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Reply to the Commentary on "Granular discharge rate for submerged hoppers"

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The commentary of Staron [1] raises interesting points and helps frame additional lines of research. Based on her initial report, we have made several helpful improvements and clarifications to our manuscript [2]. Below, we respond to some of the remaining issues.

First, Staron is absolutely correct in suggesting that we do not have a clear definitive explanation for the "surge" effect shown in Fig. 1, where the discharge rate increases as filling height decreases toward zero. The interstitial fluid clearly plays a role, but there are many possibilities. It could be due to suction into the space between grains as they move apart near the exit, as Staron suggests. It could also be due to a reduction in grain–grain or grain–wall friction, or due to lubrication effects when grains come together. We hope that on-going experiments [3] on trends versus system size, etc., will help shed light on this issue. A terminal surge has now been seen for dry grains in air [3], so the effect is of broader interest. Perhaps in vacuum it would disappear altogether. In any case, theoretical and simulation input would be helpful.

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As for the Beverloo scaling, even for dry cohesionless grains, we sympathize with Staron's point that intermittent formation and break-up of force chain arches over the outlet is hard to imagine for very large apertures —where a continuum model would be more natural. We all agree that the Janssen argument for saturation of pressure versus depth does not apply, or else discharge rates would grow with hopper diameter. This has been shown directly by recent experiments with a conveyor belt [4]. Therefore, some sort of shielding of grain–grain pressure over the outlet is needed, even if transient arches do not form. How exactly the grain pressure behaves near the outlet during flow seems like a fruitful topic for study, perhaps along the lines suggested by Staron and Fig. 1 of her comment.

Finally, regarding the exit speed for grains in the submerged case, we did indeed demonstrate a Beverloo-like scaling simply by replacing the free-fall speed $\sqrt{gD}$ for the dry case with a single-grain terminal falling speed for the submerged case. As a minor remark, the terminal speed is not simply the Stokes speed, which holds only for small grains and small Reynolds numbers Re. We varied the grain size over a large enough range that the fluid drag crossed over from viscous at low Re to inertial at high Re. The latter is for large grains, in which case the fluid drag scales as speed-squared and the interstitial fluid flow must be somewhat turbulent. Nevertheless, the modified Beverloo-like scaling holds across both regimes, with the same small-hole cutoff (which is significantly larger than in the dry case). Thus, it may not be necessary to grapple with the full complexities of the interstitial fluid flow. Further experiments would be helpful, both to investigate initial transients as Staron suggests and also to systematically vary an imposed down- or up-flow of fluid through the packing along the lines illustrated in Fig. 1d of our manuscript. The reason for a small-hole cutoff must involve dynamics, and it would be interesting to know how the size depends on fluid properties.

[1] L Staron, *Commentary on "Granular discharge rate for submerged hoppers"*, Pap. Phys. **6**, 060010 (2014).

[2] T J Wilson, C R Pfeifer, N Meysingier, D J Durian, *Granular discharge rate for submerged hoppers*, Pap. Phys. **6**, 060009 (2014).

[3] J Koivisto, D J Durian, unpublished (2014).

[4] M A Aguirre, J G Grande, A Calvo, L A Pugnaloni, J-C Géminard, *Pressure independence of granular flow through an aperture*, Phys. Rev. Lett. **104**, 238002 (2010).