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SOME 2D INTERFACE CRACK AND RIGID INCLUSION PROBLEMS IN MICROPERIODICALLY LAYERED ELASTIC COMPOSITES

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This paper presents a comparative analysis of stress and displacement plane fields due to the presence of an interface Griffith crack and a ribbon absolutely rigid inclusion in a microperiodic laminated medium consisting of alternating layers of two homogeneous, isotropic and linear-elastic materials. The study is based on the approximate treatment by using the homogenized model with microlocal paramaters (cf Woźniak, 1987). Useful solutions with the standard (non-oscillatory) inverse square-root singularity are obtained and the stress intensity factors are defined as local failure parameters. Some illustrative examples are also given.

Key words: interface crack, rigid inclusion, layered composite, stress intensity factor

1. Introduction

The behaviour of interface defect under general loading conditions is of significant interest and practical importance in the failure analysis. The purpose of this contribution is to expand the research on interface cracks in microperiodic two-layered space given in our earlier papers (cf Kaczyński and Matysiak, 1988, 1989, 1995) to cover the case of interface inclusions. Two types of defects lying on one of the straight interfaces of layers in a microperiodic laminated medium are thoroughly considered; namely, that of a Griffith crack and a ribbon absolutely rigid inclusion. The general plane problem dealing with a thin-walled elastic inclusion of any stiffness was posed and studied by Yevtushenko et al. (1995). It was shown that, the problem is reduced into the solution of a complex system of integro-differential singular equations. The literature on the problems treated herein is extensive, and only those works which are pertinent to the present study will be cited.

In Section 2 we review briefly the governing equations of the homogenized model of layered body by using the microlocal parameter theory in the linear plane-strain static case. Such an approach has proved to be very useful in solving several types of boundary value problems for periodic elastic composites (see a comprehasive survey of papers in this field given by Matysiak, 1995).

In Section 3 the problem is formulae and a common method of constructing the solution in connection with an interface defect such as a rigid ribbon-like inclusion and a crack is outlined. The procedure follows along the same line of reasoning as that used in the homogeneous isotropic case described in the monograph by Berezhnitsky et al. (1983).

An analysis of the results aiming at assessment of the laminates strength degradation of due to the presence of considered defects is included in Section 4. Useful solutions are obtained with the standard crack-tip singularity, contrary to those with oscillatory behaviour arising from the conventional formulation (see, for example, Erdogan, 1972; Sih and Chen, 1981). Hence, the concept of stress intensity factors as the parameters characterising the local stresses and controlling the fracture instability may be applied. Some examples for illustrating main results have been given at the end of this section.

Some similar problem to that under study but considered in the homogeneous solids were discussed, for example, by Sih (1965), Atkinson (1973), Matysiak and Olesiak (1981), Wang et al. (1985), Tvardovsky (1990). Detailed information on inclusion problems was given by Mura (1982) and (1988).

2. Governing relations of the homogenized model

The composite being considered is a microperiodic laminated medium consisting of thin repeated fundamental layers of thickness δ which is composed of two bonded homogeneous isotropic layers denoted by 1 and 2 (see its middle cross-section given in Fig.1). In the following, all quantities pertaining to these sublayers will be denoted with the index l or (l) taking the values of 1 and 2, respectively. Let λ_l, μ_l be the Lamé constants and δ_l be the thicknesses of subsequent sublayers. A Cartesian coordinate system (x,y,z) is introduced such that the y-axis is normal to the layering and the x-axis coincides with one of the straight interfaces of the materials. Restricting the considerations to the plane-strain state (independent of the variable z), denote at the point $(x,y,0)$ the displacement vector by $[U(x,y),V(x,y),0]$ and the stresses by $\sigma_{yy}^{(l)}(x,y)$, $\sigma_{xy}^{(l)}(x,y)$, $\sigma_{xx}^{(l)}(x,y)$.

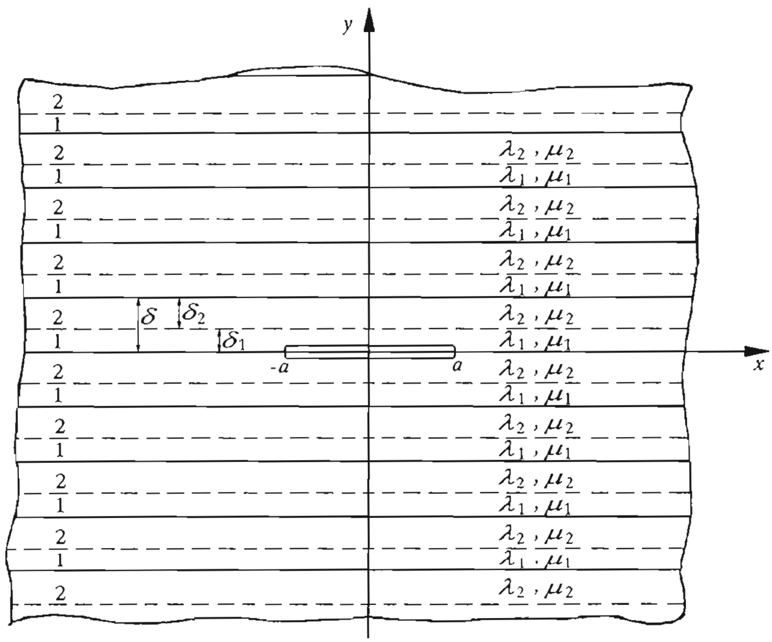

Fig. 1. Cross-section of a microperiodic composite with an interface Griffith crack

For the purpose of determining the stress and strain state in this laminated body, the homogenized model called the linear elasticity with microlocal parameters, devised by Woźniak (1987) and Matysiak and Woźniak (1988), is applied. Such an approach has proved to be useful and effective for solving a variety of boundary problems (see Matysiak, 1995). We recall below some relevant results from this theory.

The basis for microlocal modeling are some heuristic, kinematic and approximation postulates which can be written down in the following form for the stratified body under consideration

$$
\begin{aligned}
&U(x,y) = u(x,y) + h(y)p(x,y) \cong u(x,y)\\
&V(x,y) = v(x,y) + h(y)q(x,y) \cong v(x,y)\\
&\sigma_{yy}^{(l)} \cong (\lambda_l + 2\mu_l)(v_{,y} + h_{,l}\, q) + \lambda_l u_{,x}\\
&\sigma_{xx}^{(l)} \cong (\lambda_l + 2\mu_l)u_{,x} + \lambda_l(v_{,y} + h_{,l}\, q)\\
&\sigma_{xy}^{(l)} \cong \mu_l(u_{,y} + v_{,x} + h_{,l}\, p)
\end{aligned}
\tag{2.1}
$$

Here u, v and p, q are unknown functions interpreted as the macro-displacements and microlocal parameters, respectively, and h is *a priori* given δ-periodic sectionally linear shape function, defined as follows

$$
h(y) = \begin{cases} y - \frac{1}{2}\delta_1 & y \in < 0, \delta_1 > \\ \dfrac{\delta_1 - \eta y}{1 - \eta} - \frac{1}{2}\delta_1 & y \in < \delta_1, \delta > \end{cases}
\tag{2.2}
$$

where $\eta = \delta_1/\delta$.

Observe that the values of this function are small but its derivative $h'(y) = h_{,l}$ are not small, taking the values 1 for $l = 1$ and $-\eta/(1-\eta)$ for $l = 2$.

The asymptotic approach to the macro-modelling of this laminated body leads to the governing relations of certain macro-homogeneous medium (the homogenized model), given in terms of macro-displacements (after eliminating the microlocal parameters) and taking the following form (in the absence of body forces and in the static case)

$$
\begin{aligned}
&A_2 u_{,xx} + (B + C)v_{,xy} + C u_{,yy} = 0\\
&A_1 v_{,yy} + (B + C)u_{,xy} + C v_{,xx} = 0
\end{aligned}
\tag{2.3}
$$

$$
\begin{aligned}
&\sigma_{yy}^{(l)}(x,y) = B u_{,x}(x,y) + A_1 v_{,y}(x,y)\\
&{}^{(l)}_{xy}(x,y) = C\Big[u_{,y}(x,y) + v_{,x}(x,y)\Big]\\
&\sigma_{xx}^{(l)}(x,y) = D_l v_{,y}(x,y) + E_l u_{,x}(x,y)\\
&\sigma_{zz}^{(l)}(x,y) = \frac{\lambda_l}{2(\lambda_l + \mu_l)}\Big[\sigma_{xx}^{(l)}(x,y) + \sigma_{yy}^{(l)}(x,y)\Big]
\end{aligned}
\tag{2.4}
$$

in which the positive coefficients A_1, A_2, B, C, D_l, E_l, describing the material and geometric properties of the composite constituents, are given by the formulae

$$
\begin{aligned}
A_1 &= \frac{b_1 b_2}{(1-\eta)b_1 + \eta b_2} \\
A_2 &= A_1 + \frac{4\eta(1-\eta)(\mu_1-\mu_2)(\lambda_1-\lambda_2+\mu_1-\mu_2)}{(1-\eta)b_1+\eta b_2} \\
B &= \frac{(1-\eta)\lambda_2 b_1 + \eta\lambda_1 b_2}{(1-\eta)b_1+\eta b_2} \qquad C = \frac{\mu_1\mu_2}{(1-\eta)\mu_1+\eta\mu_2} \\
D_l &= \frac{\lambda_l}{b_l}A_1 \qquad E_l = \frac{4\mu_l(\lambda_l+\mu_l)+\lambda_l B}{b_l}
\end{aligned}
\tag{2.5}
$$

with $b_l = \lambda_l + 2\mu_l$, $l = 1,2$ and $\eta = \delta_1/\delta$.

3. Formulation and solution of the problem

Within the framework of the homogenized model presented in Section 2 we consider the plane boundary value problem involving an interface Griffith crack (denoted by C) or perfectly rigid line inclusion (denoted by I) occupying the region (see Fig.1)

$$
S = \{(x,y,z) : \quad -a \le x \le a \quad \wedge \quad y = 0 \quad \wedge \quad -\infty < z < \infty\} \tag{3.1}
$$

Evidently, the associated global conditions on S have to be satisfied

$$
\begin{aligned}
C &: \qquad \sigma_{yy}^{\pm} = \sigma_{xy}^{\pm} = 0 \\
I &: \qquad u^{\pm} = v^{\pm} = 0
\end{aligned}
\tag{3.2}
$$

Here and afterwards the quantities assigned with $\pm$ refer to the limiting values as $y \to 0^{\pm}$.

An efficient approach to the interface problems in question is based on the "classical" complex representation (similar to that given by Muskhelishvili, 1966) in terms of two potentials (denoted by Φ and Ω) on the x-axis (cf Kaczyński and Matysiak, 1989)

$$
\begin{aligned}
\sigma_{yy}^{\pm} - \mathrm{i}t_*\sigma_{xy}^{\pm} &= \Phi^{\pm} + \Omega^{\mp} \\
2\mu_*[u,_x^{\pm} + \mathrm{i}t_* v,_x^{\pm}] &= \kappa_*\Phi^{\pm} - \Omega^{\mp}
\end{aligned}
\tag{3.3}
$$

where

$$
t_* = \sqrt[4]{\frac{A_1}{A_2}} \qquad \mu_* = \frac{A_1 A_2 - B^2}{2(A_* - A_-)} \qquad \kappa_* = \frac{A_* + A_-}{A_* - A_-}
$$

$$
A_* = \sqrt[4]{A_1 A_2}\sqrt{\frac{(A_+ + 2C)A_-}{C}} \tag{3.4}
$$

$$
A_+ = \sqrt{A_1 A_2} + B \qquad A_- = \sqrt{A_1 A_2} - B
$$

We proceed now to the conventional approach constituting in the fact that the problem treated within the framework of homogenized model is linear, so we have the following decomposition of the stress tensor and displacements

$$
\sigma = \sigma^0 + \sigma^* \qquad (u, v) = (u^0, v^0) + (u^*, v^*) \tag{3.5}
$$

where superscripts 0 and $*$ refer to the problem of laminated composite without the crack or rigid inclusion, acted upon by an arbitrary external load and to the corresponding perturbed problem (being in the focus our attention), respectively.

Using the same technique as that developed by Berezhnitsky et al. (1983) assume first that for $|x| \leq a$

$$
\begin{aligned}
&\sigma_{yy}^{0\pm} - \mathrm{i}t_*\sigma_{xy}^{0\pm} \equiv -[P(x) \pm Q(x)] \\
&2\mu_*\Big[{u^{0\pm}}_{,x} + \mathrm{i}t_*{v^{0\pm}}_{,x}\Big] \equiv -[f'(x) \pm g'(x)]
\end{aligned} \tag{3.6}
$$

The fundamental perturbed problem reduces then to finding two single-valued, sectionally holomorphic potentials $\Phi(\cdot)$ and $\Omega(\cdot)$ satisfying, in view of Eqs (3.2) ÷ (3.6), the boundary conditions at the interface segment $(-a, a) \times \{0\}$ and at the infinity

$$
\begin{aligned}
&\Phi^\pm + \Omega^\mp = P(x) \pm Q(x) \\
&\kappa_*\Phi^\pm - \Omega^\mp = f'(x) \pm g'(x) \\
&\Phi(\infty) = \Omega(\infty) = 0
\end{aligned} \tag{3.7}
$$

The solution to the above problem involving the crack (problem C) as well as the rigid inclusion (problem I) may be written in the common form as

$$
\begin{aligned}
&\Phi(\widetilde{z}) = \frac{F(\widetilde{z}) + g}{\sqrt{\widetilde{z}^2 - a^2}} + G(\widetilde{z}) \\
&\Omega(\widetilde{z}) = -\varrho_*\Phi(\widetilde{z}) + 2\varrho_* G(\widetilde{z})
\end{aligned} \tag{3.8}
$$

where the functions (Cauchy integrals) F and G of generalized complex variable $\tilde{z}$ (for details see Kaczyński and Matysiak, 1989) and a constant g are defined in terms of the known functions $F^*(\varrho_*, t)$ and $G^*(\varrho_*, t)$ as follows

$$
\begin{aligned}
F(\tilde{z}) &= \frac{1}{4\pi\varrho_*}\int\limits_{-a}^{a}\frac{\sqrt{a^2-t^2}F^*(\varrho_*,t)}{t-\tilde{z}}\,dt \\
g &= \frac{\varrho+\varrho_*}{\varrho-\varrho_*}\int\limits_{-a}^{a}G^*(\varrho_*,t)\,dt \\
G(\tilde{z}) &= \frac{1}{4\pi i\varrho_*}\int\limits_{-a}^{a}\frac{G^*(\varrho_*,t)}{t-\tilde{z}}\,dt
\end{aligned}
\tag{3.9}
$$

provided that in the problems C and I we set

$$
\begin{array}{lll}
C: & \varrho_* = -1 & \varrho = \kappa_* \\
 & F^*(\varrho_*,t) = -2P(t) & G^*(\varrho_*,t) = -2Q(t) \\
 & & \\
I: & \varrho_* = \kappa_* & \varrho = -1 \\
 & F^*(\varrho_*,t) = 2f'(t) & G^*(\varrho_*,t) = 2g'(t)
\end{array}
\tag{3.10}
$$

The general solution obtained above for the crack and rigid line inclusion problems makes it possible to characterize the local field behaviour in the vicinity of the tips $a^{\pm}$ as well as to determine the stresses and displacements throughout the periodically layered composite.

4. Asymptotic analysis

A knowledge of stress field in the neighbourhood of the crack (inclusion) tip is essential in the failure analysis. Owing to the representation given by Eqs (3.3) the solutions to the interface problems within the framework of the proposed homogenized model are closely related to those for homogeneous isotropic bodies. It can be easily seen that, Eqs (3.8) show the classical stress singularities having the usual square-root form in contrast to these oscillatory ones existing in the interface problems. Thus, the magnitudes of the local stresses may be determined in terms of some parameters known as the stress intensity factors (SIF). For the purpose of obtaining expressions for these parameters, we follow the procedure outlined by Berezhnitsky at al. (1983) consisting in examining the singular parts of the local stresses induced by the potentials given by Eqs (3.9). The asymptotic form of the solution in the small vicinity of the tips $a^{\pm}$ on the x-axis is found to be (see the principle established by the conditions (3.10))

$$\begin{aligned}
&\begin{bmatrix} \sigma_{yy}(x,0) \\ \sigma_{xy}(x,0) \end{bmatrix} = \frac{\varrho_* - 1}{2\varrho_*} \begin{bmatrix} k_I^{\pm} \\ k_{II}^{\pm} \end{bmatrix} \frac{1}{\sqrt{2r}} + O(r^0) \qquad x = \pm a \pm r \qquad 0 < r \ll a \\
&\sigma_{xx}(x,0^{\pm}) = -c^{\pm} \frac{3 + \varrho_*}{2\varrho_*} \frac{k_I^{\pm}}{\sqrt{2r}} + O(r^0) \qquad x = \pm a \pm r \qquad 0 < r \ll a \qquad (4.1) \\
&\begin{bmatrix} u(x,0) \\ v(x,0) \end{bmatrix} = \frac{\varrho_* - \kappa_*}{2\mu_*\varrho_*} \begin{bmatrix} k_{II}^{\pm} \\ k_I^{\pm} \end{bmatrix} \sqrt{\frac{r}{2}} + O(r^{3/2}) \qquad x = \pm a \mp r \qquad 0 < r \ll a
\end{aligned}$$

where

$$c^+ = c^{(1)} \qquad c^- = c^{(2)} \qquad c^{(l)} = 1 + \frac{2\mu_l(2\lambda_l + 2\mu_l - A_+)}{(\lambda_l + 2\mu_l)A_+} \qquad (4.2)$$

and the SIFs $k_I^{\pm}$, $k_{II}^{\pm}$ (superscripts "-" and "+" refer to the left and right-hand crack (inclusion) tips, respectively) are defined by

$$k_I^{\pm} - \mathrm{i}t_* k_{II}^{\pm} = \frac{1}{2\pi\sqrt{a}} \Big[\int\limits_{-a}^{a} \sqrt{\frac{a \pm t}{a \mp t}} F^*(\varrho_*, t)\, dt \pm \mathrm{i} \frac{\varrho + \varrho_*}{\varrho - \varrho_*} \int\limits_{-a}^{a} G^*(\varrho_*, t)\, dt \Big] \qquad (4.3)$$

It is observed that the character of the above asymptotic expressions is similar to that obtained in the homogeneous isotropic case. Intensification of the local stresses is fully characterized by two parameters k_I, k_{II} determined by Eqs (4.3) (in the crack theory called the stress intensity factors and connected with the corresponding modes of crack extension). However, as it was shown in our paper (cf Kaczyński and Matysiak, 1988), the asymptotic angular distribution is different. The influence of the layering is seen in the dependence of local crack displacements and inclusion stresses on the parameter κ_* pertinent to the composite structure as indicated in Eqs (3.4). Finally, we may compare the obtained solution with the solution for a homogeneous isotropic body (characterized by the Lamé coefficients λ, μ and the Poisson ratio $\nu = (\lambda + 3\mu)/(\lambda + \mu)$) in which

$$\begin{array}{lll}
\lambda_1 = \lambda_2 = \lambda & \mu_1 = \mu_2 = \mu & \\
A_1 = A_2 = \lambda + 2\mu & B = \lambda & C = \mu \\
D_l = \lambda & E_l = \lambda + 2\mu & \\
t_* = 1 & \mu_* = mu & \kappa_* = \nu
\end{array} \qquad (4.4)$$

Comprehensive investigation of the interaction between rigid line inclusions and cracks in a homogeneous isotropic body may be found in the aforementioned monograph by Berezhnitsky et al. (1983); accurate or approximate values of the stress intensity factors for a lot of different configurations are also listed. Owing to the use of the same technique it becomes possible to obtain and compare the solutions to the corresponding problems involving the periodically layered composite under study.

As an illustration of the results obtained above we consider two simple examples of the interface problems in question for a given external loading. Attention will be focused on determination of the stress intensity factors.

4.1. Uniform tension in the y-direction

Assuming that $\sigma_{yy}(\infty) = p$, $\sigma_{xy}(\infty) = 0$ and bearing in mind Eqs (3.5) and (3.6), we obtain in Eqs (3.10)

$$
\begin{aligned}
&G^*(\varrho_*, t) = 0 \\
&F^*(\varrho_*, t) = \begin{cases} 2p & C \quad (\varrho_* = -1) \\ \dfrac{2B}{A_* - A_-} p & I \quad (\varrho_* = \kappa_*) \end{cases}
\end{aligned}
\tag{4.5}
$$

and after integration (see Eqs (4.3)) we get

$$
\begin{aligned}
&k_I^{\pm} = \begin{cases} p\sqrt{a} & C \\ \dfrac{Bp\sqrt{a}}{A_* - A_-} & I \end{cases} \\
&k_{II}^{\pm} = 0
\end{aligned}
\tag{4.6}
$$

4.2. Special system of concentrated forces

For the sake of simplicity we take only the case of external loads in the form of a system of two equal and opposite vertical concentrated forces $(0, Y_0)$ and $(0, -Y_0)$ applied at the points $(x_0, 0)$ and $(-x_0, 0)$, respectively, $(x_0 > a)$ into consideration (for detailed analysis of the stress intensity factors affected by arbitrary concentrated body forces, see Kaczyński and Matysiak, 1995).

Proceeding as in the previous case and making use of the results given in our paper, Eqs (3.10) yield

$$
\begin{aligned}
&G^*(\varrho_*,t) = 0 \\
&F^*(\varrho_*,t) = \begin{cases} \dfrac{2}{\pi}\dfrac{t_* C_* x_0 Y_0}{t^2 - x_0^2}\mathrm{i} & C \quad (\varrho_* = -1) \\ \dfrac{2}{\pi}\dfrac{t_* \mu_* D_* x_0 Y_0}{t^2 - x_0^2}\mathrm{i} & I \quad (\varrho_* = \kappa_*) \end{cases}
\end{aligned}
\tag{4.7}
$$

and the expressions for SIFs are given by

$$
\begin{aligned}
&k_I^{\pm} = 0 \\
&k_{II}^{\pm} = \begin{cases} \dfrac{C_* Y_0}{\pi\sqrt{a}}\dfrac{1}{\sqrt{d^2-1}} & C \\ \dfrac{D_* Y_0}{\pi\sqrt{a}}\dfrac{1}{\sqrt{d^2-1}} & I \end{cases}
\end{aligned}
\tag{4.8}
$$

where

$$
\begin{aligned}
&d = \frac{x_0}{a} \qquad\qquad C_* = \sqrt{\frac{A_- C}{A_1(A_1+2C)}} \\
&D_* = (\sqrt{A_1A_2} + C)\sqrt{\frac{C}{A_1A_-(A_+ + 2C)}}
\end{aligned}
\tag{4.9}
$$

Passing to the homogeneous isotropic body the results obtained coincide with those presented by Berezhnitsky et al. (1983).

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Pewne dwuwymiarowe zagadnienia szczelin i sztywnych inkluzji międzywarstwowych w mikroperiodycznych kompozytach sprężystych

Streszczenie

Praca przedstawia analizę porównującą płaski stan naprężenia i przemieszczenia wywołany istnieniem międzywarstwowej szczeliny Griffitha i doskonale sztywnej inkluzji w mikroperiodycznym ośrodku z powtarzającą się warstwą zbudowaną z dwóch jednorodnych i izotropowych materiałów sprężystych. Oparto się na przybliżonym podejściu z zastosowaniem zhomogenizowanego modelu z parametrami lokalnymi (por.

Woźniak, 1987). Otrzymano użyteczne rozwiązania ze standardową (nieoscylującą) osobliwością naprężeń i zdefiniowano lokalne parametry zwane współczynnikami intensywności naprężeń, odgrywające rolę przy analizie pękania. Podano również przykłady ilustrujące otrzymane wyniki.

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