Mathematical and Software Engineering, Vol. 3, No. 1 (2017), 139-148.
Varεpsilon Ltd, varepsilon.com

Modelling of Nigerian Residential Electricity Consumption Using Multiple Regression Model with One Period Lagged Dependent Variable

Runcie Amlabu[1], Nseobong. I. Okpura[2], Anthony Mfonobong Umoren[3]

corresponding author: umoren_m.anthony@yahoo.com

[1,2,3] Department of Electrical/Electronic and Computer Engineering, University of Uyo, Akwa Ibom, Nigeria

Abstract

This paper presents the modelling and forecasting of residential electricity consumption in Nigeria based on nine years (2006 and 2014) data and multiple regression model with one period lagged dependent variable. A Socio economic parameter (population), and climatic parameter (annual average temperature) are used as explanatory variables in modelling the and forecasting of residential electricity consumption in Nigeria. The results of the multiple regression analysis applied to the data arrived at the model with the least sum of square error as $\hat{E}_t = -36.2458 + 9.7202P_t - 12.0265T_t + 0.1540E_{t-1}$, *where t is the year;* $\hat{E}_t$ *is the predicted residential electricity demand in MW/h;* P_t *is the annual population in millions;* T_t *is the average annual temperature in °C and* E_{t-1} *is the residential electricity demand in the year before year t. The error analysis gave coefficient of determinant of 0.913, adjusted coefficient of determination of 0.86 and Root Mean Square Error of 61.86. The forecast results gave 5.11% annual average increase in the electric power demand of the residential sector with respect to the 2014 electricity consumption data. Such results presented in this paper are useful for effective planning of power supply to the residential sector in Nigeria.*

Keywords: Multiple Regression; Regression Analysis; Error Analysis; Least Square Method; Sum of Square Error; Forecast; Residential Electricity Demand

1. Introduction

Across the globe, electricity is one of the most dominant forms of energy. Some of the most common advantages of electricity as a future carrier of energy include its cleanliness, versatility, accessibility and simplicity in distribution. The flexibility of electricity as an energy carrier has contributed to technological innovation and increased industrial productivity in many countries [1]. These advantages have contributed to the increased share of electricity in the total energy consumption in many countries. Accordingly, in advanced economies, the ideal choice for the carrier of energy is electricity. Furthermore, electricity power supply has been identified as the most important commodity for the development of a nation [2,3,4,5,6]. People become easily empowered to work and develop themselves when there is electricity power supply. This development takes place from domestic level to industrial level and can also be transferred from small to medium and to large scale level.

Among other things, electricity consumption demand depends on the population and level of industrialization of a country [2,7,8,9,10,11,12]. The increased use of electricity from residential homes to industry has contributed to the increasing demand in electricity consumption worldwide. The growth in demand is even higher in developing countries as the robust economic growth boosts demand for new electrical appliances. Although the rate of growth in electricity consumption may be slower in the industrialized countries, their dependence on electricity may even be higher than the developing countries [12,13,14]. The high demand due to the heavy dependence on electricity requires planning of resources of electricity well in advance to ensure a continuous supply of electricity in the future.

In the power industry, proper planning of electric power system requires among other things, modelling of the electric power demand [15,16,17,18]. In this paper, the focus is on modelling of the residential electric power demand in Nigeria. This has become necessary in view of the perennial shortfall in power supply across Nigeria and the sweeping policy changes the government is making to address the challenges in the power sector [3,19,20]. Particularly, multiple regression model with one period lagged dependent variable is considered in the paper for modelling the residential electricity consumption in Nigeria [21,22]. The study is based on nine years (2006 and 2014) data on residential electric power demand in Nigeria. A socio-economic variable is used as parameters along with climatic variable for the modelling of the residential electricity consumption in Nigeria. The socio economic variable is population while the climatic variable is average annual temperature. Finally, performance evaluation of the selected model is conducted using statistical measures such as the Root Mean Square Error(RMSE), coefficient of determination (r^2) and adjusted coefficient of determination ($r^2{}_{adj}$).

2. Methodology

2.1. Sources of data

Nine (9) years (2006-2014) data on yearly average residential electricity consumption expressed in megawatt hour (MW/h) are obtained from Central Bank of Nigeria Statistical Bulletin [23] while data on yearly average temperature (in ^{0}C) and population are obtained from the Bulletin of the National Bureau of Statistics [24].

2.2. Description Of The Models For The Predictor and The Response Variables

Multiple regressions with one period lagged of the dependent variable is used to express E_t (the yearly average residential electricity consumption) in Nigeria as a linear function of population (P_t), temperature (T_t) and E_{t-1} (which is the electricity consumption with one period lagged of the dependent variable). In the model, E_t is defined as follows;

$$E_t = f(P_t, T_t, E_{t-1}) \quad (1)$$

$$E_t = \alpha_0 + \alpha_1 P_t + \alpha_2 T_t + \alpha_3 E_{t-1} + \varepsilon_t \quad (2)$$

where

E_t is the yearly average residential electricity consumption (in MW/h) estimated at time, t; P_t is population at time t; T_t is temperature at time t; t is time in years; α_1, α_2 and α_3 are regression coefficients. A regression coefficient in multiple regressions is the slope of the linear relationship between the dependent variable and the part of a dependent variable that is independent of all other independent variables. Specifically, α_1, α_2 and α_3 are the contributions of population (P), temperature (T) and E_{t-1} respectively and α_0 is the intercept; ε_t is a random error or residuals term. Measurement error for the dependent and independent variables, the random nature of human responses and effect of omitted variables are the main sources of the random disturbance [5].

In this study, data for the independent variable, namely, P_t and T_t are available for the years 2006 to 2014. Mathematical expression for each of the independent variables P_t and T_t are themselves obtained from models applied to the data sets of these variables over time (t). Particularly, the population for the years beyond 2014 are projected using the mathematical expression;

$$P_t = P_{t-1}(1+r)^n \tag{3}$$

where n is the number of years, P_{t-1} is previous year population, P_t is the population of the year to be estimated and r is the population growth rate of Nigeria which is given as 3.2% according to 2006 population census [4,5]. Also, temperature is predicted using simple linear regression model as follows;

$$T_t = \beta_0 + \beta_1(t) \tag{4}$$

Where β_0 and β_1 are the simple linear regression coefficients and t is time in years. The model parameters β_0 and β_1 are estimated using MATLAB and the following values are obtained; $\beta_0 = 34.602$ and $\beta_1 = 0.0613$. Therefore,

$$T_t = 34.602 - 0.0613\text{t} \tag{5}$$

Where t = time, t= 10,11,…,20.

3. Regression Analysis for the Multiple Regression Model with One Period Lagged Dependent Variable

The multiple regression model with one period lagged dependent variable (in Eq. 2) can be written in matrix form as;

$$\text{E} = \alpha \ \text{X} + \varepsilon \tag{6}$$

where

- n is the number of sampled points
- k is the number of predictor variables. In the given model, the predictors are P_t, T_t and E_{t-1}, hence, k = 3
- $E = [E_1, \dots, E_n]$ is the n × 1 column vector (or n × 1 matrix) of the response variable which is the electricity consumption
- X is an n × (k+ 1) design matrix determined by the regression model predictors whereby the values in the first column of the matrix are all 1.
- $\alpha = [\alpha_0, \dots, \alpha_k]$ is k × 1 column vector of parameters (or k × 1 matrix of predictor coefficient)
- $\varepsilon = [\varepsilon_1, \dots, \varepsilon_n]$ is an n × 1 column matrix called the error vector or vector of error terms

Let $\hat{\alpha} = [\hat{\alpha}_1, \dots, \hat{\alpha}_k]$ be the vector of least squares estimator or predictor coefficient which give the predicted dependent variable $\hat{E}$ that has the least possible value to sum of the squares error. The regression coefficients or least squares estimator, $\hat{\alpha}$ that minimize the sum of the squared errors for the multiple regression are determined by solving the least squares normal equation given as ;

$$X^T X(\hat{\alpha}) = X^T E \quad (7)$$

Then, the least squares estimator $\hat{\beta}$ is given as;

$$\hat{\alpha} = (X^T X)^{-1} (X^T E) \quad (8)$$

Let $R_d = X^T X$ and $R_d^{-1} = (X^T X)^{-1}$ and $U_d = (X^T E)$, the vector of least squares estimators

$$\hat{\alpha} = (X^T X)^{-1} (X^T E) = (R_d^{-1}) U_d \quad (9)$$

From Eq. 2, the error term ε_t can be expressed as follows

$$\varepsilon_t = E_t - \alpha_0 - \alpha_1 P_t - \alpha_2 T_t - \alpha_3 E_{t-1} \quad (10)$$

$$\varepsilon = E - \alpha X \quad (11)$$

When $\hat{\alpha}$ (the vector of least squares estimator) is considered, then

$$\varepsilon = E - \hat{\alpha} X \quad (12)$$

Let $\hat{E}$ be the predicted electricity consumption with least square sum of errors, then;

$$\hat{E} = \hat{\alpha} X \quad (13)$$

$$\varepsilon = E - \hat{E} \quad (14)$$

4. Numerical Computations, Result, and Discussion

Data on the residential electricity consumption, national population and yearly average temperature are given in Table 1.

Table 1. Nine (9) years (2006-2014) data on yearly average residential electricity consumption expressed in megawatt hour (MW/h), national population and yearly average temperature (in ^{0}C)

n	Year (t)	(P_t) Population in millions	(T_t) Temperature in °C	(E_t) Residential Electricity Consumption in MW/h
1	2006	140.43	34.64	894.11
2	2007	145.07	34.55	1151.94
3	2008	149.65	34.69	1165.5
4	2009	154.52	34.19	1104
5	2010	159.62	34.38	1365.5
6	2011	164.73	33.33	1401.01
7	2012	170.16	34.43	1437.43
8	2013	175.78	33.89	1474.81
9	2014	181.05	34.56	1513.15

(Source: Central Bank of Nigeria Statistical Bulletin and the Bulletin of the National Bureau of Statistics)

From the given data in Table 1, n = 9 and from the model (Eq. 2), the predictor variables are P_t, T_t and E_{t-1} , hence, k = 3. Then, the 9 x4 design X matrix determined by the regression model predictors is given as;

$$X = \begin{pmatrix} 1 & 140.43 & 34.64 & 34.64 \\ 1 & 145.07 & 34.55 & 1151.94 \\ 1 & 149.65 & 34.69 & 1165.50 \\ 1 & 154.52 & 34.19 & 1104 \\ 1 & 159.62 & 34.38 & 1365.5 \\ 1 & 164.73 & 33.33 & 1401.01 \\ 1 & 170.16 & 34.43 & 1437.43 \\ 1 & 175.78 & 33.89 & 1474.81 \\ 1 & 181.05 & 34.56 & 1513.15 \end{pmatrix} \tag{15}$$

Also, E (the column vector of the response variable) is given from Table 1 as;

$$E = \begin{pmatrix} 894.11 \\ 1151.94 \\ 1165.5 \\ 1104 \\ 1365.5 \\ 1401.01 \\ 1437.43 \\ 1474.81 \\ 1513.15 \end{pmatrix} \tag{16}$$

Let $R_d = X^T X$ and $R_d^{-1} = (X^T X)^{-1}$, then

$$R_d = \begin{pmatrix} 9 & 1441.01 & 308.66 & 9994.30 \\ 1441.01 & 232284.01 & 49402.13 & 1641430.37 \\ 308.66 & 49402.13 & 10587.21 & 342089.10 \\ 9994.30 & 1641430.37 & 342089.10 & 12772293.52 \end{pmatrix} \tag{17}$$

$$R_d^{-1} = \begin{pmatrix} 1004.506 & -0.3238 & -27.69 & -0.0038 \\ -0.3238 & 0.0018 & 0.0023 & -0.000044 \\ -27.69 & 0.0023 & 0.788 & 0.00026 \\ -0.0038 & -0.000044 & 0.00026 & 0.0000018 \end{pmatrix} \tag{18}$$

$$U_d = (X^T E) = \begin{pmatrix} 11507.45 \\ 1864219.44 \\ 394356.496 \\ 13445248.99 \end{pmatrix} \tag{19}$$

$$\hat{\alpha} = \left(R_d^{-1}\right) U_d = \begin{pmatrix} -36.2458 \\ 9.7202 \\ -12.0265 \\ 0.1540 \end{pmatrix} \tag{20}$$

From the numerical computations, the regression coefficients that gave the least sum of square error are :

$$\left. \begin{aligned} \hat{\alpha}_0 &= -36.2458 \\ \hat{\alpha}_1 &= 9.7202 \\ \hat{\alpha}_2 &= -12.0265 \\ \hat{\alpha}_3 &= 0.1540 \end{aligned} \right\} \tag{21}$$

Hence, the residential electricity consumption in Nigeria can be effective modelled and hence predicted by the multiple regression model with one period lagged dependent variable given as;

$$E_t = -36.2458 + 9.7202P_t - 12.0265T_t + 0.1540E_{t-1} + \varepsilon_t \quad (22)$$

Along with the regression analysis, the error analysis is also conducted to determine the goodness of fit for each of the two models. The regression goodness of fit are evaluated in terms of coefficient of determination (r^2) , adjusted coefficient of determination ($r^2{}_{adj}$) and Root Mean Square Error (RMSE). The actual and predicted yearly average residential electricity consumption in Nigeria from 2006 to 2014 are as shown in Table 2 and figure 1. The results for the error analysis are also presented in Table 2.

Table 2: The Actual and Model Predicted Residential Electricity Consumption (MWh) In Nigeria From 2006 to 2014 Along With The Goodness Of Fit Measures

S/N	Year	(Et) Actual Residential Electricity Consumption (MWh)	(Êt) Model Predicted Residential Electricity Consumption (MWh)
1	2006	894.11	912.1684
2	2007	1151.94	1096.002
3	2008	1165.5	1178.536
4	2009	1104	1233.979
5	2010	1365.5	1271.808
6	2011	1401.01	1374.363
7	2012	1437.43	1419.397
8	2013	1474.81	1486.131
9	2014	1513.15	1535.065

Goodness of Fit Measures	
RMSE	58.63386
r^2	0.911244
$r^2{}_{adj}$	0.857991

From the results in Table 1 the coefficient of determination (r^2) of 0.911244 shows that 91.12% of the variation in residential electricity consumption was accounted for by the multiple regression model with one period lagged of the dependent variable. The Root Mean Square Error (RMSE) of 58.63386 is achieved by the model with adjusted coefficient of determination($r^2{}_{adj}$) of 0.857991. Both r^2 and $r^2{}_{adj}$ are very high which shows good prediction results and that the model is not over fitted.

Also, from the model in Eq. 22 the contribution of population ($\hat{\alpha}_1$ = 9.7202) is positive meaning that as population increases, residential electricity consumption also increases. On the other hand, based on the model, as the temperature increases, the residential electricity consumption decreases.

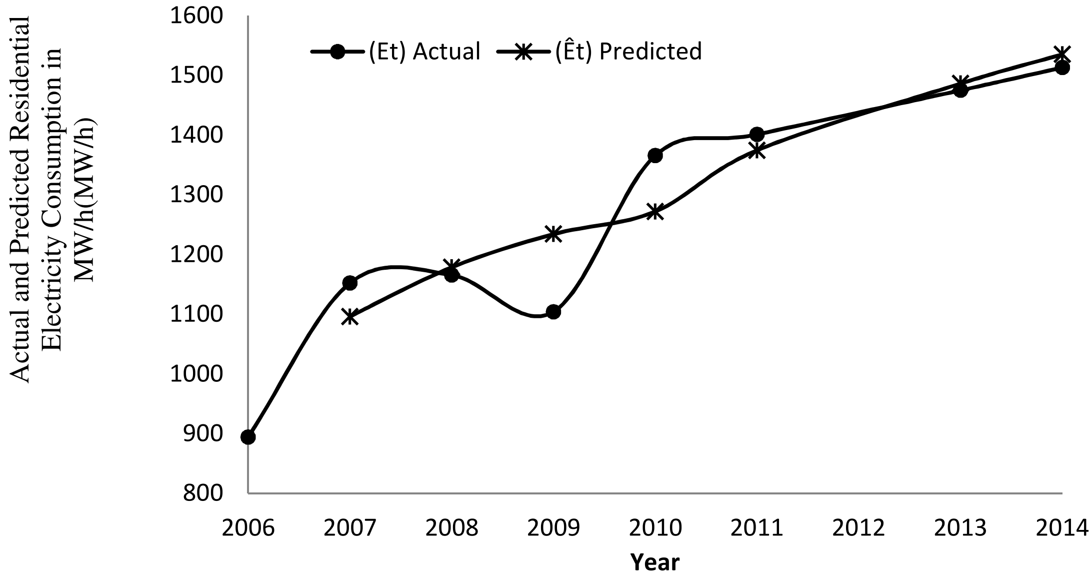

Figure 1. The actual and predicted yearly average residential electricity consumption in Nigeria from 2006 to 2014

Furthermore, the result of the Nigerian residential electricity consumption forecasted by the multiple regression model with one period lagged of the dependent variable is given in Table 3 and Figure 2.

Table 3. Forecasted Population, Temperature and Nigerian Residential Electricity Consumption

Year	**Population (Million)**	**Temperature (^{0}C)**	**Forecasted Residential Electricity Consumption (MW/h)**	**Percentage Increase over The Previous Year (%)**	**Percentage Increase over 2014 (The Year Last With Available Data (%)**
2014	181.05	34.56	1513.15		
2015	186.84	33.99	1604.108642	6.011211182	6.011211182
2016	192.82	33.93	1676.968372	4.542069539	10.82631411
2017	198.99	33.87	1748.889014	4.288729782	15.57935525
2018	205.36	33.81	1822.609437	4.215271662	20.45133906
2019	211.93	33.74	1898.671472	4.173249269	25.47807369
2020	218.71	33.68	1977.015351	4.126247229	30.65560923
		Mean	1788.043715	4.55946311	5.109268204

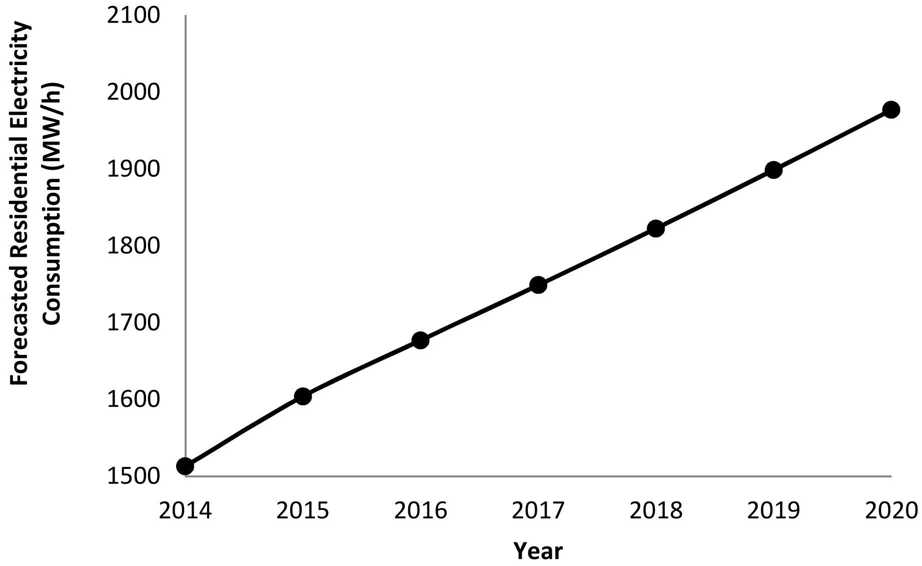

Figure 2. Forecasted Residential Electricity Consumption (MW/h)

The forecasted residential electricity consumption in Nigeria shows that there is an annual average of 1788.04 MW/h of electricity consumption by the residential sector in Nigeria. The consumption pattern shows 4.31% annual average increase in electricity consumption over the previous year which is equivalent to 4.56% annual average increase in electricity consumption over the year 2014 electricity consumption data. In essence, the authorities responsible for electricity supply to the residential sector can plan for 5.11% annual average increase in the electric power demand of the residential sector with respect to the 2014 electricity consumption data. Over the six years period (2014 to 2020) there is a total of 30.66% increase in residential electricity consumption over the year 2014 electricity consumption data.

5. Conclusion

The paper presented the residential demand for electricity in Nigeria, employing annual data over the period 2006 –2014. Multiple regression model with one period lagged dependent variable is applied to estimate residential electricity consumption and to forecast medium-term residential demand for electricity. The results of the multiple regression analysis applied to the data arrived at the model with the least sum of square error as $\hat{E}_t = -36.2458 + 9.7202P_t - 12.0265T_t + 0.1540E_{t-1}$, where t is the year ; $\hat{E}_t$ is the predicted residential electricity demand in MW/h; P_t is the annual population in millions; T_t is the average annual temperature in °C and E_{t-1} is the residential electricity demand in the year before year *t*.

The error analysis gave high coefficient of determinant, high adjusted coefficient of determinant and reasonably low and acceptable value of Root Mean Square Error. In addition, six years ahead forecast showed gave the average annual percentage increment in residential electricity consumption in Nigeria. Such results presented in this paper are useful for effective planning of power supply to the residential sector in Nigeria.

References

[1] Mohamed, E. (2004). Forecasting Electricity Consumption: A comparison of growth curves, econometrics and ARIMA models for selected countries and world region. M.Eng Thesis, Department of Electronic Engineering, University of Canterbury, New Zealand.

[2] Oke, M.O. and Bamigbola, OM. (2013) Minimization of losses on electric power transmission lines, J. Math. Theor. Model. 3(7), 28-31.

[3] Awosope, C.A. (2014). Nigeria Electricity Industry: Issues, Challenges and Solutions, Public Lecture Series. Vol. 3, No. 2.

[4] Oyedepo, S.O. (2012). On energy for sustainable development in Nigeria. *Renewable and Sustainable Energy Reviews*, *16*(5), 2583-2598.

[5] Adenikinju, A. (2008). Efficiency of the Energy Sector and its Impact on the Competitiveness of the Nigerian Economy. *International Association for Energy Economics*, *4*, 27-31.

[6] Frederick, D., & Selase, A.E. (2014). The Effect of Electric Power Fluctuations on the Profitability and Competitiveness of SMEs: A Study of SMEs within the Accra Business District of Ghana. *Journal of Competitiveness*, *6*(3), 32-48.

[7] De Sherbinin, A., Carr, D., Cassels, S., & Jiang, L. (2007). Population and environment. *Annual review of environment and resources*, *32*, 345-373.

[8] Oyedepo, S.O. (2012). Energy and sustainable development in Nigeria: the way forward. *Energy, Sustainability and Society*, *2*:15.

[9] Filippini, M., & Pachauri, S. (2004). Elasticities of electricity demand in urban Indian households. *Energy policy*, *32*(3), 429-436.

[10] Awan, A.G. Relationship between Environment and Sustainable Economic Development: A Theoretical Approach to Environmental Problems. *International Journal of Asian Social Sciences* 3.3 (2013): 741-761.

[11] Reddy, A. K., Annecke, W., Blok, K., Bloom, D., Boardman, B., Eberhard, A., & Ramakrishna, J. (2000). *World Energy Assessment: Energy and the Challenge of Sustainability.* Energy and social issues, Chapter 2, 39-60.

[12] Ahuja, D., Tatsutani, M. (2009) Sustainable energy for developing countries. *SAPI EN. S. Surveys and Perspectives Integrating Environment and Society*, (2.1).

[13] Dahlman, C. (2007). Technology, globalization, and international competitiveness: Challenges for developing countries. *Industrial Development for the 21st Century*, 29-83.

[14] Krymm, R., Lane, J. A., and Zheludev, I.S. (1977). Future Trends in Nuclear Power. *IAEA Bulletin, Twentieth Anniversary Issue*, *19*(4), 47-54.

[15] Beasley, B., & Morris, D.F. (2012). Modeling the Electricity Sector: A Summary of Recent Analyses of New EPA Regulations. *Resources for the Future Discussion Paper*, RFF DP 12-52.

[16] Drouineau, M., Mazauric, V., Maïzi, N., and Assoumou, E. (2010). The cost of reliability of supply in future power systems. In *3rd IAEE Rio 2010 International Conference" The Future of Energy: Global Challenges, Diverse Solutions"*, 12.

[17] Andersson, G. (2008). Modelling and analysis of electric power systems. *ETH Zurich,* Lecture 227-0526-00.

[18] Camacho, E.F., Samad, T., Garcia-Sanz, M., & Hiskens, I. (2011). Control for renewable energy and smart grids. *The Impact of Control Technology, Control Systems Society*, 69-88.

[19] Usman, Z. G., & Abbasoglu, S. (2014). An overview of power sector laws, policies and reforms in Nigeria. *Asian Transactions on Engineering*, *4*(2), 6-12.

[20] Babalola, R.O. (1999). *Technical Efficiency and Impact of Ownership on*

Performance: The Case of the Nigerian Electricity Industry (Doctoral dissertation, University of Surrey).

[21] Achen, C.H. (2001). Why Lagged Dependent Variables Can Suppress the Explanatory Power of Other Independent Variables. *Ann Arbor*, *1001*, 48106-1248.

[22] Wooldridge, J.M. (2015). *Introductory econometrics: A modern approach*. Nelson Education.

[23] Annual Reports-Central Bank of Nigeria, Available at: https://www.cbn.gov.ng/documents/annualreports.asp. Accessed on 10th July 2015.

[24] National Bureau of Statistics. Available at: http://www.nigerianstat.gov.ng/. Accessed on 10th July 2015.