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EXPERIMENTAL VERIFICATION OF STABILITY OF OPTIMAL COMPRESSION HELICAL SPRINGS

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In the paper, the experimental verification of stability of the optimal compression springs is presented. The optimal cylindrical spring is simply supported at both ends and the distribution of helix angle (variable pitch) ensures the maximal critical force. The theoretical results of optimization presented by Krużelecki (1995), (1996) are compared with the experimental ones and buckling experiments confirm the theory presented.

Key words: optimal helical spring, stability, experiment

1. Introductory remarks

In the papers by Krużelecki (1995), (1996) the problems of optimization of helical springs under compression against instability are presented. The first paper deals with variational whereas the latter one with parametrical optimization of geometry of a simply supported spring loaded by an axial compressive force. An initial distribution of helix angle and radius of a spring are sought for which ensure maximal value of the critical force under equality and inequality constraints. It turned out, that the optimal distribution of a helix angle as well as a radius of a spring are described by piece-wise constant functions. The main profit of optimization consists in optimal distribution of the helix angle $\alpha_0(\xi)$ along the spring axis. The influence of optimal variable radius $R_0(\xi)$ on the critical axial force is much lower.

In most structural optimization problems the improvments achieved are on the level of several per cent. In these springs the results are much better. Taking the critical force as optimality criterion and the volume of spring material as a basic constraint, typical improvements (increment of the critical force) reach over 100%. Moreover, it is possible to obtain qualitatively new results, especially for springs with small slenderness ratios, for which a classical spring can buckle. After optimization such a spring does not lose its stability at all.

After obtaining such good theoretical results of optimization, it has been decided to verify them experimentally. The experimental verification of the theoretical results of optimization consists here in comparison between the critical forces for the optimal spring and for the reference spring (with constant α_0), satisfying the imposed constraints, with the corresponding theoretical solutions. This verification is also treated as a very important argument for introducing of the optimally designed springs into engineering practice.

2. Geometry of the springs under consideration and their characteristics

As the reference spring, which is used to compare the geometry and results with the optimal spring, we assume a spring with the mean radius $R_0 = 0.015\,\mathrm{m}$, initial pitch $\bar{h}_0 = 0.014\,\mathrm{m}$, initial number of coils $n_0 = 12$ and total length $H_0 = 0.168\,\mathrm{m}$. This spring is made of a steel wire, of the Young modulus $E = 2.06 \cdot 10^5\,\mathrm{MPa}$, Poisson ratio $\nu = 0.3$, and admissible stress $\tau_0 = 750\,\mathrm{MPa}$ and subjected to a heat treatment. The diameter of wire is cross-section $\bar{d} = 0.0015\,\mathrm{m}$. It determines the constant helix angle $\alpha_0 = 0.1475\,\mathrm{rad}$ and the dimensionless volume of spring material $v = V_m/H_0^3 = 0.000426$. The compression rigidity is defined by

$$\bar{C}_0 = \frac{P}{f} \tag{2.1}$$

where f is the axial reduction of spring length, equal to $\bar{C}_0 = 153.9\,\mathrm{N/m}$.

Optimization of a spring was carried out using the "recipe" presented by Krużelecki (1995) and (1996). It was decided that we considered a cylindrical spring ($R_0 = \mathrm{const}$) and only a helix angle was subject to optimization. The equality constraints are connected here with the volume of spring material V_m, compression rigidity $\bar{C}_0$ and slenderness ratio of spring H_0/R_0. The inequality constraints are geometrical conditions bounding the helix angle α_0, namely $0.07417 \leq \alpha_0 \leq 0.28878$, and they are the active conditions of optimization in this case. The lower $\alpha_{0l} = 0.07417$ and upper $\alpha_{0u} = 0.28878$ bounds, respectively, are assumed in such a way that the pitch referring to them for the sectors of spring near to the supports equals a half ($\bar{h}_{01} = 0.007\,\mathrm{m}$) whereas in the middle sector the pitch ($\bar{h}_{02} = 0.028\,\mathrm{m}$) equals twice the pitch $\bar{h}_0$

of the reference spring. Then, neither the strength condition nor the closing up condition of neighbouring coils are active. As a result of optimization the spring of pich $\bar{h}_{02} = 0.028\,\mathrm{m}$ ($\alpha_{02} = 0.28878$) and four coils ($n_{02} = 4$) in the middle sector of a spring and a pitch $\bar{h}_{01} = 0.007\,\mathrm{m}$ ($\alpha_{01} = 0.07417$) and eight coils ($n_{01} = 8$) in two sectors close to the supports was obtained. A length of a *smoth* transitions between sectors with different helix angles α_{01} and α_{02} is negligibly small.

Geometry of these springs was examined. The pitch and radius along the spring axis, and also the number of coils were measured. Some initial imperfections of geometry were found.

For the reference spring we obtained $n_0 = 12.5$. The diameter of spring was practically constant, $D_0 = 2R_0 = 0.03\,\mathrm{m}$ whereas the pitch $\bar{h}_0$ was variable along the spring axis. A mean value of pitch $\bar{h}_0 = 0.013728\,\mathrm{m}$ and it leads to the mean value of helix angle $\alpha_0 = 0.1446\,\mathrm{rad}$ and to the total length $H_0 = 0.1716\,\mathrm{m}$. The theoretical compression rigidity $\bar{C}_0 = 147.6\,\mathrm{N/m}$ and dimensionless volume $v = 0.000416$. The distribution of helix angle α_0 obtained by measurements of the pitch is shown in Fig.1. Some deviations from the assumed value of α_0 can be observed.

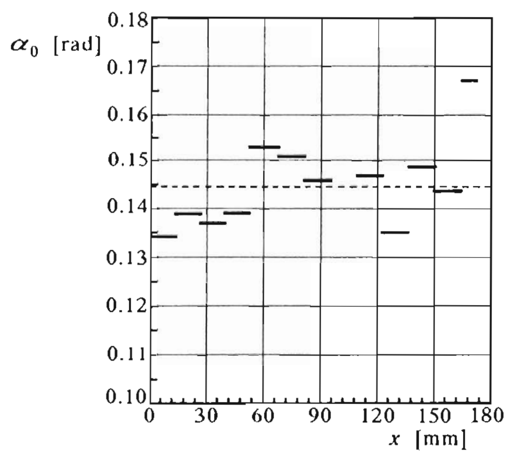

Fig. 1. Measured distribution of helix angle for the reference spring

For the optimal spring the total number of coils connected with the sectors near supports equals $n_{01} = 9.0$ whereas the middle sector, where the helix angle is larger, has $n_{02} = 4.25$. The diameter of the optimal spring is constant, $D_0 = 2R_0 = 0.03\,\mathrm{m}$. A pitch in each sector of a spring varies; its mean value in the sectors near the supports is $\bar{h}_{01} = 0.007339\,\mathrm{m}$, and in the middle sector $\bar{h}_{02} = 0.025141\,\mathrm{m}$. The mean values of helix angle in each sector are: $\alpha_{01} = 0.0777\,\mathrm{rad}$ and $\alpha_{02} = 0.2607\,\mathrm{rad}$, and the total length $H_0 = 0.1729\,\mathrm{m}$. For such a geometry of spring the theoretical compression rigidity $\bar{C}_0 = 139.8\,\mathrm{N/m}$ and dimensionless volume $v = 0.0004326$. In Fig.2 the distribution of helix angle for this spring is presented.

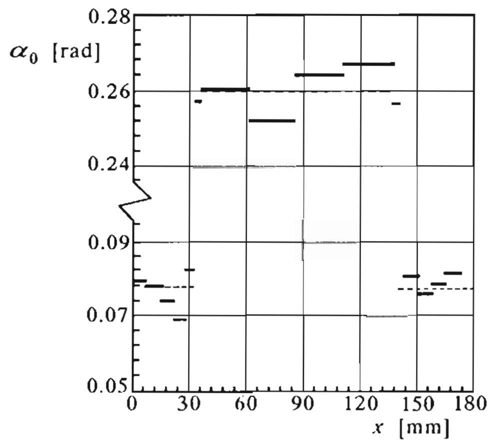

Fig. 2. Measured distribution of helix angle for the optimal spring

Before the buckling experiments are conducted we should find real experimental copression rigidities for both springs. These tests were done using the INSTRON testing machine. The whole range of compressive force is $0 \div 10\,\mathrm{N}$ and the accuracy equals 1%. The force of 10 N leads to 40% reduction of the spring length which is much larger than the value predicted in buckling experiments. In Fig.3 the curves $P = P(f)$ for both springs are shown. They are obtained from compression tests. The characteristics are linear in the whole range of the applied force. For a spring with constant α_0 it leads to the compression rigidity $\bar{C}_0 = 149.1\,\mathrm{N/m}$ whereas for the optimal spring $\bar{C}_0 = 139.3\,\mathrm{N/m}$. It should be emphasized that the theoretical and experimental compression rigidities are practically the same. Because of the different volumes of spring material V_m and the different H_0 for both springs, the dimensionless compression rigidities

$$c_0 = \frac{\bar{C}_0 H_0^5}{E V_m^2} \tag{2.2}$$

are also a little bit different, namely for the reference spring $c_0 = 0.0241$ and for the optimal spring $c_0 = 0.0209$.

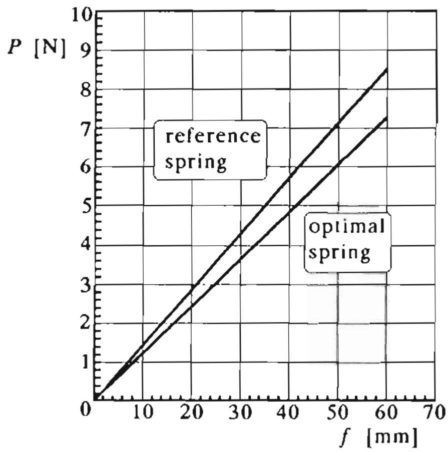

Fig. 3. Experimental characteristics for the reference and optimal springs, respectively

The parameters of the tested springs are shown in Table 1.

Table 1. Parameters of the tested springs

Type of a spring	R_0 [m]	H_0 [m]	R_0/H_0	α_0		n_0		$v \cdot 10^4$	C_0 [N/m]	c_0
				α_{01}	α_{02}	n_{01}	n_{02}			
Optimal	0.0015	0.1729	0.0868	0.0777	0.2607	9.0	4.25	4.33	139.3	0.0209
Reference	0.0015	0.1716	0.0874	0.1446		12.5		4.16	149.1	0.0241

3. Buckling experiments

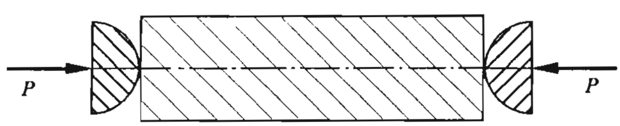

Fig. 4. Practical realization of supports in buckling experiments

The buckling experiments were carried out using the INSTRON testing machine with the assumed range of the compressive force P: $0 \div 10\,\text{N}$. Tested springs are assumed to be the simply supported ones. This type of spring support is realized by taking the beams with semicircular cross-sections which transmit compressive forces from the testing machine to the flat ends of a spring, as it is shown in Fig.4. The loading (compressive force) is realised by an axial displacement of one end of the spring with the second end fixed. The compressive force and axial displacement (reduction of the length) are recorded whereas a lateral dispacement is not recorded in tests. The optimal spring after buckling fixed in the testing machine is shown in Fig.5.

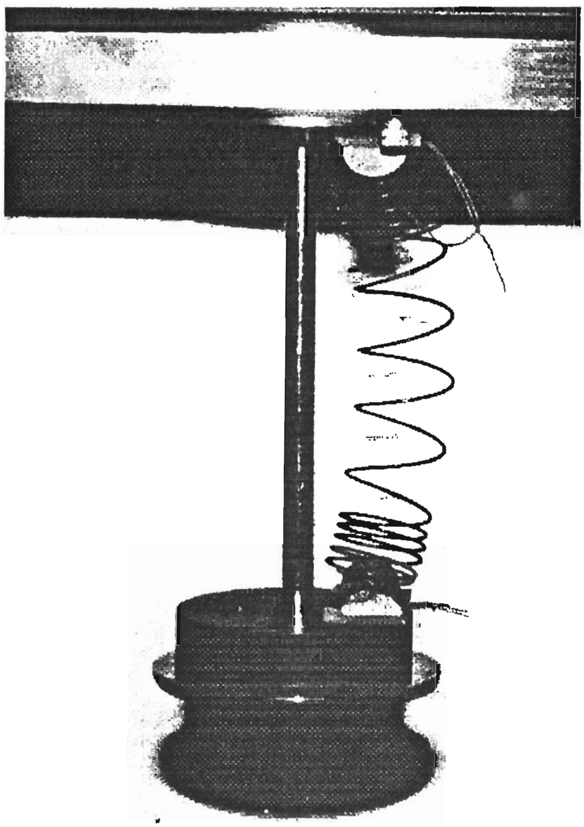

Fig. 5. Optimal spring after buckling

Buckling of the spring manifests by occuring a corner at the curve $P = P(f)$. In Fig.6, there are presented the $P = P(f)$ curves obtained experimentally for the reference and optimal springs, respectively. For the optimal spring we have obtained the critical force $P_{cr} = 3.83\,\mathrm{N}$, or in the dimensionless notation

$$p_{cr} = 4\pi \frac{H_0^4 P}{E V_m^2} = 0.0422 \tag{3.1}$$

and the axial critical displacement $f_{cr} = 0.0188\,\mathrm{m}$. The reference spring shows weaker resistance to buckling, namely $P_{cr} = 2.35\,\mathrm{N}$ ($p_{cr} = 0.02765$) and $f_{cr} = 0.0155\,\mathrm{m}$. The postbuckling path remains stable for both springs; *an additional reduction of the spring length required the appropriate a compressive force increment.*

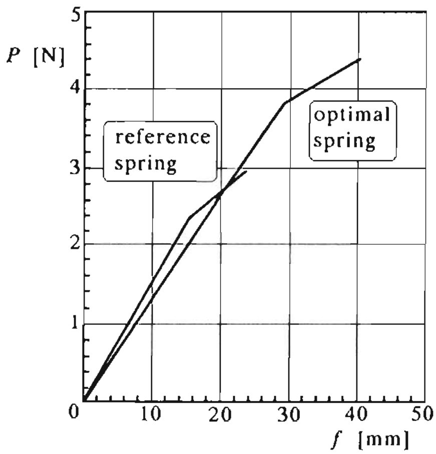

Fig. 6. Experimental $P = P(f)$ curves

To conduct experiments of stability of helical springs subject to compression, and first of all, for visualisation of the optimization effects by direct comparison between the behaviour of both springs the simple frame was designed (Fig.7). The tested springs (or spring) are placed between the lower fixed beam and the upper beam movable in the vertical direction only. For a given constant compression rigidity (both springs have the same $\bar{C}_0$) and for assumed vertical displacement of the upper beam, the forces acting on springs are known and they have the same magnitudes. So, this simple equipment neither needs direct measurement of the force nor a special measurement of the displacement. It allows for very good estimation of the critical force P_{cr} and the critical reduction f_{cr} the spring length. The springs under compression are shown in Fig.7. The reference spring has been buckled alredy and its latteral displacements are quite large whereas the optimal spring is still in a pre-critical state.

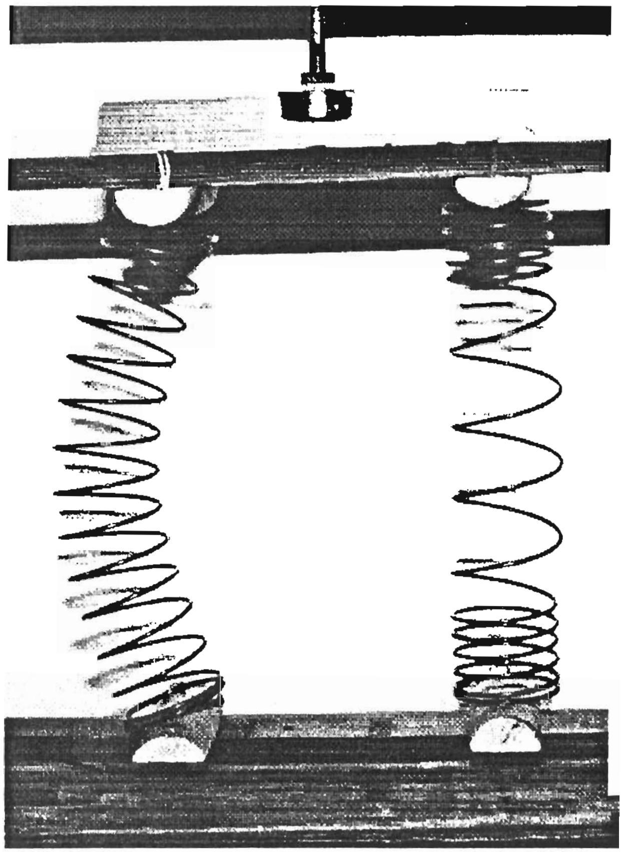

Fig. 7. Optimal and reference springs under compression

A direct quantitative comparison between the effects of optimization can be done if the equality constraints imposed on the structure are satisfied. In the case of optimization of the helical springs subject to compression against instability (cf Krużelecki, 1995, 1996) these constraints are connected with the volume of a material spring v, slenderness ratio H_0/R_0 and compression rigidity c_0. As it is shown in Table 1, these quantities are a little bit different for both springs. The optimal spring has a larger v and H_0 but a smaller compression rigidity c_0 than the reference spring. Because of that, we have to recalculate the force p (reduction of length f).

Employing the definitions of dimensionless force p and dimensionless compression rigidity c_0 we can eliminate the volume V_m of spring material. Then, we have

$$p = 4\pi c_0 \frac{f}{H_0} \tag{3.2}$$

and from Eq (3.2) the total differential takes the form

$$dp = p\Big(\frac{dc_0}{c_0} + \frac{df}{f} - \frac{dH_0}{H_0}\Big) \tag{3.3}$$

For small changes of c_0 and H_0 (V_m has been alredy eliminated) we can estimate their influence on the load p (or on reduction of the length f) of the spring. When the increments of c_0 and H_0 are larger, then the appropriate formula takes the form

$$\frac{\Delta p}{p} = \Big(1 + \frac{\Delta c_0}{c_0}\Big)\frac{1 + \Delta f/f}{1 + \Delta H_0/H_0} - 1 \tag{3.4}$$

which for small increments Δ leads to Eq (3.3).

We obtained the theoretical predictions of the buckling loads for both the real springs under consideration; namely, for the optimal spring $p_{cr} = 0.03928$ ($f_{cr} = 0.02605$ m) whereas for the reference spring $p_{cr} = 0.02723$ ($f_{cr} = 0.01548$ m).

Taking the reference spring with constant α_0 as a basic structure, we can find what change in the force p magnitude causes the increment of $dc_0/c_0 = -0.1328$ and the increment of $dH_0/H_0 = 0.0076$ when the spring is subject to the same reduction of length f, ($df = 0$). In this case $dp/p = -0.1414$ what means that the reference spring, which has the same parameters as the optimal one, loses 14.16% of its critical force (for the same f). The new recalculated critical force $p_{cr} = 0.02337$ and we should compare it with the critical force obtained for the optimal spring ($p_{cr} = 0.03928$). It gives the hipher critical force of over 68%.

Similarly, we can find the influence of parameters c_0 and H_0 deviation on the critical reduction of the spring length f_{cr}. In this case, on the assumption that $dp = 0$, we have $df/f = 0.1264$. It means that for the reference spring of the same parameters as the optimal one, $f_{cr} = 0.01744$ m whereas for the optimal spring $f_{cr} = 0.02605$ m. It gives the improvement of about 50%.

It is also interesting to compare the theoretical results with those obtained from the experiments. The both types of results are presented in Table 2.

Table 2. Results of calculations and experiments

Type of a spring	p_{cr}		f_{cr}/H_0	
	theor.	exp.	theor.	exp.
Optimal	0.03928	0.0422	0.1507	0.1666
Reference	0.02723	0.0276	0.0902	0.0903

A good agreement between the theoretical and experimental results is observed. For the reference spring the results are practically the same. For the optimal spring the difference between the critical forces is on the level of a few per cent only.

4. Conclusions

The theoretical results of optimization of the helical springs presented by Krużelecki (1995) and (1996) look very promissing. They are confirmed by the buckling experiments. This experimental verification gives also an additional argument for introducing the optimally designed helical springs into engineering practice.

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Eksperymentalna weryfikacja stateczności optymalnych ściskanych sprężyn śrubowych

Streszczenie

W pracy przedstawiono doświadczalną weryfikację stateczności optymalnych sprężyn śrubowych. Optymalna sprężyna podparta jest swobodnie na obu końcach. Posiada ona taki rozkład kąta wzniosu, który zapewnia jej maksymalną siłę krytyczną. Wyniki teoretyczne optymalizacji pokazane w pracach Krużeleckiego (1995) i (1996) porównano z eksperymentalnymi. Doświadczenia potwierdziły uzyskane rezultaty teoretyczne.

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