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BEHAVIOUR OF ACCELERATION WAVES IN A VISCOELASTIC POROUS MEDIUM WITH A STRUCTURE. Part I – PROPAGATION CONDITIONS AND GROWTH EQUATIONS

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The behaviour of acceleration wave in a fluid-saturated porous medium with a structure determined by two parameters has been studied. The considerations are based on the description of a geometric structure of such a medium proposed by Kubik (1986a), who characterized the structure of a porous medium by two parameters: volume porosity and a structural permeability tensor.
Using the theory of singular surfaces the propagation conditions of acceleration wave are formulated and the growth equations are derived.

Key words: fluid-saturated porous solid, geometric structure, acceleration wave

1. Introduction

One of the interesting direction of development of mechanics of fluid-saturated porous media in the last years have been efforts to provide description of a porous medium structure by two parameters. These investigations were initiated by Derski (1978), and have been continued by Kowalski (1986) and Kubik (1986a,b). The complete description of a porous medium structure by two parameters has been proposed by Kubik in his papers cited above. The author characterized the medium structure by two parameters: volume porosity f_v, well known in soil mechanics and in the theory of consolidation, and the structural permeability tensor $\mathbf{P}$ defined in Kubik (1986a). In the case of medium with an isotropic pore structure, instead of the tensor $\mathbf{P}$ its mean value λ, called a structural permeability parameter appears as the second parameter in the description of a medium structure.

This kind of description was applied to study the influence of a structure on the propagation of harmonic wave (cf Kubik and Kaczmarek (1988)). Propagation of a weak discontinuity wave in the case of undeformable solid skeleton and reflection and refraction of the wave on the interface of two media were considered by Cieszko (1989). The investigations of the behaviour of acceleration wave in a fluid-saturated elastic porous medium with a structure determined by two parameters are presented by Dzięcielak (1995).

The aim of this paper is to study the influence of medium structure on the propagation of acceleration wave in the fluid-saturated porous medium with a viscoelastic solid skeleton.

2. Fluid-saturated porous medium with a structure characterized by two parameters

In mechanics of soils and rocks a structure of a medium is usually described by a volume porosity f_v. Other scalar parameter which is sometimes used to characterize the porosity of medium is the surface porosity f_s. If the porous medium contains pores having the form of parallel tubes then surface porosity and volume porosity are the same, but if the pores are of arbitrary shapes and cross-sections and the distribution of channel directions is arbitrary then usually $f_v > f_s$ (cf Kubik and Rybicki (1980)), In this case the pore structure is well characterized by two parameters: the volume porosity f_v and the second order permeability tensor $\mathbf{P}$ defined by Kubik (1986a).

We focus our attention on an isotropic pore structure. In such a case $\mathbf{P} = \lambda \mathbf{1}$, where λ stands for the effective surface porosity and is called the structural permeability parameter (cf Kubik (1986b)). Thus, in this paper the isotropic porous skeleton is characterized by two scalar parameters: the volume porosity f_v and the structural permeability parameter λ. The parameter λ (or $\kappa = \lambda/f_v$) is the measure of inhomogeneity of the fluid micro-velocity in its relative flow and is limited by

$$0 < \lambda \ll f_v \qquad \text{or} \qquad 0 < \kappa \ll 1 \tag{2.1}$$

A detailed discussion of the structural permeability parameter can be found in Kubik (1992).

We consider a fluid-saturated porous medium with a deformable solid skeleton. This medium may be divided into components from either physical or kinematic viewpoint (cf Kubik (1992)). Physical components of the medium are: the porous solid skeleton moving at a velocity $\boldsymbol{v}^s$ and the fluid moving at a velocity $\boldsymbol{v}^f$. The partial densities of these two components are $\bar{\rho}^s$ and $\bar{\rho}^f$, respectively, and are related to mass densities of the solid skeleton ρ^s and of the fluid ρ^f by the formulas (cf Kubik (1992))

$$\bar{\rho}^s = (1 - f_v)\rho^s \qquad \bar{\rho}^f = f_v\rho^f \tag{2.2}$$

The masses of physical components are conserved, thus the partial density of each physical component satisfies the continuity equation of the form

$$\frac{\partial \bar{\rho}^s}{\partial t} + \operatorname{div}(\bar{\rho}^s \boldsymbol{v}^s) = 0 \qquad \frac{\partial \bar{\rho}^f}{\partial t} + \operatorname{div}(\bar{\rho}^f \boldsymbol{v}^f) = 0 \tag{2.3}$$

The kinematic components of a medium result from the description of a solid skeleton structure by two parameters. In such a case we have the following two kinematic components: the solid skeleton and the fluid associated with it of partial density $\rho^{(1)} = \bar{\rho}^s + (1-\kappa)\bar{\rho}^f$, moving at the velocity $\boldsymbol{v}^{(1)} = \boldsymbol{v}^s$, and the free fluid of partial density $\rho^{(2)} = \kappa\bar{\rho}^f$, moving at its own velocity (cf Kubik (1992))

$$\boldsymbol{v}^{(2)} = \boldsymbol{v}^s + \frac{\boldsymbol{v}^f - \boldsymbol{v}^s}{\kappa} \tag{2.4}$$

The continuity equations written for the kinematic components have the form (cf Kubik (1992))

$$\frac{\partial \rho^{(1)}}{\partial t} + \operatorname{div}(\rho^{(1)}\boldsymbol{v}^{(1)}) = g^{(1)} \qquad \frac{\partial \rho^{(2)}}{\partial t} + \operatorname{div}(\rho^{(2)}\boldsymbol{v}^{(2)}) = g^{(2)} \tag{2.5}$$

where $g^{(1)}$ and $g^{(2)}$ are the mass supply terms satisfying the condition

$$g^{(1)} + g^{(2)} = 0$$

These terms are defined by the rate of change of the fluid partial density and the pore structure parameters and have the form

$$g^{(1)} = -g^{(2)} = g = \frac{D^{(1)}}{Dt}[(1-\kappa)\bar{\rho}^f] \tag{2.6}$$

where

$$\frac{D^{(i)}}{Dt} = \frac{\partial}{\partial t} + \boldsymbol{v}^{(i)} \cdot \operatorname{grad} \qquad i = 1,2$$

from which it follows that virtual components interchange their masses which therefore are not conserved.

The equations of motion for the kinematic components are

$$\operatorname{div}\mathbf{T}^{(1)} + \rho^{(1)}\boldsymbol{b} + \boldsymbol{\pi} = \rho^{(1)}\frac{D^{(1)}}{Dt}\boldsymbol{v}^{(1)} + \frac{1}{2}g(\boldsymbol{v}^{(2)} - \boldsymbol{v}^{(1)}) \tag{2.7}$$

$$\operatorname{div}\mathbf{T}^{(2)} + \rho^{(2)}\boldsymbol{b} - \boldsymbol{\pi} = \rho^{(2)}\frac{D^{(2)}}{Dt}\boldsymbol{v}^{(2)} + \frac{1}{2}g(\boldsymbol{v}^{(2)} - \boldsymbol{v}^{(1)}) \tag{2.8}$$

where

- $\mathbf{T}^{(1)}, \mathbf{T}^{(2)}$ – stress tensors for kinematic components
- $\rho^{(1)}\boldsymbol{b}, \rho^{(2)}\boldsymbol{b}$ – external body forces
- $\boldsymbol{\pi}$ – viscous interaction force and force.

The term $0.5g(\boldsymbol{v}^{(1)} - \boldsymbol{v}^{(1)})$ results from the mass exchange between kinematic components. This physical effect stems from the influence of pore structure on the relative fluid flow. The stress tensors $\mathbf{T}^{(1)}$ and $\mathbf{T}^{(2)}$ are related to the stress tensors for physical components $\mathbf{T}^{s}$ in the solid skeleton and $\mathbf{T}^{f}$ in the fluid, respectively, by

$$\mathbf{T}^{(1)} = \mathbf{T}^{s} + (1-\kappa)\mathbf{T}^{f} \qquad\qquad \mathbf{T}^{(2)} = \kappa\mathbf{T}^{f} \tag{2.9}$$

Substituting Eqs (2.6) and (2.9) into Eqs (2.7) and (2.8) we obtain the equations of motion for the physical components as follows (cf Kubik (1986b))

$$\operatorname{div}\mathbf{T}^{s} + \bar{\rho}^{s}\boldsymbol{b} + \boldsymbol{r}^{s} = \bar{\rho}^{s}\frac{D\boldsymbol{v}^{s}}{D^{s}t} \tag{2.10}$$

$$\begin{aligned}\operatorname{div}\mathbf{T}^{f} + \bar{\rho}^{f}\boldsymbol{b} + \boldsymbol{r}^{f} &= \bar{\rho}^{f}\frac{D\boldsymbol{v}^{f}}{D^{f}t} + \\ &+\operatorname{div}\Big[(\bar{\rho}^{f} - \rho^{(2)})(\boldsymbol{v}^{s} - \boldsymbol{v}^{f})\otimes(\boldsymbol{v}^{s} - \boldsymbol{v}^{f}) + \rho^{(2)}(\boldsymbol{v}^{(2)} - \boldsymbol{v}^{f})\otimes(\boldsymbol{v}^{(2)} - \boldsymbol{v}^{f})\Big]\end{aligned} \tag{2.11}$$

where

$$\frac{D}{D^{\Gamma}t} = \frac{\partial}{\partial t} + \boldsymbol{v}^{\Gamma}\cdot\operatorname{grad} \qquad\qquad \Gamma = s, f$$

$\bar{\rho}^{s}\boldsymbol{b}$, $\bar{\rho}^{f}\boldsymbol{b}$ are the body forces in the solid skeleton and in the fluid, respectively, $\boldsymbol{r}^{s}$ and $\boldsymbol{r}^{f}$ represent the internal force interaction between physical components and take the form (cf Kubik (1986b))

$$\begin{aligned}\boldsymbol{r}^{s} &= -\boldsymbol{r}^{f} = \boldsymbol{\pi} + \operatorname{div}[(1-\kappa)\mathbf{T}^{f}] + (\bar{\rho}^{f} - \rho^{(2)})\Big(\boldsymbol{b} - \frac{D\boldsymbol{v}^{s}}{D^{s}t}\Big) + \\ &- \frac{1-\kappa}{2}(\boldsymbol{v}^{(2)} - \boldsymbol{v}^{s})\Big(\frac{D\bar{\rho}^{f}}{D^{s}t} + \bar{\rho}^{f}\operatorname{div}\boldsymbol{v}^{s}\Big)\end{aligned} \tag{2.12}$$

Additional two terms on the right hand-side of Eq (2.11) and three terms on the right-hand side of Eq (2.12) result from the mass exchange between the kinematic components of the medium.

The force $\boldsymbol{\pi}$ of internal interaction in Eq (2.12) must be determined by a constitutive relation. Variables in this relation can be, for example, the fluid partial density $\bar{\rho}^f$, the left Cauchy-Green deformation tensor $\mathbf{B}$, the rate of deformation $\mathbf{D}$, the relative velocity $\boldsymbol{u} = \boldsymbol{v}^f - \boldsymbol{v}^s$, and the relative spin tensor $\mathbf{W}$. We restrict our further considerations to the constitutive relation

$$\boldsymbol{\pi} = \boldsymbol{\pi}(\bar{\rho}^f, \mathbf{B}, \boldsymbol{u}) \tag{2.13}$$

assuming that $\boldsymbol{\pi}$ is the continuous function of its arguments.

3. Propagation conditions of an acceleration wave

The acceleration wave is a propagating singular surface of order two, across which the motion and its first derivatives, i.e. velocities and deformation gradients of the components are continuous, but the second (and higher) order derivatives, i.e. accelerations and rates of deformation of the components suffer from finite jump discontinuities. We define amplitudes of this wave by the relations

$$u^2\boldsymbol{a}^s = \left[\frac{D\boldsymbol{v}^s}{D^s t}\right] \qquad\qquad u^2\boldsymbol{a}^f = \left[\frac{D\boldsymbol{v}^f}{D^f t}\right] \tag{3.1}$$

where u is the speed of wave displacement in a medium and the square brackets denote the finite jump across a singular surface determined by

$$\Big[\mathsf{H}\Big] = \mathsf{H}^- - \mathsf{H}^+ \tag{3.2}$$

where H^- and H^+ are definite limits of the function H behind and in front of the singular surface, respectively.

The mass continuity equations written for a singular surface Cf Dzięcielak (1980), are of the form

$$\Big[\bar{\rho}^s(u - \boldsymbol{v}^s\boldsymbol{n})\Big] = 0 \qquad\qquad \Big[\bar{\rho}^f(u - \boldsymbol{v}^f\boldsymbol{n})\Big] = 0 \tag{3.3}$$

where $\boldsymbol{n}$ is the unit normal to the singular surface; thus in the case of acceleration wave the partial densities of both the constituents are continuous. For the acceleration wave all arguments of the interaction force $\boldsymbol{\pi}$, see Eq

(2.13), are continuous, therefore $\big[\boldsymbol{\pi}\big] = \mathbf{0}$. Additionally we assume the continuous body forces within a medium. Using the continuity equations (2.3) and the well-known geometric and kinematic compatibility conditions (cf Chen (1976); Thomas (1961); Kosiński (1981)), we obtain, from the equations of motion (2.10) and (2.11), the acceleration wave propagation conditions in a fluid-saturated porous medium with a structure described by two parameters, in the form

$$
\begin{aligned}
&\Big[\mathrm{div}\mathbf{T}^s\Big] + (1-\kappa)\Big[\mathrm{div}\mathbf{T}^f\Big] = \\
&= u^2[\bar{\rho}^s + (1-\kappa)\bar{\rho}^f]\boldsymbol{a}^s + \frac{1-\kappa}{2\kappa}(\boldsymbol{v}^f - \boldsymbol{v}^s)[(\boldsymbol{a}^f - \boldsymbol{a}^s)\boldsymbol{n}]\frac{\bar{\rho}^f}{U^s}
\end{aligned}
\tag{3.4}
$$

$$
\begin{aligned}
&\kappa\Big[\mathrm{div}\mathbf{T}^f\Big] = \bar{\rho}^f u^2[\boldsymbol{a}^f - (1-\kappa)\boldsymbol{a}^s] + \bar{\rho}^f u^2(\boldsymbol{v}^s - \boldsymbol{v}^f)\Big\{\frac{1-\kappa}{2\kappa}[(\boldsymbol{a}^f - \boldsymbol{a}^s)\boldsymbol{n}]\frac{1}{U^s} + \\
&+\frac{1-\kappa}{\kappa}\Big[\frac{(\boldsymbol{a}^f\boldsymbol{n})}{U^f}\Big(\frac{\boldsymbol{v}^f}{U^f} - \frac{\boldsymbol{v}^s}{U^s}\Big) + 2\Big(\frac{\boldsymbol{a}^f}{U^f} - \frac{\boldsymbol{a}^s}{U^s}\Big)\Big]\boldsymbol{n}\Big\}
\end{aligned}
\tag{3.5}
$$

Let us notice that in the propagation conditions two unknown amplitudes $\boldsymbol{a}^s$ and $\boldsymbol{a}^f$ of the wave and three unknown speeds: the speed of displacement u and two local speeds of propagation U^s with respect to the solid skeleton and U^f with respect to the fluid, determined by the relations

$$
U^s = u - \boldsymbol{v}^s\boldsymbol{n} \qquad\qquad U^f = u - \boldsymbol{v}^f\boldsymbol{n} \tag{3.6}
$$

have appeared. As usual the problem can be reduced to the following three unknowns: two amplitudes $\boldsymbol{a}^s$ and $\boldsymbol{a}^f$ and speed of displacement u.

Description of the porous medium structure by two parameters f_v and λ introduces additional terms into the propagation conditions (3.4) and (3.5). These terms, resulting from the interaction force between components and the exchange of momentum between kinematic components disappear when $\lambda = f_v$, i.e. in the case of one parameter description a pore structure.

The propagation conditions (3.4) and (3.5) indicate that in the case of description of a porous medium structure by two parameters, the speeds of displacement depend on the relative motion of constituents. This result is new and is not observed if a structure of a medium is described by one parameter. The question arises: is the influence of the fluid and solid skeleton relative motion considerable or may be omitted from the practical point of view? Usually, in the considered media (for example in oil-saturated sandstone or, generally, in fluid-saturated rocks and soils), velocities of the solid skeleton and the fluid are much smaller then the speed of displacement. The terms on the right-hand sides of Eqs (3.4) and (3.5) containing the relative velocity of both the constituents are divided by the local speeds of displacement. Thus, the terms multiplied by the relative velocity of constituents may be neglected and the propagation conditions can be reduced to the following, simpler form

$$\begin{aligned} &\Big[\operatorname{div}\mathbf{T}^s\Big] + (1-\kappa)\Big[\operatorname{div}\mathbf{T}^f\Big] = u^2[\bar{\rho}^s + (1-\kappa)\bar{\rho}^f]\boldsymbol{a}^s \\ &\Big[\operatorname{div}\mathbf{T}^f\Big] = \frac{1}{\kappa}\bar{\rho}^f u^2[\boldsymbol{a}^f - (1-\kappa)\boldsymbol{a}^s] \end{aligned} \tag{3.7}$$

The propagation conditions (3.7) are used in our further considerations to study the influence of the fluid-satureted porous medium structure on the speeds of displacement of acceleration wave.

4. Growth equations

For the governing differential equations of the amplitudes to be derived, we consider time derivatives of the equations of motion (2.10) and (2.11) on either sides of the wave. Using the relations (2.4) and (2.12) and assuming the continuous distribution of body forces across the wave, we have the conditions

$$\begin{aligned} &\Big[\frac{D(\operatorname{div}\mathbf{T}^s)}{D^s t}\Big] + (1-\kappa)\Big[\frac{D(\operatorname{div}\mathbf{T}^f)}{D^s t}\Big] + \Big[\frac{D\pi}{D^s t}\Big] = \Big[\frac{D}{D^s t}\Big(\bar{\rho}^s\frac{D\boldsymbol{v}^s}{D^s t}\Big)\Big] + \\ &+(1-\kappa)\bar{\rho}^f\Big[\frac{D^2\boldsymbol{v}^s}{D^s t^2}\Big] + \frac{1-\kappa}{\kappa}\Big[\frac{D}{D^s t}\Big[\Big(\frac{D\bar{\rho}^f}{D^s t} + \bar{\rho}^f\operatorname{div}\boldsymbol{v}^s\Big)(\boldsymbol{v}^f - \boldsymbol{v}^s)\Big]\Big] \end{aligned} \tag{4.1}$$

$$\begin{aligned} &\kappa\Big[\frac{D(\operatorname{div}\mathbf{T}^f)}{D^f t}\Big] - \Big[\frac{D\pi}{D^f t}\Big] = \Big[\frac{D}{D^f t}\Big(\bar{\rho}^f\frac{D\boldsymbol{v}^f}{D^f t}\Big)\Big] - (1-\kappa)\bar{\rho}^f\Big[\frac{D^2\boldsymbol{v}^s}{D^s t D^f t}\Big] + \\ &-\frac{1-\kappa}{2\kappa}\Big[\frac{D}{D^f t}\Big\{\Big(\frac{D\bar{\rho}^f}{D^s t} + \bar{\rho}^f\operatorname{div}\boldsymbol{v}^s\Big)(\boldsymbol{v}^f - \boldsymbol{v}^s) - \operatorname{div}\Big[\bar{\rho}^f(\boldsymbol{v}^f - \boldsymbol{v}^s)\otimes(\boldsymbol{v}^f - \boldsymbol{v}^s)\Big]\Big\}\Big] \end{aligned} \tag{4.2}$$

The jumps of functions that appeared on the right-hand side of Eqs (4.1) and (4.2) indicate that amplitudes of the wave depend on the mechanical state (initial disturbances) of medium in front of the wave, independently of the constitutive relations; this results from the well-known formula

$$\Big[\mathsf{GH}\Big] = \Big[\mathsf{G}\Big]\Big[\mathsf{H}\Big] + \Big[\mathsf{G}\Big]\mathsf{H}^+ + \Big[\mathsf{H}\Big]\mathsf{G}^+ \tag{4.3}$$

In our further considerations we assume that the medium is undisturbed in front of the wave. To obtain the growth equations, the jumps in Eqs (4.1) and (4.2) must be determined. Temporarily we do not formulate the constitutive relations for stresses $\mathbf{T}^s$, $\mathbf{T}^f$ and internal interaction force $\boldsymbol{\pi}$. We are interested in the influence of the medium structure on acceleration wave amplitudes thus we neglected the influence of geometry, which means that we confine ourselves to the velocities of components independent of surface parameters. Using the geometrical condition of compatibility (cf Chen (1976))

$$\Big[\text{grad}\mathsf{H}\Big] = \Big[\boldsymbol{n}\text{grad}\mathsf{H}\Big] \otimes \boldsymbol{n} \tag{4.4}$$

and the kinematical conditon of compatibility

$$\frac{\delta\Big[\mathsf{H}\Big]}{\delta t} = \Big[\frac{D\mathsf{H}}{Dt}\Big] + U^l\Big[\text{grad}\mathsf{H}\Big]\boldsymbol{n} \tag{4.5}$$

where $\delta/\delta t$ denotes the displacement derivative introduced by Thomas (1961) and U^l is the local speed of propagation, we arrive at the condition of compatibility

$$\Big[\text{div}\mathsf{H}\Big] = \frac{1}{U^l}\Big(\frac{\delta\Big[\mathsf{H}\Big]}{\delta t} - \Big[\frac{D\mathsf{H}}{Dt}\Big]\Big)\boldsymbol{n} \tag{4.6}$$

which is useful for calculation of jumps in Eqs (4.1) and (4.2). From Eqs (2.3) the jumps of the rate of partial densities change follow

$$\Big[\frac{D\bar{\rho}^s}{D^s t}\Big] = \frac{u^2}{U^s}\bar{\rho}^s\boldsymbol{a}^s\boldsymbol{n} \qquad\qquad \Big[\frac{D\bar{\rho}^f}{D^f t}\Big] = \frac{u^2}{U^f}\bar{\rho}^f\boldsymbol{a}^f\boldsymbol{n} \tag{4.7}$$

To calculate the jumps on the right-hand sides of Eqs (4.1) and (4.2) we use the definitions (3.1) of acceleration wave amplitudes, the condition (4.6) and the geometrical and kinematical conditions of compatibility of the second order (cf Kosiński (1981); Thomas (1961)), yielding the following growth equations

$$\begin{aligned}
&\Big[\frac{D(\text{div}\mathbf{T}^s)}{D^s t}\Big] + (1-\kappa)\Big[\frac{D(\text{div}\mathbf{T}^f)}{D^s t}\Big] + \Big[\frac{D\boldsymbol{\pi}}{D^s t}\Big] = \\
&= u^2[\bar{\rho}^s + (1-\kappa)\bar{\rho}^f]\Big(\boldsymbol{c}^s + 2\frac{\delta\boldsymbol{a}^s}{\delta t}\Big) + \bar{\rho}^s U^s u^2(\boldsymbol{a}^s\boldsymbol{n})\boldsymbol{a}^s + \\
&+\frac{1-\kappa}{2\kappa}\bar{\rho}^f u^2\frac{\boldsymbol{v}^f - \boldsymbol{v}^s}{U^s}\Big[\boldsymbol{c}^f - \boldsymbol{c}^s - \frac{\delta\boldsymbol{a}^s}{\delta t} + \Big(2 - \frac{U^s}{U^f}\Big)\frac{\delta\boldsymbol{a}^f}{\delta t}\Big]\boldsymbol{n} + \\
&+\frac{1-\kappa}{2\kappa}\bar{\rho}^f u^2\Big\{\frac{U^s}{u}(\boldsymbol{a}^f\boldsymbol{n})(U^f\boldsymbol{a}^f - u\boldsymbol{a}^s) - \frac{u}{U^s}(\boldsymbol{a}^s\boldsymbol{n})(U^f\boldsymbol{a}^f - u\boldsymbol{a}^s) +
\end{aligned} \tag{4.8}$$

$$+(\boldsymbol{v}^f - \boldsymbol{v}^s)[(\boldsymbol{a}^f\boldsymbol{n}) - (\boldsymbol{a}^f\boldsymbol{n})(\boldsymbol{a}^s\boldsymbol{n})]\Big\}$$

$$\kappa\Big[\frac{D(\mathrm{div}\mathbf{T}^f)}{D^f t}\Big] - \Big[\frac{D\pi}{D^f t}\Big] = -(1-\kappa)\bar{\rho}^f u^2\Big(\boldsymbol{c}^s + 2\frac{\delta \boldsymbol{a}^s}{\delta t}\Big) +$$
$$+\bar{\rho}^f u^2\Big(\boldsymbol{c}^f + 2\frac{\delta \boldsymbol{a}^f}{\delta t}\Big) + \frac{1-\kappa}{\kappa}\bar{\rho}^f\frac{u^2}{U^s}\Big\{\frac{U^s}{U^f}\Big(\boldsymbol{c}^f + \frac{\delta \boldsymbol{a}^f}{\delta t}\Big) - \Big(\boldsymbol{c}^s + \frac{\delta \boldsymbol{a}^s}{\delta t}\Big) +$$
$$-\frac{\boldsymbol{v}^f - \boldsymbol{v}^s}{2}\Big[\frac{U^s}{U^f}\Big(\boldsymbol{c}^f + \frac{\delta \boldsymbol{a}^f}{\delta t}\Big) - \Big(\boldsymbol{c}^s + \frac{\delta \boldsymbol{a}^s}{\delta t}\Big)\Big]\cdot\boldsymbol{n}\Big\} + \bar{\rho}^f U^f u^2(\boldsymbol{a}^f\boldsymbol{n})\boldsymbol{a}^f + \tag{4.9}$$
$$+\frac{1-\kappa}{\kappa}\bar{\rho}^f u^3\Big\{\Big[\frac{(\boldsymbol{a}^f\boldsymbol{n})}{U^f}u\Big(\frac{\boldsymbol{v}^f}{U^f} - \frac{\boldsymbol{v}^s}{U^s}\Big) + u\Big(\frac{\boldsymbol{a}^f}{U^f} - \frac{\boldsymbol{a}^s}{U^s}\Big)\Big]\boldsymbol{n}\Big\}(\boldsymbol{a}^s - \boldsymbol{a}^f) +$$
$$-\frac{1-\kappa}{\kappa}\bar{\rho}^f u^3\Big[(\boldsymbol{a}^f\boldsymbol{n})\frac{U^f}{u}(\boldsymbol{v}^s - \boldsymbol{v}^f) + u(\boldsymbol{a}^s - \boldsymbol{a}^f)\Big]\Big[\Big(\frac{\boldsymbol{a}^s}{U^s} - \frac{\boldsymbol{a}^f}{U^f}\Big)\boldsymbol{n}\Big] +$$
$$-\frac{1-\kappa}{\kappa}\bar{\rho}^f u^2(\boldsymbol{v}^f - \boldsymbol{v}^s)[(\boldsymbol{a}^f\boldsymbol{n})^2 - (\boldsymbol{a}^f\boldsymbol{n})(\boldsymbol{a}^s\boldsymbol{n})]$$

where $\boldsymbol{c}^f$ and $\boldsymbol{c}^s$ are the induced discontinuities associated with the acceleration wave. Some terms are small in comparison with the other ones on the right-hand side of Eqs (4.8) and (4.9). For example in Eq (4.8) the second term on the right-hand side is multiplied by $(\boldsymbol{v}^f - \boldsymbol{v}^s)/U^s$, thus this one is small in comparison with the first one. If we neglect such terms and take into account that in real media $U^s \approx u$ and $U^f \approx u$ we obtain the growth equations in the form

$$\Big[\frac{D(\mathrm{div}\mathbf{T}^s)}{D^s t}\Big] + (1-\kappa)\Big[\frac{D(\mathrm{div}\mathbf{T}^f)}{D^s t}\Big] + \Big[\frac{D\pi}{D^s t}\Big] = u^2[\bar{\rho}^s + (1-\kappa)\bar{\rho}^f]\Big(\boldsymbol{c}^s + 2\frac{\delta \boldsymbol{a}^s}{\delta t}\Big) + \tag{4.10}$$
$$+u^3\Big\{\frac{1-\kappa}{2\kappa}\bar{\rho}^f(\boldsymbol{a}^f - \boldsymbol{a}^s)[(\boldsymbol{a}^f - \boldsymbol{a}^s)n] + \bar{\rho}^s\boldsymbol{a}^s(\boldsymbol{a}^s\boldsymbol{n})\Big\}$$

$$\kappa\Big[\frac{D(\mathrm{div}\mathbf{T}^f)}{D^f t}\Big] - \Big[\frac{D\pi}{D^f t}\Big] = \bar{\rho}^f u^2\Big(\boldsymbol{c}^f + 2\frac{\delta \boldsymbol{a}^f}{\delta t}\Big) - (1-\kappa)\bar{\rho}^f u^2\Big(\boldsymbol{c}^s + 2\frac{\delta \boldsymbol{a}^s}{\delta t}\Big) + \tag{4.11}$$
$$+\bar{\rho}^f u^3\Big\{-\frac{5(1-\kappa)}{2\kappa}(\boldsymbol{a}^f - \boldsymbol{a}^s)[(\boldsymbol{a}^f - \boldsymbol{a}^s)n] + \boldsymbol{a}^f(\boldsymbol{a}^f n)\Big\}$$

In the second part of the paper these growth equations will be used to study the influence of a medium structure on the amplitudes of acceleration waves.

5. Concluding remarks

The propagation conditions and the growth equations have resulted from the above considerations. In the second part of this paper these equations will be used to study the influence of medium structure on the propagation of acceleration waves in the linear viscoelastic fluid- saturated porous solid.

The propagation conditions (3.4) and (3.5) indicate that independently of the constitutive relations, the speeds of displacement depend on the relative motion of constituents. This result is new and is not observed if a medium structure is described by one parameter.

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Zachowanie się fali przyspieszenia w lepkosprężystym ośrodku porowatym ze strukturą. Część I – warunki propagacji i równania amplitud

Streszczenie

W pracy bada się zachowanie się fali przyspieszenia w nasyconym cieczą ośrodku porowatym o strukturze opisanej przez dwa parametry: porowatość objętościową i tensor strukturalnej przepuszczalności. Podstawą rozważań jest opis takiego ośrodka zaproponowany przez Kubika (1986a).

Korzystając z teorii powierzchni osobliwych sformulowano warunki propagacji i wyprowadzono równanie amplitudy fali przyspieszenia.

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