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ANALYSIS OF THERMAL STRESSES TO 2D PLANE THERMOELASTIC INHOMOGENEOUS STRIP

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Abstract. This paper deals with study of the plane elasticity of thermoelastic problems for inhomogenous strip. Here, the original problems are reduced to set the governing equations in the volterra integral equations by making the use of direct integration method. Further using the iteration technique the numerical calculations have been performed. The stress distribution obtained and calculated numerically and shown graphically.

1. Introduction

A large development of the subject, thermoelasticity is motivated by various fields of engineering sciences, during the last few decades. The main physical drawback in the theory of uncoupled thermoelasticity is that an elastic body has no effect on the temperature and vice versa. The interest of researchers to study elasticity and thermo-elasticity problems has grown very fast due to wide applications to real world.

Biot [3] derived the equation of thermal conductivity by including the coupling between thermal fields and strain fields. A novel work done by Lord et.al. [10] introduced two generalizations to the coupled theory of thermoelasticity and given successful alternate to Fourier's law in heat conduction. Tokovyy et.al. in [15] emphasized on analytical treatment of the one dimensional and two dimensional elasticity and thermoelasticity problems using direct integration method, for a long hollow cylinder and a long annular radially non-homogeneous cylinder respectively. Babich et al. [2] solved the plane problem of a horizontal concentrated load by using the linearized elasticity theory from an infinite inhomogeneous stringer to an elastic infinite strip with initial stresses clamped at one edge. The problem is reduced to system of integro-differential equations which then solved by means of Fourier Transform. Manthena et. al. [12] analysed the same problems for a mixture of metals like copper and zinc. Jafari et al. [6] discussed the stress analysis in an orthotropic infinite plate with a circular hole using complex variable technique to the two dimensional thermoelastic problem. Kalynyak et al.[7] focused on development by Prof. Vihak in the field of direct and inverse problems of heat conduction and thermomechanics which are important in investigating problems of thermal power engineering. By considering an inverse thermoelastic problem in [8] Prof. Kalyanyak discussed the presence of a stationary temperature field for a long rectangular beam of inhomogeneous nature. Mahakalkar et. al. [11] studied thermoelastic transient heat conduction problem with internal heat by using classical method. They investigated results on temperature distibution, thermal deflection and stresses by integral transform. Iqbal Kaur et.al.[5] studied recent thermoelastic theories and models related to micro-nano beams and bars, their uses and limitations.

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Tokovyy [16, 17] discussed an analytical solution of plane themroelasticity inhomogeneous problem for planes, half-planes and strips with the aid of direct integration method, here, the governing equations reduced to an integral equations which are then solved by using iteration method which produces a solution of the problem in explicit form. And extended their work in [18] in terms of stresses for a infinite strip for a case of inhomogeneous isotropic material. Solution is found using Fourier transforms and iteration method. Kushnir [9] used direct integration method for generalization of the original equations of solution of 2D problems of thermoelasticity for solids with corner points and they are reduced to a governing integrodifferential equations for a key function, an explicite form solution is found. Tianhu [19] investigated the magneto-thermoelastic response of a homogeneous and isotropic finite thin slim strip subjected to a moving heat source by using Lord-Shulman theory and Laplace transform. Vigak [20, 21, 22] has been developed a method to find solution of the elasticity problems in a semi-plane using the method of direct integration of equilibrium equation. Equilibrium conditions for tractions and compatibility equations for the displacements has been found correct. In [23] Vigak et. al. invented a new analytic method for solving quasi-static thermoelastic problem for stresses in rectangular region, the initial problem is reduced to governing integral differential equations for stress components. The solution is obtained as the series expansion according to Saint-Venant's principle. Youssef et al.[24] developed a new model of three dimensional generalized thermoelasticity problem by using classic L-S model. The double Fourier transform and Laplace technique had been applied to the governing equations subjected to rectangular traction free surface, with the study of the temperature analysis, stresses, strain and displacement in a three dimensional half-space. Zhihe et. al. [25] emphasizes on characterization of FGM strip using thermoelastic problem.

In the thermoelasticity one can determine the stresses produced due to the temperature field and moreover to find the temperature distribution by internal forces which vary with time. Our intent of this paper is to extend our own work [1, 4] for obtaining an analytical solutions to the thermoelastic problems which occures in isotropic and inhomogeneous strip under some thermal condition applied.

2. Problem Formulation

Consider, 2D plane thermoelastic problem in the strip of inhomogeneous isotropic material with infinite width $R = \{(x, y) \in (-\infty, \infty) \times (-a, a)\}$, where $a > 0$ is dimensionless parameter.
Thermoelastic equillibrium of plane R is ruled by the equillibrium equations,

$$\begin{cases} \dfrac{\partial \sigma_{xx}}{\partial x} + \dfrac{\partial \sigma_{xy}}{\partial y} + F_x = 0, \\ \dfrac{\partial \sigma_{xy}}{\partial x} + \dfrac{\partial \sigma_{yy}}{\partial y} + F_y = 0, \end{cases} \tag{2.1}$$

strain-compability equations,

$$\frac{\partial^2 \epsilon_{yy}}{\partial x^2} + \frac{\partial^2 \epsilon_{xx}}{\partial y^2} = \frac{\partial^2 \epsilon_{xy}}{\partial x \partial y}, \tag{2.2}$$

stress strain relations,

$$\begin{cases} \epsilon_{xx} = \dfrac{\sigma_{xx}}{E_1(x)} - \dfrac{v_1(x)\sigma_{yy}}{E_1(x)} + \alpha(x)T(x, y), \\ \epsilon_{yy} = \dfrac{\sigma_{yy}}{E_1(x)} - \dfrac{v_1(x)\sigma_{x}}{E_1(x)} + \alpha(x)T(x, y), \\ \epsilon_{xy} = \dfrac{\sigma_{xy}}{G(x)}, \\ G_1(x) = \dfrac{E_1(x)}{2(1 + v_1(x))}. \end{cases} \tag{2.3}$$

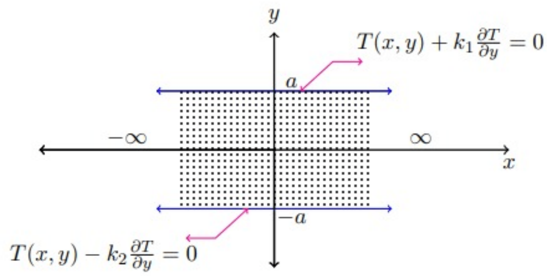

FIGURE 1. Schematic diagram of strip under consideration.

Here, $\sigma_{xx}, \sigma_{yy}, \sigma_{xy}$ are stress-tensor components, $\epsilon_{xx}, \epsilon_{yy}, \epsilon_{xy}$ denotes strain components, F_x, F_y are stress dimensional projections of forces in dimensionless components respectively, and E_1, G_1, α, v_1 denotes Young's modulus, shear modulus, coefficient of thermal expansion and poisson's ratio.
Due to temperature distribution, the normal and shearing stresses arise on the boundarres $y = \pm a$ in the strip R,

$$\begin{aligned} \sigma_{yy}(x,-a) &= -p_1(x), & \sigma_{yy}(x,a) &= p_2(x), \\ \sigma_{xy}(x,-a) &= -q_1(x), & \sigma_{xy}(x,a) &= q_2(x). \end{aligned} \tag{2.4}$$

The two dimensional steady-state temperature $T(x,y)$ can be found from the heat conduction equation [13]

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = -w(x,y), \tag{2.5}$$

under conditions imposed on the boundary in the region $-\infty \le x \le \infty$

$$\begin{cases} \dfrac{\partial T}{\partial x} = 0 \;\; at \;\; x = \pm\infty, \\ T(x,y) + k_1 \dfrac{\partial T}{\partial y} = 0 \;\; at \;\; y = a, \\ T(x,y) - k_2 \dfrac{\partial T}{\partial y} = 0 \;\; at \;\; y = -a, \end{cases} \tag{2.6}$$

where, $w(x,y) = \frac{q(x,y)}{k}$ and $q(x,y)$ denoting the heat generated due to internal heat generated and k_1, k_2 are coefficient of thermal conductivity.

Using equilibrium condition (2.1) we have

$$\frac{\partial^2 \sigma_{xx}}{\partial x^2} - \frac{\partial^2 \sigma_{yy}}{\partial y^2} + \frac{\partial F_x}{\partial x} - \frac{\partial F_y}{\partial y} = 0.$$

Differentiating first equation in (2.1) with respect to x and second equation with respect to y and subtracting, we get

$$\frac{\partial^2 \sigma_{xx}}{\partial x^2} + \frac{\partial F_x}{\partial x} = \frac{\partial^2 \sigma_{yy}}{\partial y^2} + \frac{\partial F_y}{\partial y}.$$

Adding $\frac{\partial^2 \sigma_{yy}}{\partial x^2}$ on both sides which yields

$$\Delta \sigma_{yy} = \frac{\partial^2 \sigma}{\partial x^2} + \frac{\partial F_x}{\partial x} - \frac{\partial F_y}{\partial y} \tag{2.7}$$

where

$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

denotes the two-dimensional Laplace differential operator. Putting stress-strain relations (2.3) and equillibrium conditions (2.1), equation (2.2) can be written as

$$\Delta\left[\frac{(1-v_1)}{2G}\sigma+\alpha(1+v_1)T\right]=\frac{\sigma_{yy}}{2}\frac{d^2}{dy^2}\left(\frac{1}{G}\right)-F_y\frac{d}{dy}\left(\frac{1}{G}\right)-\frac{1}{2G}\left(\frac{\partial F_x}{\partial x}+\frac{\partial F_y}{\partial y}\right). \tag{2.8}$$

The equations (2.7) and (2.8) are bounded by two boundary conditions (2.4) for σ_{yy} and for their derivatives which satisfies equilibrium condition (2.1) at $y=\pm a$:

$$\begin{cases}\dfrac{\sigma_{yy}}{\partial y}=-\dfrac{\partial q_1}{\partial x}-F_y(x) \ \text{ at } y=-a,\\ \dfrac{\sigma_{yy}}{\partial y}=-\dfrac{\partial q_2}{\partial x}-F_y(x) \ \text{ at } y=a.\end{cases} \tag{2.9}$$

The shear stress is found by integrating the equilibrium conditions which gives

$$2\sigma_{xy}=q_1+q_2-\int_{-a}^{a}(\frac{\partial\sigma_{xx}}{\partial x}+F_x)sgn(y-\xi)d\xi-\int_{-a}^{a}(\frac{\partial\sigma_{yy}}{\partial y}+F_y)sgn(x-\eta)d\eta \tag{2.10}$$

where,

$$sgn=\begin{cases}1, & \text{for } x>0,\\ 0, & \text{for } x=0,\\ -1, & \text{for } x<0.\end{cases}$$

3. Solution of Thermoelastic Problem

To find the solution of the formulated problem, we apply the Fourier transform [14] with respect to x defined by

$$\bar{f}(y;\omega)=\int_{-\infty}^{\infty}f(x,y)\exp(-i\omega x)dx \tag{3.1}$$

where $f(x,y)$ is an arbitrary function, $i^2=-1$; ω is a parameter. We choose σ_{yy} and σ to be the governing functions.

To calculate the key stresses, we apply the Fourier transform (3.1) to equation (2.7) to get

$$\left(\frac{d^2}{dy^2}-\omega^2\right)\bar{\sigma}_{yy}=-\omega^2\bar{\sigma}+i\omega\bar{F}_x-\frac{d}{dy}\bar{F}_y. \tag{3.2}$$

Applying Fourier integral transform (3.1) to equation (2.8) with the conditions in (2.4), we obtain

$$\begin{aligned}\left(\frac{d^2}{dy^2}-\omega^2\right)\left[\frac{(1-v_1)}{2G}\bar{\sigma}+\alpha(1+v_1)\bar{T}\right]&=\frac{\bar{\sigma}_{yy}}{2}\frac{d^2}{dy^2}\left(\frac{1}{G}\right)-\bar{F}_y\frac{d}{dy}\left(\frac{1}{G}\right)\\&\quad-\frac{1}{2G}\left(\frac{i\omega\bar{F}_x}{\partial x}+\frac{\partial\bar{F}_y}{\partial y}\right),\end{aligned} \tag{3.3}$$

$$\begin{aligned}&\bar{\sigma}_{yy}(x,-a)=-\bar{p}_1, && \bar{\sigma}_{yy}(x,a)=\bar{p}_2\\ &\frac{\partial\bar{\sigma}_{yy}}{\partial y}(x,-a)=-i\omega\bar{q}_1-\bar{F}_y(x,-a), && \frac{\partial\bar{\sigma}_{yy}}{\partial y}(x,a)=i\omega\bar{q}_2+\bar{F}_y(x,a).\end{aligned} \tag{3.4}$$

The solution of differential equation (3.2) is

$$\bar{\sigma}_{yy}=c_1cosh\omega y+c_2sinh\omega y+\frac{1}{\omega}\int_{-a}^{y}\left(i\omega\bar{F}_x-\frac{d\bar{F}_y}{dy}-\omega^2\bar{\sigma}\right)sinh(\omega(y-\xi))d\xi \tag{3.5}$$

where c_1 and c_2 are the constants of integration. Using the first two boundary conditions in (2.4), the solution can be expressed as

$$\begin{aligned}\bar{\sigma}_{yy} = -\bar{p}_2 cosh(\omega(y+a)) - \left(i\bar{q}_2 + \frac{\bar{F}_x(x,-a)}{\omega}\right) sinh(\omega(y+a)) \\ + \frac{1}{\omega}\int_{-a}^{y}\left(i\omega\bar{F}_x - \frac{d\bar{F}_y}{dy} - \omega^2\bar{\sigma}\right) sinh(\omega(y-\xi))d\xi.\end{aligned} \tag{3.6}$$

It satisfies two integral conditions:

$$\begin{aligned}\int_{-a}^{a}\bar{\sigma} sinh\omega\xi d\xi = i(\bar{q}_1+\bar{q}_2)\frac{sinh\omega a}{\omega} + (\bar{p}_2-\bar{p}_1)\frac{cosh\omega a}{\omega} + \left(\bar{F}_y(x,a)+\bar{F}_y(x,-a)\right)\frac{sinh\omega a}{\omega^2} \\ + \frac{1}{\omega}\int_{-a}^{a}\left(i\bar{F}_x - \frac{1}{\omega}\frac{d\bar{F}_y}{d\xi}\right) sinh\omega\xi d\xi,\end{aligned} \tag{3.7}$$

and

$$\begin{aligned}\int_{-a}^{a}\bar{\sigma} cosh\omega\xi d\xi = i(\bar{q}_1-\bar{q}_2)\frac{cosh\omega a}{\omega} - (\bar{p}_2+\bar{p}_1)\frac{sinh\omega a}{\omega} + \left(\bar{F}_y(x,a)-\bar{F}_y(x,-a)\right)\frac{cosh\omega a}{\omega^2} \\ + \frac{1}{\omega}\int_{-a}^{a}\left(i\bar{F}_x - \frac{1}{\omega}\frac{d\bar{F}_y}{d\xi}\right) cosh\omega\xi d\xi.\end{aligned} \tag{3.8}$$

Hence, the solution of equation (3.3) can be found as

$$\begin{aligned}\bar{\sigma} = \frac{2G_1}{1-v_1}\Big[c_1 cosh\omega y + c_2 sinh\omega - \alpha(1+v_1)\bar{T} + \Theta(y) + \Phi(y) + Q(y) + \Psi(y) \\ + \int_{-a}^{y}\bar{\sigma}(\eta)K(y,\eta)d\eta\Big]\end{aligned} \tag{3.9}$$

where,

$$\Theta(y) = -\frac{\bar{p}_2}{2\omega}\int_{-a}^{y}\frac{d^2}{d\xi^2}\left(\frac{1}{G(\xi)}\right) cosh(w(a+\xi))sinh(w(y-\xi))d\xi,$$

$$\Phi(y) = -\frac{i\bar{q}_2}{2\omega}\int_{-a}^{y}\frac{d^2}{d\xi^2}\left(\frac{1}{G(\xi)}\right) sinh(w(a+\xi))sinh(w(y-\xi))d\xi,$$

$$Q(y) = \frac{1}{2\omega}\int_{-a}^{y}\frac{d^2}{d\xi^2}\left(\frac{1}{G(\xi)}\right) sinh(\omega(y-\xi))\int_{-a}^{\xi}\left(i\bar{F}_x - \frac{1}{\omega}\frac{d\bar{F}_y}{d\xi}\right) sinh(\omega(\xi-\eta))d\eta d\xi,$$

$$\begin{aligned}\Psi(y) = -\frac{1}{\omega}\int_{-a}^{y}\left(\bar{F}_y(\xi)\left(\frac{1}{G(\xi)}\right) + \frac{1}{2G_1(\xi)}(is\bar{F}_x + \frac{d\bar{F}_y}{d\xi})\right) sinh(\omega(y-\xi))d\xi \\ - \bar{F}_y(-a)\frac{\bar{q}_2}{2\omega^2}\int_{-a}^{y}\frac{d^2}{d\xi^2}\left(\frac{1}{G(\xi)}\right) sinh(w(a+\xi))sinh(w(y-\xi))d\xi,\end{aligned}$$

$$K(y,\eta) = \int_{-\eta}^{y}\frac{d^2}{d\xi^2}\left(\frac{1}{G(\xi)}\right) sinh(w(y-\xi))sinh(w(\xi-\eta))d\xi.$$

Different types of techniques can be used to determine the solution of equation (3.9). Here we use the method of resolvent kernel

$$\begin{aligned}\bar{\sigma} &= \tfrac{2G_1}{1-v_1}\Big[c_{1n} cosh\omega y + c_{2n} sinh\omega - \alpha(1+v_1)\bar{T} + \Theta(y) + \Phi(y) + Q(y) + \Psi(y) \\ &\qquad + \textstyle\int_{-a}^{y}\bar{\sigma}_n(\eta)K(y,\eta)d\eta\Big].\end{aligned}$$

The resolvent kernel is calculated as

$$\Re(y,\eta) = \sum_{n=0}^{\infty} K_{n+1}(y,\eta) \tag{3.10}$$

where

$$\begin{cases} K_1(y,\xi) = K(y,\xi), \\ K_{n+1} = \int_{-a}^{y} K(y,\xi) K_n(\xi,\eta) d\eta, \;\; n = 1, 2, \cdots . \end{cases}$$

We have a fact that the recurring kernels $K_{n+1} \longrightarrow 0$ as $n \longrightarrow \infty$ which shows the initial condition for convergence holds . Consequently, for a natural number N,

$$\Re(y,\xi) \approx \Re_N(y,\xi) = \sum_{n=0}^{N} K_{n+1}(x,\xi) \tag{3.11}$$

We construct a solution of equation (3.9) by using resolvent-kernel technique, yields,

$$\bar{\sigma} = \frac{2G_1}{1-v_1}\left[Cs_1(y) + Ds_2(y) + \tau(y) - \alpha(1+v_1)\bar{T}\right] \tag{3.12}$$

where,

$$\begin{aligned}
s_1(y) &= \cosh \omega y + \int_{-a}^{y} \cosh \omega\xi \Re(y,\xi) d\xi, \\
s_2(y) &= \sinh \omega y + \int_{-a}^{y} \sinh \omega\xi \Re(y,\xi) d\xi, \\
\tau(y) &= \Theta(y) + \Phi(y) + Q(y) + \Psi(y) + \int_{-a}^{y} \left(\Theta(\xi) + \Phi(|xi) + Q(\xi) + \Psi(\xi)\right) \Re(y,\xi) d\xi, \\
C &= \frac{D_2R_1 - D_1R_2}{D_2D_3 - D_1^2}, \\
D &= \frac{D_3R_2 - D_1R_1}{D_2D_3 - D_1^2}, \\
R_1 &= \frac{1}{2}\int_{-a}^{a} \bar{\sigma} \cosh \omega\xi \Re(y,\xi) d\xi, \\
R_2 &= \frac{1}{2}\int_{-a}^{a} \bar{\sigma} \sinh \omega\xi \Re(y,\xi) d\xi, \\
D_1 &= \int_{-a}^{a} \frac{G(\xi)}{1-v_1(\xi)} \sinh \omega\xi \cosh \omega\xi d\xi, \\
D_2 &= \int_{-a}^{a} \frac{G(\xi)}{1-v_1(\xi)} \sinh^2 \omega\xi d\xi, \\
D_3 &= \int_{-a}^{a} \frac{G(\xi)}{1-v_1(\xi)} \cosh^2 \omega\xi d\xi.
\end{aligned}$$

Integral conditions of $\bar{\sigma}$ can be computed using equation (3.7)-(3.8) as

$$\begin{aligned}
\bar{\sigma}_{yy} = &-\bar{p}_2 cosh(\omega(y+a)) - \left(i\bar{q}_2 + \frac{\bar{F}_x(x,-a)}{\omega}\right) sinh(\omega(y+a)) \\
&+ \omega \int_{-a}^{y} \frac{1}{1-v_1}\left[2G\left(C\cosh\omega\xi + D\sinh\omega\xi - \tau(\xi)\right) + \alpha(\xi)E_1(\xi)\right] sinh(\omega(y-\xi)) d\xi.
\end{aligned} \tag{3.13}$$

After the total plane stress is found in the form of equation (3.12), the normal stress $\bar{\sigma}_{yy}$ can be found by using equation (3.6). Then the stress $\bar{\sigma}_{xx}$ is computed by

$$\bar{\sigma}_{xx} = \bar{\sigma} - \bar{\sigma}_{yy}. \tag{3.14}$$

If E_1, G_1 and v_1 are constant, then equation (3.12) gives the same expressions for σ and σ_{yy}. Then, equation (3.9) takes the form

$$\begin{aligned}\bar{\sigma} = E_1 \Big[&Ccosh\omega y + Dsinh\omega - \alpha(1+v_1)\bar{T} + \Theta(y) + \Phi(y) + Q(y) + \Psi(y) \\ &+ \int_{-a}^{y} \bar{\sigma}(\eta) K(y,\eta) d\eta \Big]\end{aligned} \tag{3.15}$$

which is an analytic solutions to the given thermoelastic problem in R. The shear stress can be determined from equation (2.1) as

$$\bar{\sigma}_{xy} = \frac{i}{\omega}\left(\frac{d\bar{\sigma}_{yy}}{dy} + \bar{F}_y\right),$$

which yields

$$\begin{aligned}\bar{\sigma}_{xy} = -i \Bigg[&\bar{p}_2 \sinh(\omega(y+a)) - i\left(i\bar{q}_2 + \frac{\bar{F}_x(x,-a)}{\omega}\right)\cosh(\omega(y+a)) \\ +\omega &\int_{-a}^{y} \frac{1}{1-v_1}\left[2G\left(C\cosh\omega\xi + D\sinh\omega\xi - \tau(\xi)\right) + \alpha(\xi)E_1(\xi)\right]\cosh(\omega(y-\xi))d\xi + \bar{F}_y \Bigg].\end{aligned} \tag{3.16}$$

We can see that, the stresses $\bar{\sigma}$ and $\bar{\sigma}_{xy}$ are depending only on Poisson's ratio which is deviating in y-coordinate. The stress-tensor components (3.13) – (3.16) can be found by means of the inversion formula

$$f(x,y) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \bar{f}(y,\omega)\exp(i\omega x) d\omega$$

4. **Numerical Results**

Consider a strip loaded by

$$p_1 = p_2 = A\mu(x) \quad , \quad \mu(x) = \exp(-bx^2) \tag{4.1}$$

where, $q_1 = q_2 = 0$ at $F_x = F_y = 0$ for $T = 0$. Here, A, b are constants and $a > 0$. Numerical computations have been done using Python programming language. Let, $G = G_0\exp(ky)$, $v_1 =$ constant, where, k is constant and $G_0 = E_0/2(1+v_0)$. Distribution of a function $\mu(x)$ for $b = 2$ is depicted in Figure 2. It shows equation (4.1) gives smooth curve and highest value for $x \to 0$ and vanishes for $x \to \pm\infty$ rapidly, which makes the equation more useful to verify analytic solution.

Introduce the parameter

$$v_0 = 1 + \frac{1}{1-by}$$

Figure 3 demonstrates the distribution of dimensionless stresses in R for different valus of b. The solid line curve shows the case of the homogeneous material properties i.e. b=0. The case $b = 0.5$ and $b = 1$ corresponds to dotted and dashed lines respectively on stress distribution. As the expection the curves are symmetric about $y = 0$ which clearly shows an effect of inhomogeneity. Thus, transversal stress $\bar{\sigma}_{yy}$ should have maximum value at $x = 0$ which is shifted towards the direction of inhomogeneity increase.

5. **Conclusions**

This article develops an approach for solving an analytic solution of the plane two dimensional thermoelastic problems in terms of inhomogeneous isotropic strip. In this study we arrive at following conclusions.

- The original thermoelastic problem is reduced to that of solution of integral equations using direct integration method.

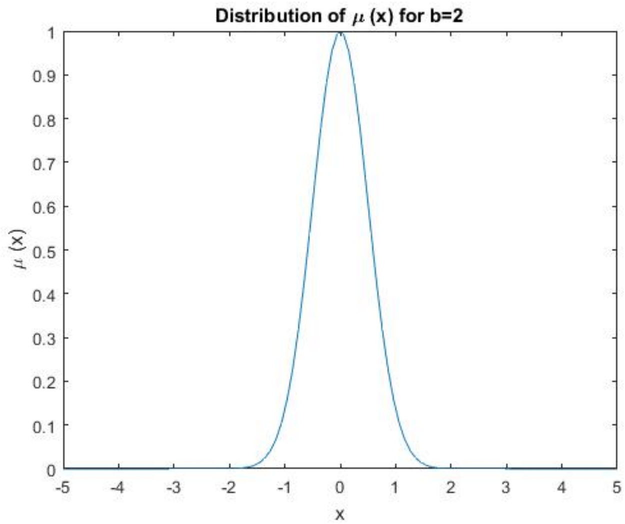

FIGURE 2. Distribution of μ(x)

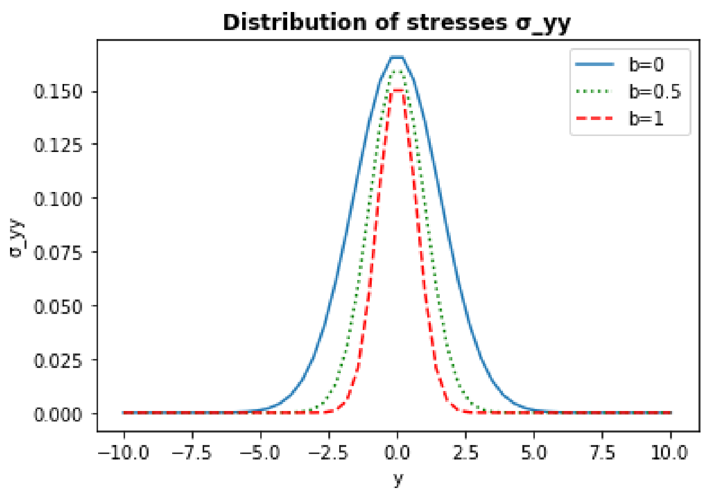

FIGURE 3. Distribution of $\bar{\sigma}_{yy}$ for b=0, 0.5, 1

- It provides the solution of volterra integral equation of second kind which is then solved by resolvent kernel method which provides an efficient technique for analysis of inhomogeneous thermoelastic problems in terms of stress components in the strip R.
- The presesnted technique can be applied without any restrictions for material properties .
- One can solve corresponding inverse thermoelastic problem in displacements using constructed solutions.
- In the present article, analytical solution of thermal stresses is constructed by assuming the fact that, stresses are vanishing at infinity. We can see that, same technique can be used for problems with different loading conditions, instead of Fourier transform.

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