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RECENT ADVANCES IN THE BOUNDARY ELEMENT METHOD IN POLAND

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This paper is a short survey of the recent advances in the boundary element method in Poland. Attention is focused on some problems of computational mechanics where the contribution of Polish researchers to the development of the boundary element method is leading and substantial. A list of over 280 references is included.

Key words: the boundary element method, boundary integral equations

1. Introduction

The boundary element method (BEM) is a well established computer method which is rapidly gaining more acceptance within the engineering profession. It is treated as an alternative to the much-developed finite element method (FEM). The aspects of the BEM which lead to its widespread engineering application are: (i) reduced modelling requirements for the surface of the body, (ii) reduced problem size for comparable accuracy to be attained and (iii) the potential for substantial gains in accuracy when compared to the FEM.

The basic idea of the BEM is to transform a differential description, usually given in terms of partial differential equations, into a corresponding integral description of the boundary effects, in the form of boundary integral equations (BIEs). This integral description leads to a formulation of the problem on a lower dimensional level. Therefore, only the boundary needs to be discretized. A direct consequence of this a substantial reduction of the set of algebraic equations, to a few degree of freedom only. This method is also developed in

Poland and a lot of scientific works in the form of papers, books, Ph.D. theses and D.Sc. (habilitation) have been worked out by Polish researchers.

The issues covered in these works concerning the most significant problems in applied mechanics include:

- Heat transfer problems
- Treatment of domain integrals
- Uncertain (stochastic and fuzzy) problems
- Radiation problems
- Sensitivity analysis and optimization
- Inverse problems
- Other problems (non-linear problems, dynamics, viscoelastic and themoelastic problems, contact problems, fracture mechanics, aerodynamics, acoustics, numerical and computational aspects, coupling with other methods).

The paper aims at giving a brief review of several important areas which have been developed by Polish researchers.

2. Brief review of the BEM

The main feature of the BEM is the fact that is not based on a differential problem description (e.g. Navier's equation in the case of elasticity) but on an integral problem formulation transformed to the boundary. This boundary integral formulation can be deduced in different ways:

- Green's third identity
- Betti's reciprocal work theorem
- weighted residual method.

The last way of derivation gives a better insight into the approximative character of the BEM and permits a straightforward extension to more complex differential equations. Consider the Navier's equation of elastic equilibrium given in terms of the shear modulus μ, Poisson ratio ν, the displacement vector $\boldsymbol{u}$ and a body force vector $\boldsymbol{b}$

$$\mathbf{L}\boldsymbol{u} + \boldsymbol{b}(\boldsymbol{x}) = \mathbf{0} \qquad \boldsymbol{x} \in \Omega \tag{2.1}$$

with the boundary conditions

$$\begin{aligned} \boldsymbol{u}(\boldsymbol{x}) &= \overline{\boldsymbol{u}}(\boldsymbol{x}) \qquad \boldsymbol{x} \in \Gamma_u \\ \boldsymbol{p}(\boldsymbol{x}) &= \overline{\boldsymbol{p}}(\boldsymbol{x}) \qquad \boldsymbol{x} \in \Gamma_p \end{aligned} \tag{2.2}$$

where

$$\mathbf{L} = \mu\nabla^2(\cdot) + \frac{\mu\nabla(\cdot)}{1-2\nu} \qquad \Gamma \equiv \partial\Omega = \Gamma_u \cup \Gamma_p \qquad \Gamma_u \cap \Gamma_p = \emptyset \tag{2.3}$$

An approximate solution gives rise to the errors appearing Eqs (2.1) and (2.2). These errors can be minimized by writing the following weighted residual statement

$$\int\limits_{\Omega} \mathbf{L}\boldsymbol{u}\mathbf{U}^* \, d\Omega = \int\limits_{\Gamma} (\overline{\boldsymbol{u}} - \boldsymbol{u})\mathbf{P}^* \, d\Gamma + \int\limits_{\Gamma} (\boldsymbol{p} - \overline{\boldsymbol{p}})\mathbf{U}^* \, d\Gamma \tag{2.4}$$

where the displacement field $\mathbf{U}^*$, corresponding to a weighting field, is the fundamental solution

$$\mathbf{L}\mathbf{U}^* + \delta\mathbf{I} = \mathbf{0} \tag{2.5}$$

and the tractions $\mathbf{P}^*$ and $\boldsymbol{p}$ are the boundary stresses on the boundary Γ corresponding to the displacement fields $\mathbf{U}^*$ and $\boldsymbol{p}$.

After integrating by parts and taking the limit $\boldsymbol{x} - \Gamma$, finally, the boundary integral equation is obtained

$$\boldsymbol{c}\boldsymbol{u} + \int\limits_{\Gamma} \mathbf{P}^*\boldsymbol{u} \, d\Gamma = \int\limits_{\Gamma} \mathbf{U}^*\boldsymbol{p} \, d\Gamma + \int\limits_{\Omega} \mathbf{U}^*\boldsymbol{b} \, d\Omega \tag{2.6}$$

where $\boldsymbol{c}$ depends on a local geometry of the boundary Γ and is $1/2\mathbf{I}$ for smooth boundaries.

Eq (2.6) is the boundary integral equation (BIE) which constraints the boundary traction and displacement solution to the boundary value-problem, Eqs (2.1) and (2.2).

For the numerical solution of Eq (2.6) the boundary surface Γ is discretized into a number of boundary elements. Thus, Eq (2.6) after discretization, nodal collocation and separation of the unknown $\mathbf{X}$ from the known $\mathbf{Y}$ nodal quantities (displacements and tractions) takes the matrix form

$$\mathbf{AX} = \mathbf{BY} \tag{2.7}$$

where the influence matrices $\mathbf{A}$ and $\mathbf{B}$ consist of integrals over various boundary elements with integrands the fundamental tensors $\mathbf{U}^*$ and $\mathbf{P}^*$ multiplied by the spatial shape functions and the Jacobian between global and local coordinates.

The stress at an arbitrary point $\boldsymbol{x} \in \Omega$ is given by

$$\sigma = \int_{\Gamma} \mathbf{S}^* \boldsymbol{u} \, d\Gamma + \int_{\Gamma} \mathbf{D}^* \boldsymbol{p} \, d\Gamma + \int_{\Omega} \mathbf{D}^* \boldsymbol{b} \, d\Omega \tag{2.8}$$

where $\mathbf{D}^*$ and $\mathbf{S}^*$ are given by the appropriate derivatives of $\mathbf{U}^*$ and $\mathbf{P}^*$ and can be calculated numerically after solving Eq (2.7).

Fundamentals of the BEM and its applications Eq (2.7) various fields of engineering mechanics are presented in [79,107].

3. Heat transfer problems

The problems of the BEM application to numerical modelling of steady and non-steady diffusion have been developed in Poland very extensively by several researchers but the main contribution has been made by R. Białecki, E. Majchrzak, B. Mochnacki and A.J. Nowak.

A typical partial differential equation describing a heat transfer processes proceeding in the domain Ω is of the form

$$x \in \Omega: \quad c(T)\Big[\frac{\partial T(x,t)}{\partial t} + \boldsymbol{w} \cdot \mathrm{grad}T(x,t)\Big] = \mathrm{div}\Big[\lambda(T)\mathrm{grad}T(x,t)\Big] + q_V(x,t) \tag{3.1}$$

where

c, λ	–	thermophysical parameters of the domain Ω (specific heat per unit volume and thermal conductivity)
$\boldsymbol{w}$	–	velocity field (in the case of heat conduction as a rule $\boldsymbol{w} = \mathbf{0}$)
q_V	–	source function
T, x, t	–	temperature, spatial co-ordinate and time, respectively.

In the case of constant parameters one obtains the following simpler form of the above equation

$$x \in \Omega: \quad c\Big[\frac{\partial T(x,t)}{\partial t} + \boldsymbol{w}\cdot \mathrm{grad}T(x,t)\Big] = \lambda \mathrm{div}\Big[\mathrm{grad}T(x,t)\Big] + q_V(x,t) \quad (3.2)$$

The energy equation is supplemented by a boundary condition in the general form

$$x \in \Gamma: \quad \Phi\Big[T(x,t), \boldsymbol{n}\cdot \mathrm{grad}T(x,t)\Big] = 0 \quad (3.3)$$

where $\boldsymbol{n}\cdot\mathrm{grad}T$ is the normal derivative. In particular, the Dirichlet, Neumann or Robin conditions can be taken into account.

The initial condition $T(x,0) = T_0(x)$ is also given.

A typical approach to the problem formulated (assuming the constant values of thermophysical parameters and $\boldsymbol{w} = \boldsymbol{0}$) consists in the application of the weighted residual method criterion, i.e.

$$\int_0^{t^F}\int_\Omega \Big[a\mathrm{div}[\mathrm{grad}T(x,t)] - \frac{\partial T(x,t)}{\partial t} + \frac{q_V(x,t)}{c}\Big] T^*(\xi,x,t^F,t)\, d\Omega dt = 0 \quad (3.4)$$

where

a	–	diffusion coefficient, $a = \lambda/c$
$[0, t^F]$	–	time interval considered
ξ	–	point at which the concentrated heat source is applied
T^*	–	fundamental solution.

For the domain Ω oriented in a rectangular co-ordinate system T^* is a function of the form

$$T^*(\xi,x,t^F,t) = \frac{1}{[4\pi a(t^F-t)]^{d/2}} \exp\Big[-\frac{r^2}{4a(t^F-t)}\Big] \quad (3.5)$$

where r is the distance between points ξ and x, d is the dimension of the problem (1D, 2D or 3D).

In the stage of numerical realization, at first, the time grid must be introduced

$$0 = t^0 < t^1 < \ldots < t^{f-1} < t^f < \ldots < t^F < \infty$$

and at this stage two approaches can be taken into account.

The basic idea of the so-called 1st scheme of the BEM consists in the 'step by step' integration with respect to time and then the boundary integral equation resulting from the weighted residual method criterion can be written in the form

$$
\begin{aligned}
&B(\xi)T(\xi,t^f)+\frac{1}{c}\int\limits_{t^{f-1}}^{t^f}\int\limits_{\Gamma}T^*(\xi,x,t^f,t)q(x,t)\,d\Gamma dt=\\
&=\frac{1}{c}\int\limits_{t^{f-1}}^{t^f}\int\limits_{\Gamma}Q^*(\xi,x,t^f,t)T(x,t)\,d\Gamma dt+\\
&+\int\limits_{\Omega}T^*(\xi,x,t^f,t^{f-1})T(x,t^{f-1})\,d\Omega+\frac{1}{c}\int\limits_{t^{f-1}}^{t^f}\int\limits_{\Omega}T^*(\xi,x,t^f,t)q_V)\,d\Omega dt
\end{aligned}
\tag{3.6}
$$

where $q=-\lambda\boldsymbol{n}\cdot\mathrm{grad}T$, $Q^*=-\lambda\boldsymbol{n}\cdot\mathrm{grad}T^*$, $B(\xi)\in(0,1)$.

This equation constitutes a basis for construction of a numerical algorithm (boundary and interior discretization, numerical integration, etc.).

In the case of the 2nd scheme of the BEM the integration process starts from $t=0$ and then the knowledge of successive pseudo-initial conditions is needless, but temporary values of boundary temperatures and heat fluxes for $t=t^0$, $t=t^1$, ... , $t=t^{f-1}$ must be 'registered'.

Within the scope of problems discussed above the following new results have been obtained.

- Numerical modelling of heat diffusion for different forms of the source function [204,263].
- Approximate solution of energy equation with substantial derivative ($\boldsymbol{w}\neq\boldsymbol{0}$) [203,225,226].
- Non-linear problems (non-linear material and non-linear boundary conditions [39,40,42,232].
- The methods of energy equation linearization [216,231]. It should be pointed out that the fundamental solution is known only for the case of constant thermophysical parameters and in order to use the BEM for numerical modelling of non-steady and non-linear thermal diffusion problems certain additional procedures (at a stage of numerical computations) must be introduced. The new algorithms has been worked out which supplement the basic BEM algorithm and allow one to take into account nonlinearities in the differential equation, namely, the temperature field correction method [200], the generalized alternating phase truncation method [212,227], the artificial heat source method [228].

- Application of Green's function [43,44,46].
- Application of the BEM algorithm to the domain oriented in Cartesian co-ordinate system in the case of domains oriented in cylindrical or spherical co-ordinate systems [217].
- Application of the combined BEM-FEM algorithm to numerical modelling of diffusion problems [205,206,214,224].
- Non-homogeneous domains and composition of the 1st and the 2nd schemes of the BEM [201,202,213,215].

The other approach to the non-steady diffusion problems consists in the approximation of time derivative appearing in the energy equation by an corresponding differential quotient. So, the following form of the weighted residual method criterion is considered

$$\int_{\Omega} \Big\{ \lambda \mathrm{div}[\mathrm{grad} T(x,t^f)] - c\frac{T(x,t^f) - T(x,t^{f-1})}{\Delta t} + q_V(x,t) \Big\} T^*(\xi,x)\, d\Omega = 0 \tag{3.7}$$

where $T^*(\xi,x)$ for 1D, 2D and 3D problems can be found in [200]. This method is called the BEM using discretization in time. The basic idea of the algorithm is known, while the research works deal with application of the method to numerical simulation of different technical problems described by more complex mathematical models [209,210,223].

3.1. BEM in modelling of technological processes

The papers worked out in this area are devoted, first of all, to the foundry and casting problems. The solidification process proceeding in the system casting-mould belongs to the group of so-called moving-boundary problems. In the case of pure metal the kinetics of solidification is determined by the well known Stefan condition, while the solidification of alloys proceeds in the temperature interval, and then the mushy zone sub-region must be taken into account. The numerical algorithm using the BEM for the Stefan problem solution was presented, among others, in [200,202]. Considering the mushy zone problems a mathematical model called the 'fixed domain method' was very often applied [200÷202,209,228,230]. InWithin the scope of the problems discussed a numerous numerical algorithms have been worked out and the review of them was presented in [215]. The Stefan problem and also mushy zone problem belong to the group of so-called first generation models of solidification.

The second generation models take into account the crystallization process in a microscopic scale. The laws determining the nucleation and grains growth are introduced into the mathematical description of the process and the source function in relevant differential equation is constructed on the basis of 'microscopic' considerations. The results obtained in this field are presented in [208,219÷222]. Casting problems were considered in [23÷27].

3.2. Numerical modelling of bio-heat transfer

The heat transfer processes proceeding in a biological tissues are described by the energy equation in which the source function is due to metabolic and perfusion processes. The tissue can be subjected to an external influences, for instance low or high temperature and these problems are most essential from the practical point of view. So, one of the problems which can be considered consists in numerical modelling of freezing processes (effect of low temperature), while the other is connected with the action of high temperature (burns). The team worked out the numerical algorithms (on the basis of the BEM) both in the case of freezing of tissue simulation [211,218] and also for the prediction of burn grade [207,229].

A few doctoral theses have been done, namely [200,224,232,263] – the above works deal with the theory and application of the BEM to numerical modelling of different technical problems associated with heat and mass transfer processes.

4. Radiation problems

As the governing equation of radiation is an integral one, the idea of using the BEM for solving the radiation problems naturally arises and was proposed by R. Białecki. Application of the BEM to solution of heat radiation problems has been developed in papers [5,7÷11,14÷17,27,28,33,37,40,44] and the books [13,29]. The BEM proved to be efficient and easy to implement.

The BEM formulation of the integral equation of heat radiation leads to the following equation

$$
\begin{aligned}
&q_V^r(\boldsymbol{p}) + 4a(\boldsymbol{p})e_p[T^m(\boldsymbol{p})] = \\
&= a(\boldsymbol{p}) \int_\Gamma \Big\{ e_b[T(\boldsymbol{r})] + \frac{1-\varepsilon(\boldsymbol{r})}{\varepsilon(\boldsymbol{r})} q^r(\boldsymbol{r}) \Big\} \tau(\boldsymbol{r},\boldsymbol{p}) K_r(\boldsymbol{r},\boldsymbol{p})\, d\Gamma(\boldsymbol{r}) + \qquad (4.1)\\
&+ a(\boldsymbol{p}) \int_\Gamma \Big\{ \int_{L_{rp}} a(\boldsymbol{r}') e_b[T^m(\boldsymbol{r}')] \tau(\boldsymbol{r}',\boldsymbol{p})\, dL_{rp}(\boldsymbol{r}') \Big\} K_r(\boldsymbol{r},\boldsymbol{p})\, d\Gamma(\boldsymbol{r})
\end{aligned}
$$

where

a_V^r	–	radiative heat source
a	–	absorption coefficient
$e_b[T^m]$	–	emissive power (temperature) of the medium filling the enclosure
q^r	–	radiative heat flux on the walls of the enclosure
τ	–	transmissivity
ε	–	emissivity
dL_{rp}	–	infinitesimal path along the line sight.

The kernel K_r exhibits singular behaviour as the distance between the observation point $\boldsymbol{p}$ and the current point $\boldsymbol{r}$ tends to zero.

Eq (4.1) is not an integral equation with respect to the radiative heat source q_V^r. Once the temperatures of both the medium and the bounding surface, as well as the radiative heat fluxes are known, the radiative heat source can be computed explicitly by carrying out appropriate integrations. The BEM approach of solving the heat radiation problems can be interpreted as a development and improvement of the known classic Hottel's zoning method. The superior numerical behaviour of the BEM is attributed mainly to two features: (i) conversion of the volume integrals into surface ones and (ii) using well established discretization techniques as used other applications of the BEM.

5. Treatment of domain integrals

The BEM formulations of the boundary-value problems with occurring body forces $\boldsymbol{b}(\boldsymbol{x})$, $\boldsymbol{x} \in \Omega$, internal sources $q_V(\boldsymbol{x})$, $\boldsymbol{x} \in \Omega$, or nonhomogeneous conditions contain both boundary and domain integrals. Domain integrals not only detract from the elegance of formulation but first of all affect numerical efficiency. This is why a substantial amount of research has been carried out in order to convert the domain integrals occurring in BEM equations into the boundary integrals. Several methods have been proposed so far and some of them have been originated and developed by A.J. Nowak.

The problem of treatment of domain integrals can be simply explained for the Poisson equation

$$\nabla^2 u + \frac{1}{k} q_v(\boldsymbol{x}) = 0 \qquad \boldsymbol{x} \in \Omega \tag{5.1}$$

where for a steady-state heat conduction u is temperature, k is the thermal conductivity and q_V is a known function which describes the internal heat source.

Eq (5.1) is equivalent to the boundary integral formulation

$$kcu + \int\limits_{\Gamma} Q^* u \, d\Gamma = \int\limits_{\Gamma} U^* q \, d\Gamma - \int\limits_{\Omega} U^* q_V \, d\Omega \tag{5.2}$$

where the fundamental solution U^* satisfies the following equation

$$\nabla^2 U^* = \delta \tag{5.3}$$

where

$$q = -k\frac{\partial u}{\partial n} \qquad\qquad Q^* = -k\frac{\partial U^*}{\partial n}$$

The domain integral in Eq (5.2)

$$D = \int\limits_{\Omega} U^* b \, d\Omega \tag{5.4}$$

causes that discretization of this equation can not be restricted to the boundary Γ only.

The transformation methods which have been proposed so far fall into one of the following main group:

- Methods related to particular solutions
- Methods related to the Galerkin approaches.

Particular solution satisfies Eq (5.1) and the body forces can be expressed in terms of a particular Laplacian solution

$$b = -k\nabla^2 \widehat{u} \tag{5.5}$$

and the domain integral after integrating by parts takes the form

$$D = -kc\hat{u} + \int_{\Gamma} (U^*\hat{q} - Q^*\hat{u})\, d\Gamma \tag{5.6}$$

In the case when a particular solution is not known, the following global interpolation form of the body force can be proposed

$$b = \frac{1}{k} q_V = \sum_j f_j \alpha_j \tag{5.7}$$

where f_j is are arbitrary approximating functions and α_j are unknown coefficients.

This approach is widely known as the Dual Reciprocity Method (DRM).

For given f_j a particular solution $\hat{u}$ is received from

$$\nabla^2 \hat{u} = f_j \tag{5.8}$$

Finally, BIE (5.2) takes the form

$$kcu + \int_{\Gamma} Q^* u\, d\Gamma = \int_{\Gamma} U^* q\, d\Gamma + \sum_j \Big[\int_{\Gamma} (U^*\hat{q}_j - Q^*\hat{u}_j)\, d\Gamma - kc\hat{u}_j \Big] \alpha_j \tag{5.9}$$

The DRM has been used for solution of incompressible fluid flow problems [49,277,278,], dynamic problems [114,150,170].

The second group of methods is related to the Galerkin approach. In this case the domain integral can be expressed as follows

$$D = \int_{\Omega} V^* b\, d\Omega = \int_{\Gamma} \Big(b\frac{\partial U^*}{\partial n} - \frac{\partial b}{\partial n} \Big)\, d\Gamma \tag{5.10}$$

where $\nabla^2 V^* = U^*$.

The Multiple Reciprocity Method (MRM) developed by A.J.Nowak generalizes this concept by introducing a set of the so-called higher order fundamental solutions

$$\begin{aligned} \nabla^2 U^*_{j+1} &= U^*_j \qquad j = 0, 1, 2, \ldots \\ Q^*_j &= -k\frac{\partial U^*_j}{\partial n} \end{aligned} \tag{5.11}$$

as well as a sequence of the source function Laplacians

$$\nabla^2 b_j = b_{j+1} \qquad j = 0, 1, 2, ... \qquad\qquad (5.12)$$
$$w_j = -k \frac{\partial b_j}{\partial n}$$

As a result this approach leads to the exact boundary-only formulation of the problem

$$D = \frac{1}{k} \sum_j \int_\Gamma (Q^*_{j+1} b_j - U^*_{j+1} w_j)\, d\Gamma \qquad (5.13)$$

where $b_0 = b$ and $U^*_0 = U^*$.

The detailed discussion about the MRM can be found in books [246,256,257] and papers [48,49,233÷245,247÷249,253÷255,258].

6. Uncertain (stochastic and fuzzy) problems

The nature of uncertainty can be discussed under assumptions: stochastic uncertainty and fuzzy uncertainty. The prediction of these types of uncertainty is difficult and present methods tend to concentrate on random uncertainty. There is, however, a fundamental difference between the natures of stochastic uncertainty and fuzzy uncertainty.

The Stochastic Boundary Element Method (SBEM) is a computer method which account for stochastic uncertainties in boundary conditions, material properties and geometry of the boundary. But if the underlying structure is not probabilistic, e.g. because of subjective choices, then it may be appropriate to use fuzzy numbers instead of real random variables. This leads to fuzzy boundary-value problems and in consequence to the Fuzzy Boundary Element Method (FBEM).

Stochastic and fuzzy problems are, as a general rule, need more fire-consuming computation and burdensome than deterministic problems. The SBEM and FBEM, which reduce the size of the problem by one and require solution only for stochastic or fuzzy boundary variables, appear promising in various uncertain problems and belong to the rapidly advancing fields of computational mechanics.

The application of the BEM to stochastic problems was initiated by Burczyński for stochastic boundary-value problems of elastostatics [51]. Later the SBEM was used to stochastic potential problems [56], stochastic heat conduction problems [147,148] and dynamical problems [54,55,57,60,69,71,76,77].

The SBEM was also extended to the problems with random media [59,71], stochastic boundaries and sensitivity analysis and identification [58,61,63,66,72,74,75,78,80,81,84].

When stochastic boundary conditions

$$\begin{aligned} \boldsymbol{u}(x) &= \overline{u}(x,\gamma) \qquad \boldsymbol{x} \in \Gamma_u \\ \boldsymbol{p}(x) &= \overline{\boldsymbol{p}}(x,\gamma) \qquad \boldsymbol{x} \in \Gamma_p \end{aligned} \tag{6.1}$$

are taken into account then the following stochastic BIE is obtained

$$\boldsymbol{c}\boldsymbol{u}(x,\gamma) + \int_\Gamma \mathsf{P}^*(x,\boldsymbol{y})u(y,\gamma)\, d\Gamma(y) = \int_\Gamma \mathsf{U}^*(x,y)\boldsymbol{p}(\boldsymbol{y},\gamma)\, d\Gamma(y) \tag{6.2}$$

where γ is an elementary event.

After discretization of the problem the covariance matrix of unknown stochastic displacements and tractions is expressed by

$$\mathsf{K}_X = \mathsf{A}^{-1}\mathsf{B}\mathsf{K}_Y\mathsf{B}^\top(\mathsf{A}^{-1})^\top \tag{6.3}$$

where K_Y is the covariance matrix of given boundary conditions.

In the case of dynamical problems it is convenient to use spectral densities and the BIE's in the Fourier transform domain.

For random media the stochastic elastic moduli can be expressed in the form

$$\mathsf{C}(x,\gamma) = \mathsf{C}^0 + \widetilde{\mathsf{C}}(x,\gamma) \tag{6.4}$$

and a stochastic BIE takes the form

$$\begin{aligned} \boldsymbol{c}\boldsymbol{u}(x,\gamma) &+ \int_\Gamma \mathsf{P}^*(x,\boldsymbol{y})u(y)\, d\Gamma(y) = \\ &= \int_\Gamma \mathsf{U}^*(x,y)\boldsymbol{p}(y)\, d\Gamma(y) - \int_\Omega \mathsf{E}^*(x,y)\widetilde{\sigma}(\boldsymbol{y},\gamma)\, d\Omega(y) \end{aligned} \tag{6.5}$$

where U^*, P^* and E^* are fundamental solutions for the mean value of the elastic moduli tensor $\mathsf{C}^O = \langle \mathsf{C}(x,\gamma)\rangle$ and

$$\widetilde{\sigma}(\boldsymbol{y},\gamma) = \widetilde{\mathsf{C}}(\boldsymbol{y},\gamma)\varepsilon(\boldsymbol{y},\gamma) \qquad \boldsymbol{y} \in \Omega \tag{6.6}$$

Eq (6.6) is similar to that for deterministic problems with addition of a new stochastic term only, which depends on the elastic moduli tensor $\widetilde{\mathsf{C}}(\boldsymbol{y},\gamma)$ characterizing the random fluctuations of medium.

The possibility of application of the BEM to fuzzy problems was originated by Burczyński and Skrzypczyk [127]. They used the FBEM to potential problems [128÷132,265] and elastostatics [267] with fuzzy boundary conditions and the possibility of utilization for a fuzzy domain was also explored [266].

7. Sensitivity analysis and optimization

The shape determination of structural components plays an essential role in mechanical designing and the problem of shape sensitivity analysis and optimal design is much more complicated than a typical conventional analysis.

The BEM is an exceptionally natural and convenient numerical technique for shape sensitivity analysis and optimal design. Application of the BEM to shape optimization and sensitivity analysis was initiated by Burczyński and Adamczyk [88]. Later several original works have been elaborated by Burczyński and his co-workers [61÷64,66÷68,70,73,82,83,86,91,94÷97,100,102÷106, 116÷120,122,150,166÷169,171], Grabacki [172÷175], Krzesiński [185,186], Wilczyński [279÷281]. It is known that for any arbitrary functional, e.g. in the integral form

$$J = \int_{\Omega^*} \Psi(\sigma, \varepsilon, \boldsymbol{u})\, d\Omega + \int_{\Gamma^*} \phi(\boldsymbol{u}, \boldsymbol{p})\, d\Gamma \tag{7.1}$$

where Ψ is an arbitrary function of stresses σ, strains ε and displacements $\boldsymbol{u}$ within the domain $\Omega^* = \Omega(\boldsymbol{a})$, and ϕ is an arbitrary function of displacements $\boldsymbol{u}$ and tractions $\boldsymbol{p}$ on the boundary $\Gamma^* = \Gamma(\boldsymbol{a})$, the first derivative with respect to the shape parameters $\boldsymbol{a} = (a_r)$ can be expressed analytically

$$\begin{aligned} \frac{DJ}{Da_r} &= \int_{\Gamma} \Big[\Psi - \sigma\varepsilon^a + \boldsymbol{b}\boldsymbol{u}^a + (\phi + \boldsymbol{p}\boldsymbol{u}^a)_{,n} K\Big] n_k v_k^r \, d\Gamma + \\ &+ \int_{\Gamma_1} \Big(\frac{\partial\phi}{\partial\boldsymbol{u}} - \boldsymbol{p}^a\Big)\Big(\frac{D\boldsymbol{u}^0}{Da_r} - \boldsymbol{u}^0_{,k} v_k^r\Big)\, d\Gamma_1 + \\ &+ \int_{\Gamma_2} \Big(\frac{\partial\phi}{\partial\boldsymbol{p}} + \boldsymbol{u}^a\Big)\Big(\frac{D\boldsymbol{p}^0}{Da_r} - \boldsymbol{p}^0_{,k} v_k^r\Big)\, d\Gamma_2 + \int_{L} \|\phi + \boldsymbol{p}\boldsymbol{u}^a\| v_\nu^r \, dL \end{aligned} \tag{7.2}$$

where $\|\phi + \boldsymbol{p}\boldsymbol{u}^a\| = (\phi + \boldsymbol{p}\boldsymbol{u}^a)^+ - (\phi + \boldsymbol{p}\boldsymbol{u}^a)^-$ represents the discontinuity of $(\phi + \boldsymbol{p}\boldsymbol{u}^a)$ along the curve L, which separates two parts of the boundary Γ_u and Γ_p, $\boldsymbol{n} = [n_k]$ is the unit normal vector, K is the mean curvature of the boundary, $v_k^r = \partial g_k / \partial a_r$ is a velocity transformation field which is associated with a shape design parameter a_r and $g_k(\boldsymbol{x}) = g_k(\boldsymbol{x}, \boldsymbol{a})$ is the transformation field which modifies the shape of the boundary Γ.

The co-ordinates of boundary nodes, control points of Bezier functions or B-splines or some dimensions of the body can be chosen as shape parameters.

The analytical expression for sensitivity of the functional J depends on solutions for the primary system (PS): $\boldsymbol{u}$, ε, σ and the adjoint system (AS): $\boldsymbol{u}^a$, ε^a and σ^a. The adjoint system is an elastic body with identical configuration and physical properties as the primary system but with other boundary conditions

$$
\begin{aligned}
\boldsymbol{u}^{a0} &= -\frac{\partial \phi(\boldsymbol{u}, \boldsymbol{p})}{\partial \boldsymbol{p}} \qquad \text{on } \Gamma_u \\
\boldsymbol{p}^{a0} &= \frac{\partial \phi(\boldsymbol{u}, \boldsymbol{p})}{\partial \boldsymbol{u}} \qquad \text{on } \Gamma_p
\end{aligned}
\tag{7.3}
$$

and with initial strains ε^{ai}, stresses σ^{ai} fields and body forces $\boldsymbol{b}^a$ specified within the domain Ω

$$
\varepsilon^{ai} = \frac{\partial \Psi(\sigma, \varepsilon, \boldsymbol{u})}{\partial \sigma} \qquad \sigma^{ai} = \frac{\partial \Psi(\sigma, \varepsilon, \boldsymbol{u})}{\partial \varepsilon} \qquad \boldsymbol{b}^{ai} = \frac{\partial \Psi(\sigma, \varepsilon, \boldsymbol{u})}{\partial \boldsymbol{u}} \tag{7.4}
$$

The BIEs for primary and adjoint systems have the form

$$
\begin{aligned}
&\boldsymbol{c}(\boldsymbol{x})\boldsymbol{u}^w(\boldsymbol{x}) = \int_\Gamma \Big[\mathsf{U}^*(\boldsymbol{x}, \boldsymbol{y})\boldsymbol{p}^w(\boldsymbol{y}) - \mathsf{P}^*(\boldsymbol{x}, \boldsymbol{y})\boldsymbol{u}^w(\boldsymbol{y}) \Big] \, d\Gamma(\boldsymbol{y}) + \mathsf{B}^w(\boldsymbol{x}) \\
&w = (PS), (AS)
\end{aligned}
\tag{7.5}
$$

where B^w depends on the body forces in the case of primary system and on the initial strains ε^{ai}, stresses σ^{ai} and body forces $\boldsymbol{b}^{ai}$ in the case of adjoint system.

It is seen that sensitivities of J depend only on boundary state variables of the primary system and the adjoint system. This fact gives significant advantages in numerical calculations by means of the BEM.

The problem of shape optimal design consists in finding the optimum shape design parameters $\boldsymbol{a}_{op}$ according to a prescribed optimality criterion. The functional J, Eq (7.1), can express arbitrary objective or constraint functionals.

A well-posed optimal shape design problem stated as:

minimize an objective functional $J_0(\boldsymbol{a})$ with the behaviour constraints J_α, $\alpha = 1, 2, ..., A$ imposed, expressed in terms of stresses, strains, displacements and with upper bound of the cost of the structure J_c, that is

$$J_0(\boldsymbol{a}) \to \min_{a} \tag{7.6}$$

subjected to the constraint

$$J_\alpha - c_\alpha \leq 0 \qquad \alpha = 1, 2, ..., A \tag{7.7}$$

where c_α are given constant.

If the cost of the structure is treated as proportional to the material volume or weight, one can write

$$J_c = \int_\Omega C \, d\Omega \tag{7.8}$$

where C is a specific cost of the material.

In a typical mathematical programming application the search for the optimum shape parameters $\boldsymbol{a}_{op}$ is based on a construction of an iterative process of the type

$$\boldsymbol{a}^{(i+1)} = \boldsymbol{a}^{(i)} + \beta^{(i)} \boldsymbol{h}^{(i)} \tag{7.9}$$

where $\boldsymbol{h}^{(i)}$ is the vector determining the direction of motion from point $\boldsymbol{a}^{(i)}$ to $\boldsymbol{a}^{(i+1)}$ and $\beta^{(i)}$ is a numerical factor whose value determines the length of the step in the direction of $\boldsymbol{h}^{(i)}$.

There are several numerical optimization techniques which enable one to construct the iterative process (7.9). There are effective methods in which the vector $\boldsymbol{h}^{(i)}$ depends only on gradients of the objective and constraint functionals. In this case the sensitivity information can be directly applied.

The BEM formulation offers distinct advantages: (i) in the iterative shape optimal design process one uses only the values defined on the modified boundary, (ii) if it is necessary the boundary element mesh can easily be generated and the design changes do not require a complete remeshing.

8. Inverse problems

The inverse problems are dealing with the determination of mechanical system – with unknown material properties, geometry, sources and boundary or initial conditions – from the knowledge of the responses to given excitations on its boundary. From the mathematical point of view, such problems are ill-posed and have to be overcome by development of new computational methods, introduction of new objective functionals into optimization algorithms, new sensitivity analysis methods, new regularization techniques, new experimental procedures, etc.

The BEM is a very useful computational technique for inverse problems where one should estimate unknown quantities $\boldsymbol{a} = (a_q)$, $q = 1,2,...,Q$, through the measurements of boundary state fields $\widetilde{\boldsymbol{u}}^m$ (e.g. displacements and temperature) at the boundary points $\boldsymbol{x}^m$, $m = 1,2,...,M$, where M is the total number of sensors.

In Poland the BEM has been used to inverse problems by K.Kurpisz and A.J.Nowak (for thermal problems) and T.Burczyński with co-workers (for identification of cracks and voids).

In order to solve this problem an objective functional is constructed. This functional can represent a distance norm between the measured $\widetilde{\boldsymbol{u}}^m$ and theoretical values of the state field $\boldsymbol{u}(\boldsymbol{x}_m)$ calculated at discrete boundary points $\boldsymbol{x}_m$

$$J = \frac{1}{2}\sum_{m=1}^{M}[\boldsymbol{u}(\boldsymbol{x}_m) - \widetilde{\boldsymbol{u}}^m]^2 = \int_{mg} \varphi(\boldsymbol{u})\, d\Gamma \tag{8.1}$$

where

$$\varphi(\boldsymbol{u}) = \frac{1}{2}\sum_{m=1}^{M}[\boldsymbol{u}(\boldsymbol{x}_m) - \widetilde{\boldsymbol{u}}^m]^2\delta(\boldsymbol{x} - \boldsymbol{x}^m) \tag{8.2}$$

In order to solve this problem one should find the vector $\boldsymbol{a}$ which minimizes the objective function $J = J(\boldsymbol{a})$ given by Eq (8.1).

To have a physical meaning, on the vector $\boldsymbol{a}$ are imposed some constraints, e.g. geometric constraints which can be expressed symbolically in the form

$$C_j(a_q) \leq 0 \qquad j = 1,2,...,L \qquad q = 1,2,...,Q \tag{8.3}$$

The constraints (8.3) together with minimization of the objective function J (8.1) lead to the non-linear constrained optimization problem. For transformation of this problem into an unconstrained optimization problem, one can propose the internal penalty function method.

The inverse problem is ill-posed and its solution may not be stable since small errors appear in the experimentally measured state field. It may affect a significant difference in the computed quantities. Regularization methods can reduce numerical fluctuations in the solution by modifying the objective function. The augmented regularization terms, up to the second order terms, can be expressed in the form

$$R = \gamma_0 \sum_{q=1}^{Q} \Big[a_q^{(n)}\Big]^2 + \gamma_1 \sum_{q=1}^{Q} \Big[a_q^{(n)} - a_q^{(n-1)}\Big]^2 + \gamma_2 \sum_{q=1}^{Q} \Big[a_q^{(n)} - 2a_q^{(n-1)} + a_q^{(n-2)}\Big]^2 \tag{8.4}$$

where γ_j are the regularization parameters, and n is the iteration number.

The final augmented objective function is expressed in the form

$$\widetilde{J}(\boldsymbol{a}, r) = J(\boldsymbol{a}) + P[C_j(a_q), r] + R \tag{8.5}$$

where P denotes the penalty function and depending upon the constraints C_j as well as upon an arbitrary penalty parameter r.

The vector $\boldsymbol{a}$ of unknown quantities is calculated iteratively using Eq (7.9). In order to estimate the vector $\boldsymbol{h}^{(i)}$, sensitivity information about the augmented objective function (8.5) is needed.

The most important is to find derivatives of $J(\boldsymbol{a})$ with respect to unknown quantities $\boldsymbol{a} = (a_q)$. It may be done using the material derivative adjoint variable approach. For example for dynamical geometrical inverse problems, where unknown quantities $\boldsymbol{a} = (a_q)$ describe the shape and position of unknown boundary of the void, the derivative with respect to an arbitrary shape parameter s can be expressed by

$$\frac{DJ}{Da_q} = \int_0^T \int_S [\sigma(\boldsymbol{u})\nabla \boldsymbol{u}^a - \rho \dot{\boldsymbol{u}}\dot{\boldsymbol{u}}^a] n_k v_k^q \, dS dt \tag{8.6}$$

where S denotes the boundary of the void, ρ is the mass density.

In the case of cracks it is convenient to introduce special shape transformation in the form of: (i) translation, (ii) expansion and (iii) rotation of a neighbourhood of the crack and then the sensitivity information is expressed by path-independent integrals. Among many kinds of inverse problems, issues of sensitivity analysis and identification of voids and cracks are especially suited for the boundary element treatment and have been considered in [46,47,75,81,86,87,101,108÷112,121,124÷126]. The inverse thermal problems have been considered in books [199,251] and in papers [19,20,188÷198,250]

9. Other problems

Many other problems have been considered by Polish researchers in the field of developments and applications of the BEM.

- Non-linear problems in solid mechanics: Burczyński and Adamczyk [1,90,92,93,99], Novati and Burczyński [259], Cecot and Orkisz [133, 142÷145,260]
- Dynamic problems Burczyński [52,53], Burczyński and Adamczyk [89], Fedeliński and Burczyński [170]
- Viscoelasticity and thermoelasticity: Burczyński and John [85,113, 115,179,180]
- Contact problems: Burczyński and Adamczyk [2÷4,98], Drewniak [149]
- Fracture mechanics: Fedeliński et al. [151÷165,268], Jackiewicz [76,177]
- Aerodynamics: Sygulski [269÷276]
- Acoustic scattering problems: Karafiat [183], Karafiat et al. [146,184]
- Numerical and computational aspects: Białecki et al. [18,30,34,50,261], Cecot and Orkisz [136÷139], Karafiat [181,182], Krzesiński [178,187]
- Coupling with other methods: Cecot and Orkisz [134,135,140,141].

10. Conclusions

The boundary element method has reached a level of maturity and belongs to the numerical techniques of computational mechanics which develop very quickly. Polish researchers are very active in this field. They proposed novel ideas in development and new areas of applications of the BEM. This method is very promising in various problems of mechanics and substantial energy is being devoted to a rapid expansion of the applications. However, it is still not a widely used numerical technique. This owes largely to the analytical and algorithmic complexities of the BEM, as well as to the already widely established utility of the FEM codes.

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Ostatnie osiągnięcia w metodzie elementów brzegowych w Polsce

Streszczenie

Artykul zawiera krótki przegląd ostatnich polskich osiągnięć w dziedzinie metody elementów brzegowych. Uwagę zwrócono na problemy mechaniki komputerowej, gdzie wklad polskich mechaników w rozwój metody elementów brzegowych jest wiodący i bardzo znaczący. Załączono listę ponad 280 publikacji.