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GROUP PROPERTIES AND SIMILARITY SOLUTIONS FOR THE HEAT EQUATION WITH SEMI-EMPIRICAL TEMPERATURE[1]

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The similarity solution which is invariant under the corresponding symmetry group is obtained for fairly general coefficients in the heat equation with semi-empirical temperature. The heat equation is reduced to a nonlinear ordinary differential equation and the stability of the equilibrium manifold is considered. Numerical solutions for some kinds of coefficients are obtained. Further, on the basis of the present numerical solutions an analysis of initial boundary value problem has been considered.

1. Introduction

In the last years, the group analysis has been intensively applied to nonlinear or semi-linear evolution equations of the type

$$u_{tt} - [f(u_x, u_t, u, x, t)]_x = 0$$

in the plane (x,t). The case $f = g(u)u_x$ was considered by Ames et al. (1981) and the case $f = g(u,x)u_x$ by Torrisi and Valenti (1985). The case with $f = g(u,x,t)u_x$ was considered recently by Şuhubi and Bakkaloğlu (1991).

In this paper, we express quite a general heat equation of the hyperbolic (wave) type with semi-empirical temperature developed by Kosiński and Saxton (1993) for a one-dimentional, homogeneous rigid body by means of a closed ideal of exterior differential forms over a 5-dimensional manifold. Using

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the transport property of isovectors, the components of the isovector field are constructed for fairly general coefficients of the equation given. A similarity solution to the differential equation is an invariant solution with respect to a particular symmetry group. Therefore they can be obtained by determining the invariants of the group. In this paper the symmetry group is obtained for general forms of the source function.

2. Basic equations

The heat equation for the semi-empirical temperature β is described by a second-order partial differential equation of the form (cf Kosiński and Saxton (1993))

$$- C(\beta,\beta_t)\beta_{tt} + b\nabla\beta\nabla\beta_t + a\Delta\beta + H(\beta,\beta_t) = \rho_0\vartheta^0 r \tag{2.1}$$

The coefficients in the system (2.1) are

$$\begin{aligned} C(\beta,\beta_t) &= \rho_0\tau_0\vartheta c_V(\vartheta) \\ H(\beta,\beta_t) &= \rho_0\vartheta c_V(\vartheta) f_{0,\beta}(\beta)\beta_t \\ a = k_0\vartheta^0 & \qquad\qquad b = k_0\tau_0 \end{aligned} \tag{2.2}$$

and, according to (cf Kosiński and Saxton (1993))

$$\vartheta = \vartheta(\beta,\beta_t) = \vartheta^0 \exp\Big\{\frac{1}{\vartheta^0}[f_0(\beta) - \tau_0\beta_t]\Big\}$$

where the following 3 different forms are (cf Kosiński and Saxton (1993))

$$\begin{aligned} f_0(\beta) &= -(\beta - \beta^0) =: f_{01} \quad \text{or} \\ f_0(\beta) &= \vartheta^0 \log(\beta^0\beta^{-1}) =: f_{02} \quad \text{or} \\ f_0(\beta) &= -\frac{1}{2\vartheta^0}(\beta^2 - \beta^{02}) =: f_{03} \end{aligned} \tag{2.3}$$

Here

ϑ^0	–	reference absolute temperature
β^0	–	reference semi-empirical temperature
τ_0	–	relaxation time
k_0	–	thermal conductivity at ϑ^0
c_V	–	specific heat.

For one-dimensional thermal waves the system (2.1) reduces to

$$- C(\beta,w)w_t + bpw_x + ap_x + H(\beta,w) = \rho_0\vartheta^0 r \tag{2.4}$$

where we have introduced new variables $p = \beta_{,x}$ and $w = \beta_{,t}$. In the new variables we obtain

$$
\begin{aligned}
&-C(\beta, w)w_t + bpw_x + ap_x + H(\beta, w) = \rho_0\vartheta^0 r \\
&p_t - w_x = 0
\end{aligned} \tag{2.5}
$$

Let us consider now a five-dimensional differentiable manifold $\mathcal{M} = \mathcal{R}^5$ with a coordinate cover (p, w, β, x, t), the exterior algebra $\Lambda(\mathcal{M})$ over $\mathcal{M}$ and the following exterior differential 2- and 1-forms

$$
\begin{aligned}
\omega &= C(\beta, w)dw \wedge dx + bpdw \wedge dt + adp \wedge dt + [H(\beta, w) - \rho_0\vartheta^0 r]dx \wedge dt \\
\sigma &= d\beta - pdx - wdt \\
d\sigma &= -dp \wedge dx - dw \wedge dt
\end{aligned} \tag{2.6}
$$

where $\wedge$ represents the exterior product and d represents the exterior differentiation. By taking the exterior derivative of ω, the derived form is found to be in the ideal $I(\omega, \sigma, d\sigma) \subset \Lambda(\mathcal{M})$, so that the set is closed (cf Edelen (1985)).

The set is in involution with respect to x and t, and gives back the original partial equations if we impose independence of these variables, requiring the sectioned forms to be annuled by elements of a 2-manifold in which dx and dt are independent forms.

We denote a two-dimensional submanifold of $\mathcal{M}$ with a coordinate cover (x, t) by $\mathcal{S}$ and define a map $\Phi : \mathcal{S} - \mathcal{M}$ by

$$
\Phi : \beta = \beta(x, t) \qquad p = p(x, t) \qquad w = w(x, t)
$$

Φ induces a map $\Phi^* : \Lambda(\mathcal{M}) - \Lambda(\mathcal{S})$ under which we find

$$
\begin{aligned}
&\Phi^*\omega \equiv -C(\beta, w)w_t + bpw_x + ap_x + [H(\beta, w) - \rho_0\vartheta^0 r]dx \wedge dt = 0 \\
&\Phi^*\sigma \equiv (\beta_x - p)dx + (\beta_t - w)dt = 0 \\
&\Phi^* d\sigma \equiv (p_t - w_x)dx \wedge dt = 0
\end{aligned}
$$

Therefore the solution manifold of Eq (2.5) annuls the exterior forms (2.6).

We now find the isogroup by requiring that the Lie derivatives of ω, σ and $d\sigma$ with respect to a vector field V should be in the ideal I. An isovector field of I is a vector $V \in T(\mathcal{M})$ given by

$$
\begin{aligned}
V \;&=\; V^x(p, w, \beta, x, t)\partial_x + V^t(p, w, \beta, x, t)\partial_t + V^\beta(p, w, \beta, x, t)\partial_\beta + \\
+ \;&\; V^w(p, w, \beta, x, t)\partial_w + V^p(p, w, \beta, x, t)\partial_p
\end{aligned}
$$

where $V^x, V^t, V^\beta, V^w, V^p \in \Lambda^0(\mathcal{M})$ are smooth functions over $\mathcal{M}$, such that I remains invariant under V, i.e., $\forall\alpha \in I$ we have $\pounds_V\alpha \in I$ where $\pounds_V$ denotes the Lie derivative operator with respect to V. The Lie derivative has the explicit evaluation

$$\pounds_V\alpha = V\rfloor d\alpha + d(V\rfloor\alpha) \tag{2.7}$$

where $\rfloor$ represents the interior product operator between a differential form and a vector field.

The vector field V is an isovector field of the ideal I if and only if there exist appropriate 0- and 1-forms such that the following relations are valid (cf Edelen (1985))

$$\begin{aligned} &\pounds_V\sigma = \lambda\sigma && \lambda \in \Lambda^0(\mathcal{M}) \\ &\pounds_V\omega = \mu\omega + \gamma\wedge\sigma + \nu d\sigma && \mu,\nu \in \Lambda^0(\mathcal{M}) \qquad \gamma \in \Lambda^1(\mathcal{M}) \end{aligned} \tag{2.8}$$

Indeed, the relation

$$d\pounds_V\sigma = \pounds_V d\sigma = d\lambda\wedge\sigma + \lambda d\sigma$$

shows that $\pounds_V d\sigma$ remains in I if Eq $(2.8)_1$ is already satisfied.

Let us now define a smooth function $F \in \Lambda^0(\mathcal{M})$ by

$$F = V\rfloor\sigma = V^\beta - pV^x - wV^t \tag{2.9}$$

Using the relation (2.7) we obtain for Eq $(2.8)_1$

$$V\rfloor d\sigma + dF = \lambda\sigma$$

or

$$\begin{aligned} &(-V^p + F_x)dx + (V^x + F_p)dp + (-V^w + F_t)dt + (V^t + F_w)dw + \\ &+F_\beta d\beta = \lambda(d\beta - pdx - wdt) \end{aligned} \tag{2.10}$$

Comparing both sides of Eq (2.10) we find immediately

$$\begin{aligned} &\lambda = F_\beta && V^x = -F_p && V^t = -F_w \\ &V^w = F_t + wF_\beta && V^p = F_x + pF_\beta && \end{aligned} \tag{2.11}$$

and Eq (2.9) yields

$$V^\beta = F - wF_w - pF_p \tag{2.12}$$

Therefore all components of an isovector field are derivable from a single function $F(p,w,\beta,x,t)$. After expanding of $(2.8)_2$ and equating coefficients of all 2-forms we obtain

$$
\begin{aligned}
& pA_t + \mu(H - r\rho_0\vartheta^0) - wA_x - V_\beta H_\beta - V_w H_w + \rho_0\vartheta^0(V_t r_t + V_x r_x) + \\
& \quad -aVpx - (H - r\rho_0\vartheta^0)V_{tt} + CV_{wt} - bpV_{wx} - HV_{xx} + r\rho_0\vartheta^0 V_{xx} = 0 \\
& A_p - aV_{t\beta} = 0 \\
& A_x + pA_\beta - (H - r\rho_0\vartheta^0)V_{t\beta} + CV_{w\beta} = 0 \\
& A_w - bpV_{t\beta} - CV_{x\beta} = 0 \\
& A_t + wA_\beta + aV_{p\beta} + bpV_{w\beta} + (H - r\rho_0\vartheta^0)V_{x\beta} = 0 \\
& -\mu C + pA_w + V_\beta C_\beta + V_w C_w - (H - r\rho_0\vartheta^0)V_{tw} + bpV_{tx} + \\
& \quad +CV_{ww} + CV_{xx} = 0 \\
& -\nu + \mu bp - bV_p - wA_w - aV_{pw} - bpV_{tt} - bpV_{ww} - CV_{xt} + \\
& \quad -(H - r\rho_0\vartheta^0)V_{xw} = 0 \\
& \nu + pA_p - (H - r\rho_0\vartheta^0)V_{tp} + aV_{tx} + CV_{wp} = 0 \\
& -bpV_{tp} + aV_{tw} - CV_{xp} = 0 \\
& -a\mu + wA_p + aV_{pp} + aV_{tt} + bpV_{wp} + (H - r\rho_0\vartheta^0)V_{xp} = 0
\end{aligned}
\tag{2.13}
$$

where $A_p, A_w, A_\beta, A_x, A_t$ are components of the linear form γ. Eliminating $A_p, A_w, A_\beta, A_x, A_t, \nu, \mu$ from the above equations and after that substituting Eqs (2.11) and (2.12) into the resulting expresions we finally obtain four linear second order partial differential equations to determine the single function F for given coefficients

$$
\begin{aligned}
& aFC_\beta + awC_wF_\beta - apC_\beta F_p + aC_wF_t - awC_\beta F_w - p(2a + bw)CF_{\beta p} + \\
& -a(bp^2 - 2Cw)F_{\beta w} + (H - r\rho_0\vartheta^0)CF_{pp} - bpCF_{tp} + 2aCF_{tw} + \\
& +a(H - r\rho_0\vartheta^0)F_{ww} - 2aCF_{xp} - abpF_{xw} = 0 \\[1em]
& -abpF_\beta - abF_x + (abp^2 + 2awC + b^2p^2w)F_{\beta p} - ap(2a + bw)F_{\beta w} + \\
& -bp(H - r\rho_0\vartheta^0)F_{pp} + (2aC + b^2p^2)F_{tp} - abpF_{tw} + 2a(H - r\rho_0\vartheta^0)F_{wp} + \\
& +abpF_{xp} - 2a^2F_{xw} = 0 \\[1em]
& -aH_\beta F - aH_wF_t + a(H - r\rho_0\vartheta^0 - wH_w)F_\beta + a(wH_\beta - \rho_0\vartheta^0 r_t)F_w + \\
& +a(pH_\beta - \rho_0\vartheta^0 r_x)F_p - a(ap^2 + bp^2w - Cw^2)F_{\beta\beta} + (2apH - 2apr\rho_0\vartheta^0 + \\
& +bpwH - bpr\rho_0\vartheta^0 w)F_{\beta p} + (-H^2 + 2Hr\rho_0\vartheta^0 - r^2\rho_0^2\vartheta^{02})F_{pp} + \\
& +a(-bp^2 + 2Cw)F_{t\beta} + (bpH - bpr\rho_0\vartheta^0)F_{tp} + aCF_{tt} - ap(2a + bw)F_{x\beta} +
\end{aligned}
\tag{2.14}
$$

$$+2a(H - r\rho_0\vartheta^0)F_{xp} - abpF_{xt} - a^2F_{xx} = 0$$

$$a(CF_{pp} + bpF_{wp} - aF_{ww}) = 0$$

3. Determination of the vector field

If we wish to exclude the higher-order symmetries we have to impose the condition that V^x i V^t are independent of p and w. This, in view of Eq (2.14), leads to the equations

$$F_{pp} = F_{wp} = F_{ww} = 0 \ ,$$

which has a solution of the form

$$F(p, w, \beta, x, t) = \Phi(\beta, x, t)p + \Psi(\beta, x, t)w + \Omega(\beta, x, t) \tag{3.1}$$

Then the components of isovector follow from Eqs (2.11) and (2.12)

$$\begin{aligned}
&V^x = -\Phi \qquad\qquad V^t = -\Psi \qquad\qquad V^\beta = \Omega \\
&V^w = wp\Phi_\beta + w^2\Psi_\beta + p\Phi_t + w(\Psi_t + \Omega_\beta) + \Omega_t \\
&V^p = p^2\Phi_\beta + wp\Psi_\beta + p(\Phi_x + \Omega_\beta) + w\Psi_x + \Omega_x
\end{aligned} \tag{3.2}$$

In order to determine Φ, Ψ and Ω we should substitute Eq (3.1) into Eqs $(2.14)_{1-3}$. We first consider Eq $(2.14)_2$ which is written as

$$\begin{aligned}
&(b^2p^2 + 2aC)(\Phi_t + w\Phi_\beta) - [2ap(a + bw)\Psi_\beta + abp\Psi_t + a(bw + 2a)\Psi_x] + \\
&-ab(\Omega_x + p\Omega_\beta) = 0
\end{aligned} \tag{3.3}$$

that can be satisfied if and only if

$$\begin{aligned}
&\Phi_t + w\Phi_\beta = 0 \qquad\qquad \Omega_x + p\Omega_\beta = 0 \\
&2ap(a + bw)\Psi_\beta + abp\Psi_t + a(bw + 2a)\Psi_x = 0
\end{aligned}$$

The solution to this equation is

$$\Phi = P(x) \qquad\qquad \Psi = Q = \mathrm{const} \qquad\qquad \Omega = \Omega(t) \tag{3.4}$$

Furthermore Eqs $(2.14)_1$ and $(2.14)_3$ become, respectively

$$\begin{aligned}
&a(-2CP' + \dot{\Omega}(t)C_w + \Omega(t)C_\beta) = 0 \\
&a[-apP''(x) + 2(H - \rho_0\vartheta^0 r)P'(x) + C\ddot{\Omega}(t) - \dot{\Omega}(t)H_w - \Omega(t)H_\beta + \\
&-Q\rho_0\vartheta^0 r_t - \rho_0\vartheta^0 P(x)r_x] = 0
\end{aligned} \tag{3.5}$$

From $(3.5)_1$ we have

$$\Omega = 0 \qquad \Phi = P = \text{const}$$

and if we substitute it into $(3.5)_2$ we achive

$$\rho_0 \vartheta^0 (Q r_t + P r_x) = 0 \,, \tag{3.6}$$

which is a condition for the source function

$$r(x, t) = r(Qx - Pt) \tag{3.7}$$

From the above calculations we finally have

$$F(p, w, \beta, x, t) = pP + wQ \tag{3.8}$$

The form of the function F clearly indicates that Eq (2.1) with the function r given by (3.7) does not admit higher-order symmetries. According to Eq (2.11) the components of isovector are

$$\begin{aligned} &V^x = -P \qquad &&V^t = -Q \qquad &&V^\beta = 0 \\ &V^p = 0 \qquad &&V^w = 0 \end{aligned} \tag{3.9}$$

4. Similarity solutions

The similarity solutions to the heat equation are invariant solutions under the appropriate Lie group. Therefore they are obtainable through the invariants of the group which satisfy

$$V^x \frac{\partial I}{\partial x} + V^t \frac{\partial I}{\partial t} + V^\beta \frac{\partial I}{\partial \beta} + V^w \frac{\partial I}{\partial w} + V^p \frac{\partial I}{\partial p} = 0$$

the solution to which can be formed by its characteristic fields

$$\frac{dx}{V^x} = \frac{dt}{V^t} = \frac{d\beta}{V^\beta} = \frac{dw}{V^w} = \frac{dp}{V^p}$$

Solving the equation we have the similarity variable

$$\eta = x - \lambda_0 t \tag{4.1}$$

where $\lambda_0 = P/Q$, and the similarity solution

$$\beta = \overline{\beta}(\eta) \tag{4.2}$$

If we introduce Eq (4.2) into Eq (2.1) we obtain an ordinary differential equation in the unknown function $\overline{\beta}$

$$\overline{\beta}''(\eta)[a - \lambda_0^2\overline{C}(\overline{\beta},\overline{\beta}') - b\lambda_0\overline{\beta}'(\eta)] + \overline{H}(\overline{\beta},\overline{\beta}') = r(\eta) \tag{4.3}$$

This time the coefficients (2.2) for Eq (4.3) are

$$\begin{aligned} &\overline{C}(\overline{\beta},\overline{\beta}') = \rho_0\tau_0\vartheta c_V(\vartheta) \\ &\overline{H}(\overline{\beta},\overline{\beta}') = -\lambda_0\rho_0\vartheta c_V(\vartheta) f_{0,\overline{\beta}}\overline{\beta}' \\ &a = k_0\vartheta^0 \qquad\qquad b = k_0\tau_0 \end{aligned} \tag{4.4}$$

where

$$\vartheta = \vartheta(\overline{\beta},\overline{\beta}') = \vartheta^0 \exp\Big\{\frac{1}{\vartheta^0}[f_0(\overline{\beta}) + \lambda_0\tau_0\overline{\beta}']\Big\}$$

5. Stability of the equilibrium manifold

We shall consider the system in the form

$$x_1' = g_1(x_1,x_2) \qquad\qquad x_2' = g_2(x_1,x_2) \tag{5.1}$$

where $(\cdot)' := d/d\eta$, $x_1 := \beta$, $x_2 := \beta'$ and

$$\begin{aligned} &g_1(x_1,x_2) := x_2 \\ &g_2(x_1,x_2) := \frac{-\overline{H}(x_1,x_2)}{aQ^2 - \lambda_0^2\overline{C}(x_1,x_2) - bQ^2\lambda_0 x_2} \end{aligned} \tag{5.2}$$

The system has a manifold $\mathcal{M}_e$ of equilibrium solutions defined by

$$\mathcal{M}_e := \Big\{(x_1,x_2) \in R^2 : \ x_1 \in R \setminus S_g,\ x_2 = 0\Big\} \tag{5.3}$$

where S_g denotes possible singularity of the function g.

If we put

$$x = x_e + u$$

we can reduce the question of the stability of $x_e \in \mathcal{M}_e$ to the question of the stability of the solution $u = 0$ of the variational system (cf Cesari (1971))

$$u' = \mathbf{A}u \tag{5.4}$$

where matrix $\mathbf{A}$ consists of the elements $a_{i,j} := \partial_j g_i(x_1, x_2)$ and is of the form

$$\begin{bmatrix} 0 & 1 \\ 0 & \alpha \end{bmatrix}$$

The matrix $\mathbf{A}$ has been evaluated at $x = x_e$. The characteristic roots of the matrix $\mathbf{A}$ are $\nu_1 = 0$ and $\nu_2 = \alpha$, where α can be evaluated for different functions f_{0i} Eq (2.3)

$$\begin{aligned}
\alpha_1 &:= -\frac{\exp\left(\frac{\beta^0}{\vartheta^0}\right)\lambda_0\rho_0\vartheta^0 c_V(\overline{\vartheta})}{a\exp\left(\frac{x_e}{\vartheta^0}\right)Q^2 - \exp\left(\frac{\beta^0}{\vartheta^0}\right)\lambda_0^2\rho_0\tau_0\vartheta^0 c_V(\overline{\vartheta})} \\
\alpha_2 &:= -\frac{\beta^0\lambda_0\rho_0{\vartheta^0}^2 c_V(\overline{\vartheta})}{aQ^2x_e^2 - \beta^0\lambda_0^2\rho_0\tau_0\vartheta^0 x_e c_V(\overline{\vartheta})} \\
\alpha_3 &:= -\frac{\exp\left[\frac{1}{2}\left(\frac{\beta^0}{\vartheta^0}\right)^2\right]\lambda_0\rho_0 x_e c_V(\overline{\vartheta})}{a\exp\left[\frac{1}{2}\left(\frac{x_e}{\vartheta^0}\right)^2\right]Q^2 - \exp\left[\frac{1}{2}\left(\frac{\beta^0}{\vartheta^0}\right)^2\right]\lambda_0^2\rho_0\tau_0\vartheta^0 c_V(\overline{\vartheta})}
\end{aligned} \tag{5.5}$$

where $\overline{\vartheta}$ is the value of ϑ on the manifold $\mathcal{M}_e$. The equilibruim manifold is stable if $\nu_2 \leq 0$ (cf Cesari (1971)).

6. Numerical results

The numerical solutions to Eq (4.3) have been derived for two materials: Bi and NaF for which we take the following material constants (cf Frischmuth and Cimmelli (1995))

$$k_0 = 36\frac{\mathrm{W}}{\mathrm{mK^4}} \qquad c_0 = 55\frac{\mathrm{J}}{\mathrm{m^3K^4}} \qquad \rho_0 = 9800\frac{\mathrm{kg}}{\mathrm{m^3}} \qquad \tau_0 = 5\cdot 10^{-7}\mathrm{s}$$

$$k_0 = 10\frac{\mathrm{W}}{\mathrm{mK^4}} \qquad c_0 = 2.3\frac{\mathrm{J}}{\mathrm{m^3K^4}} \qquad \rho_0 = 2790\frac{\mathrm{kg}}{\mathrm{m^3}} \qquad \tau_0 = 1\cdot 10^{-6}\mathrm{s}$$

respectively, and $\vartheta^0 = 1\,\mathrm{K}$, $\beta^0 = 1\,\mathrm{K}$ for both materials.

The numerical calculations are made for the same initial values $\overline{\beta}_0 = \overline{\beta}(0) = 0.103\,\mathrm{K}$ and $\overline{\beta}'(0) = 0.01\,\mathrm{Ks^{-1}}$ for both materials. The initial value for $\overline{\beta}_0$ was calculated from the eqiulibrium condition $f(\overline{\beta}_0, \vartheta_0) = 0$, for $\vartheta_0 = 1.64\,\mathrm{K}$, where f is the function from the kinetic equation for β (cf Kosiński and Saxton (1993)). The numerical calculations were carried out for f_{03} only and for $\lambda_0 = 1$.

We present here only results for the specific heat expressed by $c_V = c_0\vartheta^4/4$.

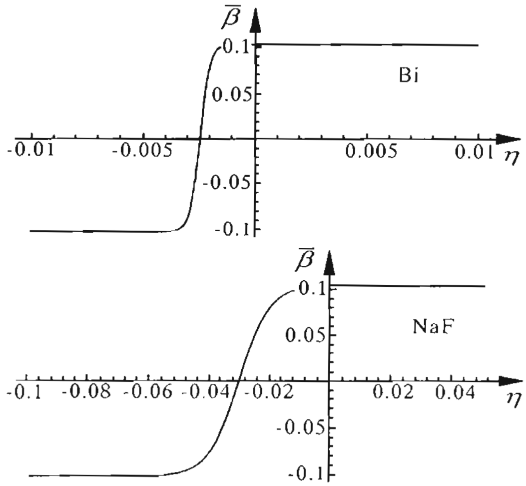

Fig. 1. Semi-empirical temperature $\overline{\beta}$ versus η

7. Wave motion

Consider the problem of one-dimensional nonlinear propagation

$$- C(\beta, \beta_t)\beta_{tt} + b\beta_x\beta_{tx} + a\beta_{xx} + H(\beta, \beta_t) = 0 \tag{7.1}$$

with the prescribed boundary conditions

$$\frac{\partial\beta}{\partial t}(0, t) = h(t) \qquad t \geq 0$$

$$\beta(x, 0) = 0 \qquad x \geq 0$$

At the wave front displacement is zero, therefore $\beta(x = X(t); t) = 0$, where $X(t)$ describes the wave-front. From the condition at the wave front we have

$\overline{\beta}(\eta_w) = 0$, where η_w is the value of the similarity variable at the wave front to be determined (cf Seshadri and Singh (1980)).

From the numerical calculation (Fig.1) we can see that $\overline{\beta}$ can be approximated by

$$\overline{\beta} = A(\eta) * \operatorname{sgn}(\eta)\,, \tag{7.2}$$

where $*$ denotes the convolution of two functions and

$$A(x) := \begin{cases} \kappa_1 - |x - \kappa_3| & |x - \kappa_3| < \kappa_2 \\ 0 & |x - \kappa_3| > \kappa_2 \end{cases}$$

If we use Eq (7.2) we may calculate the second conditon for $\overline{\beta}$: $\overline{\beta}'(0) = 2A(0)$.

The conditions

$$\overline{\beta}(\eta_w) = 0 \qquad\qquad \overline{\beta}'(0) = 2A(0)$$

with Eq (4.3) for the similarity representation of the semi-empirical temperature β lead to the wave propagation problem Eq (7.1) with the prescribed boundary conditions above, where

$$h(t) = -2\lambda_0 A(-\lambda_0 t)$$

8. Remarks

The similarity solution in the paper has been derived only for one similarity variable. Using parabolic regularization of the heat equation may lead to further groups and further similarity solutions. That will be the subject of future work.

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Własności grupowe i samopodobne rozwiązania dla równania przewodnictwa cieplnego z semi-empiryczną temperaturą

Streszczenie

W pracy zbadano własności grupowe równania przewodnictwa cieplnego z semi-empiryczną temperaturą. Dla znalezionej grupy symetrii znaleziono samopodobną reprezentację dla semi-empirycznej temperatury. Przeprowadzono analizę stabilności rozmaitości położeń równowagi dla równania opisującego rozwiązanie samopodobne. Uzyskano rozwiązania numeryczne dla rozwiązania samopodobnego i na ich bazie rozważono pewne zagadnienie brzegowo-początkowe.

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