase of ship's speed the ship's directional stability impairs. With the increase of speed the effect gets stonger an then after passing the critical speed value the ship's directional stability improves.

Shallow water exerts also an influence on ship's speed and its draught. With the decrease of depth of water region ship slows down and its draught gets larger. The larger is the ship and the higher is its speed the stronger is this effect.

References

1. ABKOWITZ M.A., 1964, Lectures on Ship Hydrodynamics Steering and Manoevuerability, Lyngby, Hydrolog Aerodynamisk Laboratorium, Report Hy-5

2. AMERONGEN J. VAN., 1982, Adaptive Steering of Ship, PhD Thesis Technische Hogeschool, Delft

3. ANOSOV A.V., LICYK A.D., 1976, *Upravlenie sudnom i ego ékspluatacya,* Izdat. Transport., Leningrad
4. ANTONENKO A.V., BERNSHTEĬN S.I., LUBKOV A.V., SIRKIN A.B., SHLEER G.E., 1978, *Adaptivnoe upravlenie kursom sudna,* Izdat. Nauka, Moskva
5. BASIN A.M., 1977, *Khodkost i upravlaemost sudov,* Izdat. Transport, Moskva
6. BASIN A.M., VELEDNICKIĬ I.O., LYAKHOVICKIĬ A.G., 1976, *Gidrodinamika sudov na melkovode,* Izdat. Sudostroenie, Leningrad
7. BECH M.T., 1968, Linearization of Ships Steering Characteristics, *Shipbuilding and Shiping Rec.,* 8
8. CLARK D., 1971, A New Non-Linear Equation for Ship Manoeuvring, *International Shipbuilding Progress,* **18**, 201
9. DUDZIAK J., 1988, *Teoria okrętu,* Wydawnictwo Morskie, Gdańsk
10. EDA H., CRANE C.L., 1965, Steering Characteristics of Ship in Calm Water and in Waves, *SNAME*
11. NOMOTO K., 1969, *Approximate Non-Linear Analysis on Steering Motion,* ITTC Rzym
12. NORRBIN N.H., 1971, Theory and Observations on the Use of a Mathematical Model for Ships Manoeuvring in Deep and Confined Waters, *SSPA,* **68**, Goeteborg
13. SOBOLEV G.V., 1976, *Upravlaemost korabla i avtomatizaciya sudovozhdeniya,* Izdat. Sudostroenie, Leningrad
14. VOĬTKUNSKIĬ I., PERSHYC R., TITOV I., 1985, *Spravochnik po teorii korabla,* Izdat. Sudostroenie, Leningrad
15. ZEMLYANOVSKIĬ D.K., 1976, *Ustoĭchivost dvizheniya i risklivost sudov,* Izdat. Transport, Moskwa

Wpływ płytkiej wody na sterowność okrętu

Streszczenie

W artykule przedstawiono analizę wpływu płytkiej wody na właściwości dynamiczne okrętu. W oparciu o równania ruchu okrętu w płaszczyźnie poziomej rozpatrzony został wpływ płytkiej wody i prędkości okrętu na wartości współczynników hydrodynamicznych kadłuba okrętu i steru. Pokazano również charakter zmian wartości współczynników równań różniczkowych opisujących ruch okrętu oraz wyniki symulacji komputerowej zmiany kursu.

Manuscript received April 24, 1996; accepted for print October 4, 1996