

Dynamics of Predator-Prey Model Interaction with Harvesting Effort

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Abstrak. Di dalam penelitian ini, kami mempelajari dan mengonstruksi suatu dinamika model mangsa-pemangsa. Kami memasukkan unsur kompetisi intraspesifik pada kedua pemangsa. Kami menformulasikan fungsi respon Holling tipe I pada masing-masing pemangsa. Kami menganggap semua populasi bernilai ekonomis sehingga dapat dipanen. Kami menganalisis solusi positifnya, keeksisan titik keseimbangannya, dan kestabilan pada titik-titik keseimbangannya itu. Kondisi kestabilan lokalnya kami peroleh dengan pendekatan kriteria Routh-Hurwitz. Kami juga mensimulasikan model tersebut. Penelitian ini bisa dikembangkan dengan formulasi fungsi respon yang berbeda dan pengoptimalan pemanenan.

Kata kunci: Mangsa-pemangsa; Intraspesifik; Pemanenan; Routh-Hurwitz

Abstract. In this research, we study and construct a dynamic prey-predator model. We include an element of intraspecific competition in both predators. We formulated the Holling type I response function for each predator. We consider all populations to be of economic value so that they can be harvested. We analyze the positive solution, the existence of the equilibrium points, and the stability of the balance points. We obtained the local stability condition by using the Routh-Hurwitz criterion approach. We also simulate the model. This research can be developed with different response function formulations and harvest optimization.

Keywords: Prey-predator; Intraspecific; Harvesting; Routh-Hurwitz

1. Introduction

Mathematical models can be used in observing individual behavior, population dynamics and population linkages in a system. Mathematical models can also be used in determining a policy. Mathematical modeling in the field of ecology is very interesting to study considering the many factors that affect the growth and life of living populations and the balance of organisms. The process of dynamics of organisms can be modeled mathematically by using differential equations involving continuous time or discrete time. One of the mathematical models used to explain this natural phenomenon is the prey-predator population model. Competition between predators and harvesting factors in populations is very important in the discipline of ecology. Many researchers can evoke interesting things from behavioral dynamics in population ecosystems.

By combining the two aspects above, namely the aspects of competition between predators and harvesting, population dynamics can be expressed in a model. One of the policies related to the use of living things is harvesting. Intraspecific competition factors are also interesting to study. Intraspecific competition is competition between predators in competing for prey. This is another factor in population dynamics that can affect the stability of a system.

There are many researchers who model prey-predator interactions. [1] examined the resistance of predators in the prey-predator model system with non-periodic solutions. [2] discusses the dynamics of the prey-predator with diseased predators. [3] in his journal discussed global dynamics of a prey-predator model with antipredator behavior and two predators. [4] discuss the dynamics of the prey-predator model by quadratic harvesting. Research from [5] discusses global dynamics and control of predator prey models with Holling type III response functions. [6] examined the effect of harvesting and competition between predators in the prey-predator model. [7] discuss a model of interaction of three species in one habitat. [8] discuss the complex dynamics of a three-species food chain model with Holling type III response functions.

Many previous researchers have examined prey-predator population models. We examine prey-predator population models with respect to intraspecific competition for predators and considering the economic value of all populations. Our study constructs the factors influencing prey-predator population dynamics as investigated by previous researchers, but we add intraspecific competition and harvesting factors simultaneously to all three populations.

2. Assumptions and model

In this model, there is an interference between predators as modeled by other researchers [1], [6], [8], [9]. There are researchers who studied the intraspecific competition coefficient [9]. Researchers frequently use the Holling-type I response function [3], [5], [8]. The response function is used by researchers in their models [10]–[13]. Researchers also use the harvesting rate [4], [6], [14], [15].

The assumptions used are:

- The prey growth rate uses the logistical growth rate,
- Predators compete with each other for prey,
- All of predators uses the Holling type I response function for predation,
- There is an intraspecific competition for each predator.
- All of population have interest economic values.

The model is formulated as follows:

$$\frac{dP}{dt} = rP \left(1 - \frac{P}{K} \right) - \alpha_1 P H_1 - \alpha_2 P H_2 - q_1 E_1 P \quad (1)$$

$$\begin{aligned}\frac{dH_1}{dt} &= e_1\alpha_1PH_1 - g_1H_1^2 - \beta_1H_1H_2 - d_1H_1 - q_2E_2H_1 \\ \frac{dH_2}{dt} &= e_2\alpha_2PH_2 - g_2H_2^2 - \beta_2H_1H_2 - d_2H_2 - q_3E_3H_2\end{aligned}$$

with initial condition

$$P(0) \geq 0, H_1(0) \geq 0, H_2(0) \geq 0. \quad (2)$$

The first predator (H_1) and second predator (H_2) are assumed to have direct access to prey (P). The effect of disturbance in the growth rate of competitors is assumed to be proportional to the density of the predator population with β_1 and β_2 respectively given disturbance rates. The parameter α_1 and α_2 represent predator rates for predator species H_1 and H_2 respectively. The parameter g_1 and g_2 represent coefficient intraspecific competition for two predators H_1 and H_2 respectively, here d_1 and d_2 are their mortality rate. The parameter e_1 and e_2 are the predator's conversion efficiency. However, the predation functions of the two predators were made different - one following a Holling type I response and the other following a Holling type II response. Besides experiencing a reduction due to the predation function, the prey population grew logistically with r as the intrinsic growth rate and K as the holding capacity. The parameter q_1 , q_2 , and q_3 are the catchability coefficient of susceptible prey, first predator, and second predator, respectively. The parameter E_1 , E_2 , and E_3 are the harvesting effort prey, first predator, and second predator, respectively.

3. Equilibrium points and stability analysis

3.1. Equilibrium points

Equilibrium points of the system (1) are given below:

- The trivial equilibrium point $T_0 = (0, 0, 0)$,
- The predator free equilibrium point $T_1 = \left(K - \frac{Kq_1E_1}{r}, 0, 0\right)$,
- The H_1 -free boundary equilibrium state

$$T_2 = \left(\frac{K(q_3E_3\alpha_2 + \alpha_2d_2 + rg_2 - q_1g_2E_1)}{Ke_2\alpha_2^2 + rg_2}, 0, \frac{Kre_2\alpha_2 - rq_3E_3 - d_2r - Ke_2\alpha_2q_1E_1}{Ke_2\alpha_2^2 + rg_2}\right)$$

- The H_2 -free boundary equilibrium state

$$T_3 = \left(\frac{K(q_2E_2\alpha_1 + \alpha_1d_1 + rg_1 - q_1g_1E_1)}{Ke_1\alpha_1^2 + rg_1}, \frac{Kre_1\alpha_1 - rq_2E_2 - d_1r - Ke_1\alpha_1q_1E_1}{Ke_1\alpha_1^2 + rg_1}, 0\right)$$

- The interior equilibrium point $T_4 = (P^*, H_1^*, H_2^*)$, where

$$\begin{aligned}P^* &= \frac{K(q_1E_1g_1g_2 + q_2E_2\alpha_2\beta_2 + q_3E_3\alpha_1\beta_1 + d_1\alpha_2\beta_2 + d_2\alpha_1\beta_1 + r\beta_1\beta_2)}{Ke_1\alpha_1\alpha_2\beta_2 + Ke_2\alpha_1\alpha_2\beta_1 + r\beta_1\beta_2 - Ke_1\alpha_1^2g_2 - Ke_2\alpha_2^2g_1 - rg_1g_2} \\ &\quad - \frac{K(q_1E_1\beta_1\beta_2 + q_2E_2\alpha_1g_2 + q_3E_3\alpha_2g_1 + d_1\alpha_1g_2 + d_2\alpha_2g_1 + rg_1g_2)}{Ke_1\alpha_1\alpha_2\beta_2 + Ke_2\alpha_1\alpha_2\beta_1 + r\beta_1\beta_2 - Ke_1\alpha_1^2g_2 - Ke_2\alpha_2^2g_1 - rg_1g_2} \\ H_1^* &= \frac{Ke_1\alpha_1q_1E_1g_2 + Ke_2\alpha_2^2q_2E_2 + Kd_1e_2\alpha_2^2 + Kre_2\alpha_2\beta_1 + g_2rq_2E_2 + rd_1g_2}{Ke_1\alpha_1\alpha_2\beta_2 + Ke_2\alpha_1\alpha_2\beta_1 + r\beta_1\beta_2 - Ke_1\alpha_1^2g_2 - Ke_2\alpha_2^2g_1 - rg_1g_2} \\ &\quad - \frac{(Ke_1\alpha_1\alpha_2q_3E_3 + Ke_2\alpha_2\beta_1q_1E_1 + Ke_1\alpha_1\alpha_2d_2 + Kre_1\alpha_1g_2 + r\beta_1q_3E_3 + rd_2\beta_1)}{Ke_1\alpha_1\alpha_2\beta_2 + Ke_2\alpha_1\alpha_2\beta_1 + r\beta_1\beta_2 - Ke_1\alpha_1^2g_2 - Ke_2\alpha_2^2g_1 - rg_1g_2} \\ H_2^* &= \frac{Ke_1\alpha_1q_1E_1\beta_2 + Ke_2\alpha_1\alpha_2q_2E_2 + Kd_1e_2\alpha_1\alpha_2 + Kre_2\alpha_2g_1 + \beta_2rq_2E_2 + rd_1\beta_2}{Ke_1\alpha_1\alpha_2\beta_2 + Ke_2\alpha_1\alpha_2\beta_1 + r\beta_1\beta_2 - Ke_1\alpha_1^2g_2 - Ke_2\alpha_2^2g_1 - rg_1g_2} \\ &\quad - \frac{(Ke_1\alpha_1^2q_3E_3 + Ke_2\alpha_2g_1q_1E_1 + Ke_1\alpha_1^2d_2 + Kre_1\alpha_1\beta_2 + rg_1q_3E_3 + rd_2g_1)}{Ke_1\alpha_1\alpha_2\beta_2 + Ke_2\alpha_1\alpha_2\beta_1 + r\beta_1\beta_2 - Ke_1\alpha_1^2g_2 - Ke_2\alpha_2^2g_1 - rg_1g_2}\end{aligned}$$

3.2. Stability analysis

The stability analysis equilibrium point of the system (1) is studied and determined. The point T_0 is trivial equilibrium point. Jacobian matrix of the model system (1) is

$$J = \begin{bmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{bmatrix} \quad (3)$$

where,

$$\begin{aligned} J_{11} &= r - \frac{2rP}{K} - \alpha_1 H_1 - \alpha_2 H_2 - q_1 E_1 \\ J_{12} &= -\alpha_1 P \\ J_{13} &= -\alpha_2 P \\ J_{21} &= e_1 \alpha_1 H_1 \\ J_{22} &= e_1 \alpha_1 P - 2g_1 H_1 - \beta_1 H_2 - d_1 - q_2 E_2 \\ J_{23} &= -\beta_1 H_1 \\ J_{31} &= e_2 \alpha_2 H_2 \\ J_{32} &= -\beta_2 H_2 \\ J_{33} &= e_2 \alpha_2 P - 2g_2 H_2 - \beta_2 H_1 - d_2 - q_3 E_3 \end{aligned}$$

Theorem 1. Equilibrium point T_1 local stable if $r > q_1 E_1$, $q_2 E_2 + d_1 > \frac{Ke_1 \alpha_1 (r - q_1 E_1)}{r}$, and $q_3 E_3 + d_2 > \frac{Ke_2 \alpha_2 (r - q_1 E_1)}{r}$.

Proof. The result of substitution equilibrium point T_1 to Jacobian Matrix (3)

$$J(T_1) = \begin{bmatrix} J_{11}^1 & J_{12}^1 & J_{13}^1 \\ J_{21}^1 & J_{22}^1 & J_{23}^1 \\ J_{31}^1 & J_{32}^1 & J_{33}^1 \end{bmatrix} \quad (4)$$

where

$$\begin{aligned} J_{11}^1 &= q_1 E_1 - r \\ J_{12}^1 &= -\frac{\alpha_1 K (r - q_1 E_1)}{r} \\ J_{13}^1 &= -\frac{\alpha_2 K (r - q_1 E_1)}{r} \\ J_{21}^1 &= 0 \\ J_{22}^1 &= \frac{Ke_1 \alpha_1 (r - q_1 E_1)}{r} - q_2 E_2 - d_1 \\ J_{23}^1 &= 0 \\ J_{31}^1 &= 0 \\ J_{32}^1 &= 0 \\ J_{33}^1 &= \frac{Ke_2 \alpha_2 (r - q_1 E_1)}{r} - q_3 E_3 - d_2 \end{aligned}$$

Characteristic equation matrix $J(T_1)$ is

$$(\lambda - q_1 E_1 + r) \left(\lambda - \frac{Ke_1 \alpha_1 (r - q_1 E_1)}{r} + q_2 E_2 + d_1 \right) \left(\lambda - \frac{Ke_2 \alpha_2 (r - q_1 E_1)}{r} + q_3 E_3 + d_2 \right) = 0 \quad (5)$$

The roots of the equation (5) is negative if $r > q_1 E_1$, $q_2 E_2 + d_1 > \frac{Ke_1 \alpha_1 (r - q_1 E_1)}{r}$, and $q_3 E_3 + d_2 > \frac{Ke_2 \alpha_2 (r - q_1 E_1)}{r}$.

Theorem 2. Equilibrium point T_2 local stable if $J_{11}^2 < 0, J_{22}^2 < 0, J_{33}^2 < 0, J_{11}^2 J_{22}^2 + J_{11}^2 J_{33}^2 + J_{22}^2 J_{33}^2 > J_{13}^2 J_{31}^2$ and $J_{13}^2 J_{31}^2 J_{22}^2 > J_{11}^2 J_{22}^2 J_{33}^2$.

Proof. The result of substitution equilibrium point T_2 to Jacobian Matrix (3)

$$J(T_2) = \begin{bmatrix} J_{11}^2 & J_{12}^2 & J_{13}^2 \\ J_{21}^2 & J_{22}^2 & J_{23}^2 \\ J_{31}^2 & J_{32}^2 & J_{33}^2 \end{bmatrix} \quad (6)$$

where

$$\begin{aligned} J_{11}^2 &= r - \frac{2r(q_3 E_3 \alpha_2 + \alpha_2 d_2 + r g_2 - q_1 g_2 E_1)}{K e_2 \alpha_2^2 + r g_2} \\ &\quad - \alpha_2 \left[\frac{K r e_2 \alpha_2 - r q_3 E_3 - d_2 r - K e_2 \alpha_2 q_1 E_1}{K e_2 \alpha_2^2 + r g_2} \right] - q_1 E_1 \\ J_{12}^2 &= -\alpha_1 \left[\frac{K(q_3 E_3 \alpha_2 + \alpha_2 d_2 + r g_2 - q_1 g_2 E_1)}{K e_2 \alpha_2^2 + r g_2} \right] \\ J_{13}^2 &= -\alpha_2 \left[\frac{K(q_3 E_3 \alpha_2 + \alpha_2 d_2 + r g_2 - q_1 g_2 E_1)}{K e_2 \alpha_2^2 + r g_2} \right] \\ J_{21}^2 &= 0 \\ J_{22}^2 &= e_1 \alpha_1 \left[\frac{K(q_3 E_3 \alpha_2 + \alpha_2 d_2 + r g_2 - q_1 g_2 E_1)}{K e_2 \alpha_2^2 + r g_2} \right] \\ &\quad - \beta_1 \left[\frac{K r e_2 \alpha_2 - r q_3 E_3 - d_2 r - K e_2 \alpha_2 q_1 E_1}{K e_2 \alpha_2^2 + r g_2} \right] - d_1 - q_2 E_2 \\ J_{23}^2 &= 0 \\ J_{31}^2 &= e_2 \alpha_2 \left[\frac{K r e_2 \alpha_2 - r q_3 E_3 - d_2 r - K e_2 \alpha_2 q_1 E_1}{K e_2 \alpha_2^2 + r g_2} \right] \\ J_{32}^2 &= -\beta_2 \left[\frac{K r e_2 \alpha_2 - r q_3 E_3 - d_2 r - K e_2 \alpha_2 q_1 E_1}{K e_2 \alpha_2^2 + r g_2} \right] \\ J_{33}^2 &= e_2 \alpha_2 \left[\frac{K(q_3 E_3 \alpha_2 + \alpha_2 d_2 + r g_2 - q_1 g_2 E_1)}{K e_2 \alpha_2^2 + r g_2} \right] \\ &\quad - 2g_2 \left[\frac{K r e_2 \alpha_2 - r q_3 E_3 - d_2 r - K e_2 \alpha_2 q_1 E_1}{K e_2 \alpha_2^2 + r g_2} \right] - d_2 - q_3 E_3 \end{aligned}$$

Characteristic equation matrix $J(T_2)$ is

$$\lambda^3 + A_1 \lambda^2 + A_2 \lambda + A_3 = 0 \quad (7)$$

where,

$$\begin{aligned} A_1 &= -(J_{11}^2 + J_{22}^2 + J_{33}^2) \\ A_2 &= J_{11}^2 J_{22}^2 + J_{11}^2 J_{33}^2 + J_{22}^2 J_{33}^2 - J_{13}^2 J_{31}^2 \\ A_3 &= J_{13}^2 J_{31}^2 J_{22}^2 - J_{11}^2 J_{22}^2 J_{33}^2 \end{aligned}$$

The roots of the equation (7) is negative if $J_{11}^2 < 0, J_{22}^2 < 0, J_{33}^2 < 0, J_{11}^2 J_{22}^2 + J_{11}^2 J_{33}^2 + J_{22}^2 J_{33}^2 > J_{13}^2 J_{31}^2$ and $J_{13}^2 J_{31}^2 J_{22}^2 > J_{11}^2 J_{22}^2 J_{33}^2$ and $A_1 A_2 > A_3$

Theorem 3. Equilibrium point T_3 local stable if $J_{11}^3 + J_{22}^3 + J_{33}^3 < 0, J_{11}^3 J_{22}^3 + J_{11}^3 J_{33}^3 + J_{22}^3 J_{33}^3 > J_{12}^3 J_{21}^3, J_{12}^3 J_{21}^3 J_{33}^3 > J_{11}^3 J_{22}^3 J_{33}^3$, and $B_1 B_2 > B_3$

Proof. The result of substitution equilibrium point T_3 to Jacobian Matrix (3)

$$J(T_3) = \begin{bmatrix} J_{11}^3 & J_{12}^3 & J_{13}^3 \\ J_{21}^3 & J_{22}^3 & J_{23}^3 \\ J_{31}^3 & J_{32}^3 & J_{33}^3 \end{bmatrix} \quad (8)$$

where

$$\begin{aligned} J_{11}^3 &= r - 2r \left[\frac{(q_2 E_2 \alpha_1 + \alpha_1 d_1 + r g_1 - q_1 g_1 E_1)}{K e_1 \alpha_1^2 + r g_1} \right] - \alpha_1 \left[\frac{K r e_1 \alpha_1 - r q_2 E_2 - d_1 r - K e_1 \alpha_1 q_1 E_1}{K e_1 \alpha_1^2 + r g_1} \right] - q_1 E_1 \\ J_{12}^3 &= -\alpha_1 \left[\frac{(q_2 E_2 \alpha_1 + \alpha_1 d_1 + r g_1 - q_1 g_1 E_1)}{K e_1 \alpha_1^2 + r g_1} \right] \\ J_{13}^3 &= -\alpha_2 \left[\frac{(q_2 E_2 \alpha_1 + \alpha_1 d_1 + r g_1 - q_1 g_1 E_1)}{K e_1 \alpha_1^2 + r g_1} \right] \\ J_{21}^3 &= e_1 \alpha_1 \left[\frac{K r e_1 \alpha_1 - r q_2 E_2 - d_1 r - K e_1 \alpha_1 q_1 E_1}{K e_1 \alpha_1^2 + r g_1} \right] \\ J_{22}^3 &= e_1 \alpha_1 P - 2g_1 \left[\frac{K r e_1 \alpha_1 - r q_2 E_2 - d_1 r - K e_1 \alpha_1 q_1 E_1}{K e_1 \alpha_1^2 + r g_1} \right] \\ &\quad - \beta_1 \left[\frac{K r e_1 \alpha_1 - r q_2 E_2 - d_1 r - K e_1 \alpha_1 q_1 E_1}{K e_1 \alpha_1^2 + r g_1} \right] - d_1 - q_2 E_2 \\ J_{23}^3 &= -\beta_1 \left[\frac{K r e_1 \alpha_1 - r q_2 E_2 - d_1 r - K e_1 \alpha_1 q_1 E_1}{K e_1 \alpha_1^2 + r g_1} \right] \\ J_{31}^3 &= 0 \\ J_{32}^3 &= 0 \\ J_{33}^3 &= e_2 \alpha_2 \left[\frac{K (q_2 E_2 \alpha_1 + \alpha_1 d_1 + r g_1 - q_1 g_1 E_1)}{K e_1 \alpha_1^2 + r g_1} \right] \\ &\quad - \beta_2 \left[\frac{K r e_1 \alpha_1 - r q_2 E_2 - d_1 r - K e_1 \alpha_1 q_1 E_1}{K e_1 \alpha_1^2 + r g_1} \right] - d_2 - q_3 E_3 \end{aligned}$$

Characteristics equation matrix $J(T_3)$ is

$$\lambda^3 + B_1 \lambda^2 + B_2 \lambda + B_3 = 0 \quad (9)$$

where,

$$\begin{aligned} B_1 &= -(J_{11}^3 + J_{22}^3 + J_{33}^3) \\ B_2 &= J_{11}^3 J_{22}^3 + J_{11}^3 J_{33}^3 + J_{22}^3 J_{33}^3 - J_{12}^3 J_{21}^3 \\ B_3 &= J_{12}^3 J_{21}^3 J_{33}^3 - J_{11}^3 J_{22}^3 J_{33}^3. \end{aligned}$$

To ensure the stability of model system with equilibrium point T_3 , the point must qualify of the Routh-Hurtwitz criteria. The equation (9) have negative roots if $J_{11}^3 + J_{22}^3 + J_{33}^3 < 0$, $J_{11}^3 J_{22}^3 + J_{11}^3 J_{33}^3 + J_{22}^3 J_{33}^3 > J_{12}^3 J_{21}^3$, $J_{12}^3 J_{21}^3 J_{33}^3 > J_{11}^3 J_{22}^3 J_{33}^3$, and $B_1 B_2 > B_3$.

Theorem 4 Equilibrium point T_4 local stable if $J_{11}^4 + J_{22}^4 + J_{33}^4 < 0$, $J_{11}^4 J_{22}^4 + J_{11}^4 J_{33}^4 + J_{22}^4 J_{33}^4 > J_{12}^4 J_{21}^4 + J_{13}^4 J_{31}^4 + J_{23}^4 J_{32}^4$, $J_{11}^4 J_{23}^4 J_{32}^4 + J_{12}^4 J_{21}^4 J_{33}^4 + J_{13}^4 J_{31}^4 J_{22}^4 > J_{11}^4 J_{22}^4 J_{33}^4 + J_{12}^4 J_{23}^4 J_{31}^4 + J_{13}^4 J_{32}^4 J_{21}^4$, and $C_1 C_2 > C_3$.

Proof. The result of substitution equilibrium point T_4 to Jacobian Matrix (3)

$$J(T_4) = \begin{bmatrix} J_{11}^4 & J_{12}^4 & J_{13}^4 \\ J_{21}^4 & J_{22}^4 & J_{23}^4 \\ J_{31}^4 & J_{32}^4 & J_{33}^4 \end{bmatrix} \quad (10)$$

where

$$\begin{aligned}
 J_{11}^4 &= r - \frac{2rP^*}{K} - \alpha_1 H_1^* - \alpha_2 H_2^* - q_1 E_1 \\
 J_{12}^4 &= -\alpha_1 P^* \\
 J_{13}^4 &= -\alpha_2 P^* \\
 J_{21}^4 &= e_1 \alpha_1 H_1^* \\
 J_{22}^4 &= e_1 \alpha_1 P^* - 2g_1 H_1^* - \beta_1 H_2^* - d_1 - q_2 E_2 \\
 J_{23}^4 &= -\beta_1 H_1^* \\
 J_{31}^4 &= e_2 \alpha_2 H_2^* \\
 J_{32}^4 &= -\beta_2 H_2^* \\
 J_{33}^4 &= e_2 \alpha_2 P^* - 2g_2 H_2 - \beta_2 H_1^* - d_2 - q_3 E_3
 \end{aligned}$$

Characteristics equation matrix $J(T_4)$ is

$$\lambda^3 + C_1 \lambda^2 + C_2 \lambda + C_3 = 0 \tag{11}$$

where,

$$\begin{aligned}
 C_1 &= -(J_{11}^4 + J_{22}^4 + J_{33}^4) \\
 C_2 &= J_{11}^4 J_{22}^4 + J_{11}^4 J_{33}^4 + J_{22}^4 J_{33}^4 - J_{12}^4 J_{21}^4 - J_{13}^4 J_{31}^4 - J_{23}^4 J_{32}^4 \\
 C_3 &= J_{11}^4 J_{23}^4 J_{32}^4 + J_{12}^4 J_{21}^4 J_{33}^4 + J_{13}^4 J_{31}^4 J_{22}^4 - J_{11}^4 J_{22}^4 J_{33}^4 - J_{12}^4 J_{23}^4 J_{31}^4 - J_{13}^4 J_{32}^4 J_{21}^4.
 \end{aligned}$$

To ensure the stability of model system with equilibrium point E_4 , the point must qualify of the Routh-Hurtwiz criteria. The equation (11) have negative roots if $J_{11}^4 + J_{22}^4 + J_{33}^4 < 0$, $J_{11}^4 J_{22}^4 + J_{11}^4 J_{33}^4 + J_{22}^4 J_{33}^4 > J_{12}^4 J_{21}^4 + J_{13}^4 J_{31}^4 + J_{23}^4 J_{32}^4$, $J_{11}^4 J_{23}^4 J_{32}^4 + J_{12}^4 J_{21}^4 J_{33}^4 + J_{13}^4 J_{31}^4 J_{22}^4 > J_{11}^4 J_{22}^4 J_{33}^4 + J_{12}^4 J_{23}^4 J_{31}^4 + J_{13}^4 J_{32}^4 J_{21}^4$, and $C_1 C_2 > C_3$.

4. Numerical Simulation

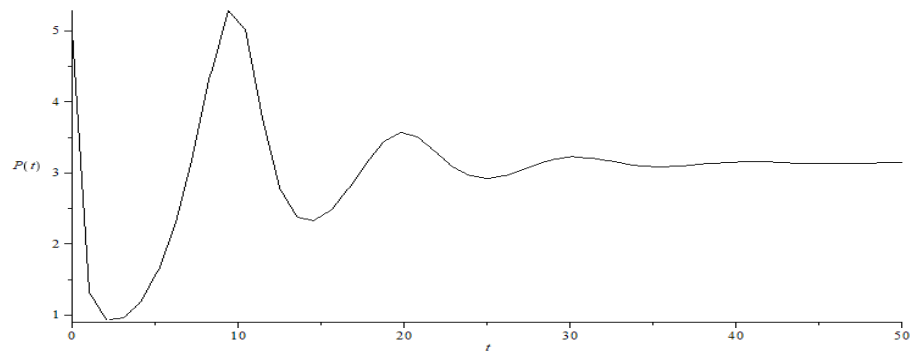
In this section we simulated the model with some parameter values. The parameter values was adopted from literature [3], [6], [8], [9], [11], [15]. We try simulated the model with some condition. The first condition with parameter with $E_1 = 0.28773096$, $E_2 = 0.24093140$ and $E_3 = 0.13141604$. The second condition with parameter $E_1 = 0.2$, $E_2 = 0.4$ and $E_3 = 0.3$. The third condition with $E_1 = 0.2$, $E_2 = 0.3$ and $E_3 = 0.4$. To see the system is in a stable state, a numerical simulation is performed with parameter estimates according to the following Table 1.

Parameter	Values
r	1
K	100
α_1	0.21
α_2	0.274
e_1	1
e_2	1
g_1	0.1
g_2	0.2
β_1	0.05
β_2	0.06
d_1	0.05
d_2	0.06
q_1	1
q_2	1
q_3	1
E_1	0.5

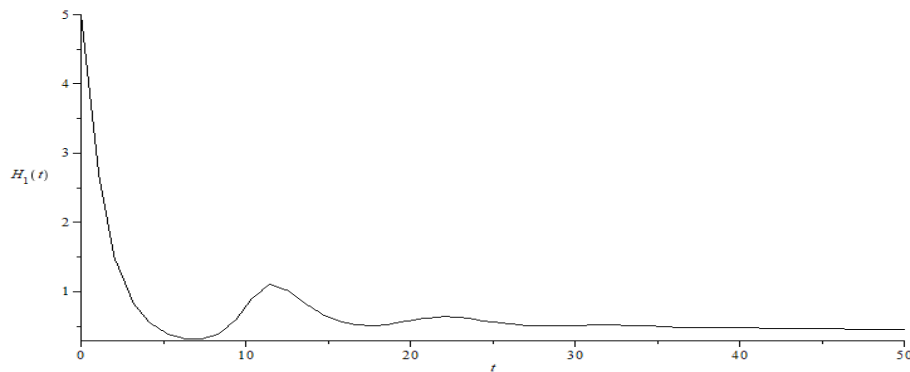
Parameter	Values
E_2	0.5
E_3	0.5

With the parameter values in **Table 1**, the simulation results are given nonnegative equilibrium points:

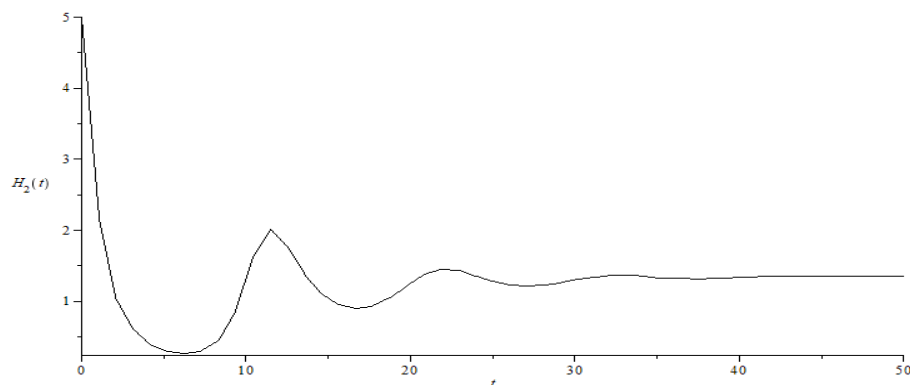
- $T_0 = (0, 0, 0)$
 $T_1 = (50, 0, 0)$,
 $T_2 = (3.288183092, 0, 1.704810836)$,
 $T_3 = (3.669623060, 2.206208426, 0)$,
 $T_4 = (3.149229952, 0.4190122146, 1.388741370)$.



(a)



(b)



(c)

Figure 1. Numerical simulation model. (a) prey population density; (b) first predator population density; (c) second predator population density

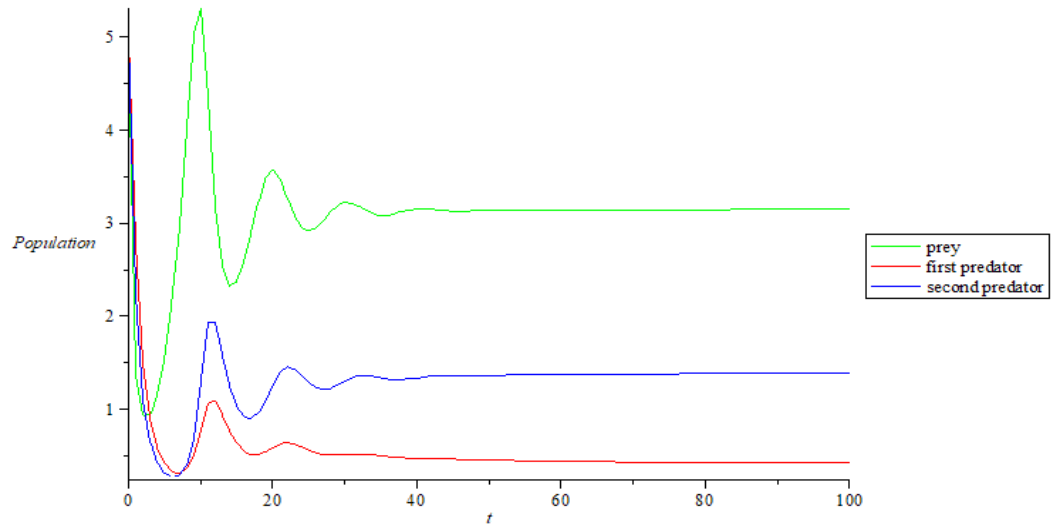


Figure 2. Time series of the model with $E_1 = 0.5$, $E_2 = 0.5$ and $E_3 = 0.5$

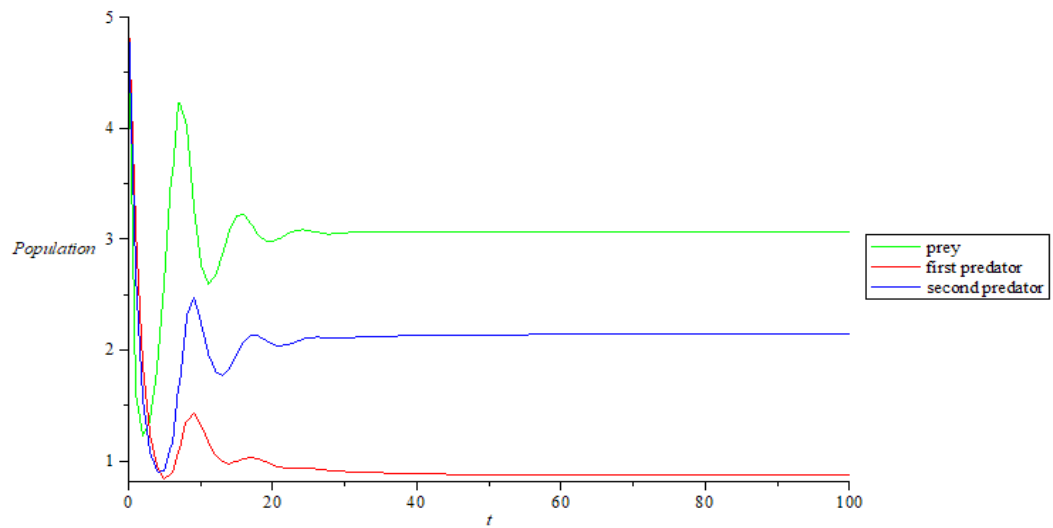


Figure 3. Time series of the model with $E_1 = 0.2$, $E_2 = 0.4$ and $E_3 = 0.3$

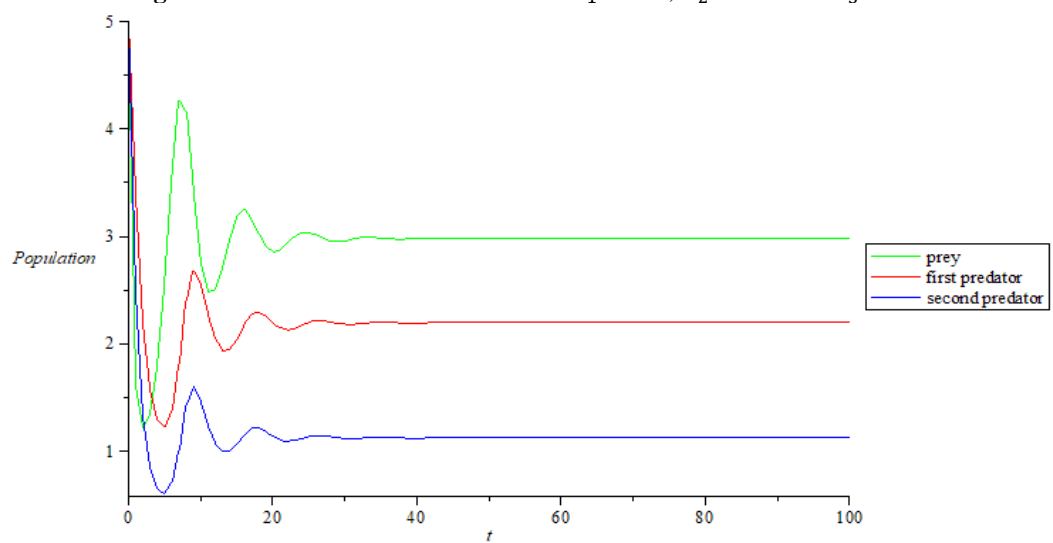


Figure 4. Time series of the model with $E_1 = 0.2$, $E_2 = 0.3$ and $E_3 = 0.4$

Figure 1 shows the existence of each population. With a combination of the parameters presented, the food supply of predators is more than the predators themselves. the system is expected to last a long time with a combination of these parameters.

Figure 2 shows the simulation results with given parameters with the same parameter ($E_1 = 0.5$, $E_2 = 0.5$ and $E_3 = 0.5$) of harvest effort for all populations. In this simulation, the system is stable and lasts a long time with the same harvesting effort conditions. To demonstrate system dynamics, we simulate models with varying combinations of harvest effort parameters.

By changing the number of harvesting business parameters to $E_1 = 0.2$, $E_2 = 0.4$ and $E_3 = 0.3$, the simulation plot will look different. **Figure 3** shows that the number of the first predator population is reduced compared to other populations because the number of harvesting efforts is also the highest.

With different parameter numbers ($E_1 = 0.2$, $E_2 = 0.3$ and $E_3 = 0.4$), **Figure 4** shows the same symptoms, the second predator population is the smallest because the number of harvesting efforts is the largest.

Apart from the stability of the system, the policy in harvesting in this study is a matter of focus. In this study, harvesting also determines the stability of a system, if the harvesting of a population changes and does not match the harvest rates in other populations, it will cause system stability disturbances. It can be seen clearly in the simulation that produces dynamic graphs in Figures 2, 3, and 4, the population density of one predator is sometimes more than other predators, and vice versa.

The results of this study are intended to show in general that harvesting efforts have a large enough impact on population sustainability and also have an impact on the system. Harvesting effort can interfere with the growth and activity of predators. We look visually in the image with the selection of different harvesting efforts. Population that is harvested in large numbers to other populations will decrease in population.

This research can be developed by considering other factors that make a system dynamic. For example, further researchers can optimize harvesting efforts so that economic benefits can be clearly measured.

5. Conclusions

In this section we will make conclusions of this research. This study focuses on the dynamics of prey-predator populations with harvesting effort for all of population. There are 5 non-negative equilibrium point of the system. The interior point is T_4 . The equilibrium point T_4 stabil if $J_{11}^4 + J_{22}^4 + J_{33}^4 < 0$, $J_{11}^4 J_{22}^4 + J_{11}^4 J_{33}^4 + J_{22}^4 J_{33}^4 > J_{12}^4 J_{21}^4 + J_{13}^4 J_{31}^4 + J_{23}^4 J_{32}^4$, $J_{11}^4 J_{23}^4 J_{32}^4 + J_{12}^4 J_{21}^4 J_{33}^4 + J_{13}^4 J_{31}^4 J_{22}^4 > J_{11}^4 J_{22}^4 J_{33}^4 + J_{12}^4 J_{23}^4 J_{31}^4 + J_{13}^4 J_{32}^4 J_{21}^4$, and $C_1 C_2 > C_3$. Harvesting effort have impact to system. Harvesting efforts will reduce the population size so that it can affect the stability of the dynamics of the prey-predator population system. The system will be stable and exist if we have control quantity of the harvesting effort. The system will be stable and exist if we have control's the harvesting effort.

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