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Reconstruction of Distorted Images

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Abstract

The non-focused images quality arising algorithm based on Winer filtration is presented in the paper. Filtration is realized in spectral domain of image.
Keywords: Filter, Image, Spectrum, Wiener.

1. Introduction

During the image registration there often appear distortions of different types depending on registering devices characteristics (permission), technical situation of location and also peculiarities of the area being registered. On images obtained by optical devices there can be violations of focal distance; there can appear diffusions when moving objects are being registered, etc.

We consider the image as a function of two variables $f(x,y)$ which is the projection of two or three dimensional fields of view, where (x,y) is a coordinate of any point of plane and $f(x,y)$ is the light intensity in the point (x,y).

We'll consider a problem of optically [1] registered distorted images reconstructon in spectral area, because optical systems focus the falling light and that can be expressed by Fourie transform, so the image reconstruction problem reduces the solving of integral equations of second order.

2. Image reconstruction

Let $g(x,y)$ be the given image and $f(x,y)$ be the reconstructing image. Then the following equation [2] takes place:

$$g(x,y) = \iint f(u,v)h(x,y,u,v)dudv, \qquad (1)$$

where the function $h(x,y,u,v)$ is called an image registering system's impulse response (output value corresponding to unit impulse).
To solve this equation we'll give some assumptions.

Definition 1: *The system is called space-invariant, if its impulse function response depends on the difference between the input(x, y) and output(x, y) planes coordinates:*

$$h(x, y, u, v) = h(x - u, y - v).$$

For such system the equation (1) will be represented as

$$g(x, y) = \iint f(u, v)h(x - u, y - v)dudv, \tag{2}$$

which is usually called a convolution. Equation (2) can also be represented as

$$g(x, y) = f(x, y) * h(x, y). \tag{3}$$

Since $f(x, y)$ is a function of image describing the range of vision, and $g(x, y)$ is a function of registered image, we can see that $h(x, y)$ is a noise describing function.
In general case linear filtration algorithms are realized by transforms of type (2) having the following discrete representation

$$g_{i,j} = \sum_{k=i-r/2}^{i+r/2} \sum_{l=j-r/2}^{j+r/2} f_{k,l} h_{k-i+{}^{r}/_{2},\, l-j+{}^{r}/_{2}}, \quad i \in [{}^{r}/_{2}, M + {}^{r}/_{2}], j \in [{}^{r}/_{2}, N + {}^{r}/_{2}]. \tag{4}$$

M is the number of image rows, N is the number of image columns, the sum includes the points of rectangular with centre(i, j) and $2r + 1$ sides. Before calculation of transform (4) all sides of image should be already widened by rectangular layers of width $r/2$.
In spectral domain the linear filtration algorithm is also based on convolution theorem, so instead of calculating by formula (4) it can be realized by the following formula:

$$G(u, v) = F(u, v)H(u, v), \tag{5}$$

where G, F, H are Fourier transforms of functions g, f, h. Note, that complex multiplication is realized by all u, v frequencies.
Now we'll represent the mathematical model of the system:
$f(x, y)$ -input image function (undistorted),
$h(x, y)$ - noise causing function,
$n(x, y)$ - total noise,
$g(x, y)$ - distorted image (fuzzified, unfocused).
So we have the following representation of the process:

$$g(x, y) = f(x, y) * h(x, y) + n(x, y). \tag{6}$$

It is required to find the impulse characteristic function which will be for the system the best reconstruction function by mean square deviation

$$\sigma = \sqrt{\frac{1}{mn}\sum_{i=0}^{m-1}\sum_{j=0}^{n-1}\left(\hat{f}_{i,j-}f_{i,j}\right)^2} \rightarrow min.$$

The problem solution for linear stationary processes was given by Wiener, the detailed proof is given in [3]. The best approximating filter's spectral representation of function $f(x,y)$ is represented as [3]

$$\hat{F}(u,v) = \frac{1}{H(u,v)} \cdot \frac{|H(u,v)|^2}{|H(u,v)|^2 + {S_n(u.v)}/{S_f(u.v)}} G(u,v), \quad (7)$$

where $S_n(u.v)$ is the spectral density of additive noise and $S_f(u.v)$- $f(x,y)$ is the spectral density of the function. Generally these values are unknown. The ratio $S_n(u.v)/S_f(u.v)$ is the inverse value of signal-noise value. Its value in time domain is considered acceptable if it is in the interval of 30-40 decibels.

The noises induced by focal distance violations on the images registered by the optical devices mainly depend on the light dispersion problem described by the following two functions:

$$h(x,y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}},$$

$$h(x,y) = \begin{cases} \frac{1}{\pi r^2}, & if\ x^2+y^2 < r^2 \\ 0, & if\ x^2+y^2 \geq r^2. \end{cases}$$

If the image doesn't include an additive noise then $n(x,y) = 0$ and the formula (7) is represented as

$$\hat{F}(u,v) = G(u,v)/H(u,v)\,, \quad (8)$$

and is called an inverse filter.

3. Inverse Filters

Indeterminacy appears when because of some device errors during image registering under some frequencies the value of denominator $H(u,v)$ of equation (8) is equal to 0. In such cases the value of spectrum corresponding to this value of image is set equal to zero. As a result, on the filtered image there appear obvious horizontal or vertical (sometimes curved) phenomena.

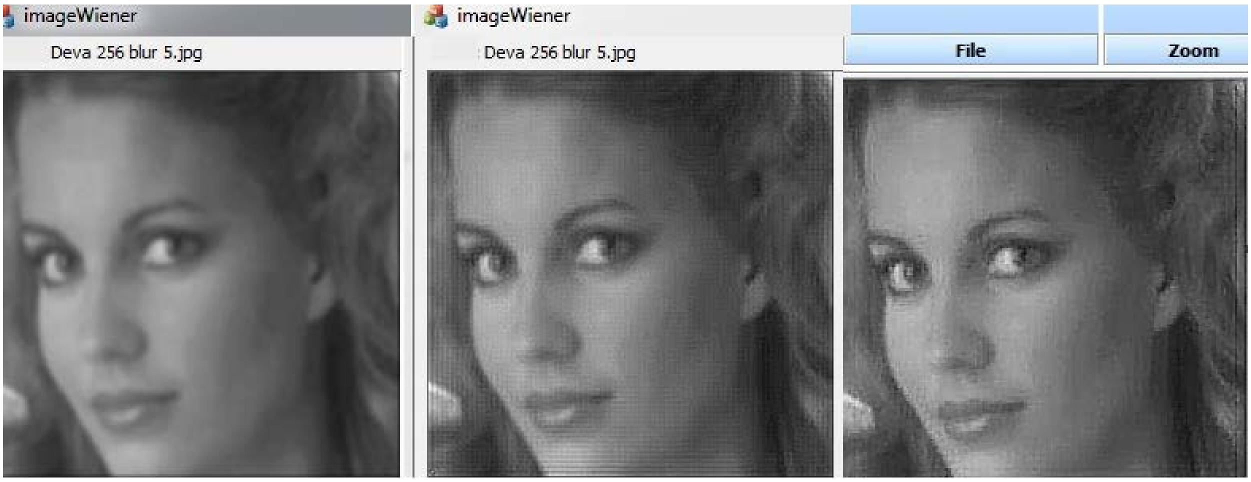
To reduce such occurrences we offer to realize the low- frequency interpolation in spectral domain:

$$\widehat{F}(u,v) = \sum_{i=u-w}^{u+w}\sum_{j=v-w}^{u+w} s_{i,j}, \quad (9)$$

where

$$s_{i,j} = \begin{cases} F(i,j)\dfrac{\sin(2\pi f i)}{i}\dfrac{\sin(2\pi f j)}{j}, & if\ i > 0, j > 0, \\ 2\pi f F(i,j), & if\ i = 0\ or\ j = 0, \\ 0, & if\ i < 0\ or\ j < 0. \end{cases}$$

In case of $i = w, j = w$, $\hat{F}(u,v) = 2\pi f,\ f \in (0;\ 0{,}5)$.
There are many internet investigations and program realizations of this problem.
I think, the system SmartDeblur-1.27-win is one of the best program realizations, but its mathematical apparatus is not presented in the work.
The program realization of the method(5)-(9) presented in this paper has been fulfilled.
The result of the system work and comparison with system SmartDeblur-1.27-win [4] are given below.



a b c

Fig. 1.

a) input image including gauss noise with domain of dispersion $\sigma = 3$ and radius $r = 5, f = 0.45$;

b) the result of program realization of developed system;

c) the result of SmartDeblur-1.27-win system work.

References

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Չֆոկուսացված պատկերների որակի բարձրացում

Ս. Ալավերդյան

Ամփոփում

Աշխատանքում ներկայացվում է չֆոկուսացված պատկերների որակի բարձրացման նոր ալգորիթմ, որը հիմնված է Վիների ֆիլտրացիայի վրա: Ֆիլտրացիան իրականացվում է պատկերի սպեկտրալ տիրույթում:

Улучшение качества расфокусированных изображений

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Аннотация

В работе представляется новый алгоритм улучшения качества расфокусированных изображений, который основан на фильтре Винера. Фильтрация выполняется в спектральной области изображения.