Arboricity and Span in M-Fuzzy Chromatic Index

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Abstract

A fuzzy matching is a set of edges in which no two edges incident on a same membership value. In this paper, Arboricity and span of a fuzzy labeling graph are defined with suitable examples. Also, the relation between Arboricity and *M*-fuzzy Chromatic Index, Span and *M*-fuzzy Chromatic Index are discussed. Some results based on these concepts are stated and proved.

Keywords: M-fuzzy coloring; M-fuzzy chromatic index; Arboricity; span annuity

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1. Introduction

In 1736, Euler first invited the concept of graph theory. The theory of graphs is extremely useful for solving combinatorial problems in different areas such as geometry, algebra, number theory, topology, operations research, optimization and computer science. Graph Theory is the main tool to constitute many real-world problems. Nowadays, graphs do not represent all systems properly due to the uncertainty or nebulousness of the framework of systems. This and many other circumstances stimulated to define fuzzy graphs.

Rosenfeld [8] first established the notion of fuzzy graphs in 1975. Crisp graph and fuzzy graph are both systematically the same, but when there is an uncertainty on points and or lines then fuzzy graph has a segregated significance. Since the world is brimming with uncertainty, so the fuzzy graph occurs in many real-life locations. Fuzzy graph theory is elaborated with a large number of branches. The fuzzy graph model is used to constitute the traffic network. Also it is used for job allocation, group structure and decision-making analysis etc. Seethalakshmi et al. [10] defined perfect fuzzy matching with some examples, and they derived that if G is a strong regular fuzzy graph with each vertex and is of degree at least two, then E is not a perfect fuzzy matching in G.

Ananthanarayanan et al. [1] discussed the coloring of fuzzy graphs using α-cut with appropriate examples. Anjaly Kishore et al.[2] defined the chromatic number of a fuzzy graph and they discussed some results based on these chromatic numbers. Arindam Deyet al.[3] explained the concept of complement fuzzy graph and their edge coloring. R. Jahir Hussain et al.[5] found the fuzzy chromatic number of some special fuzzy graphs S.Yahya Mohamed et al.[11-14] introduced the concept of fuzzy matching in fuzzy labeling graphs through some illustrations. In this paper, we introduced the new concept of Arboricity and Span of a fuzzy labeling graph G. Also, we discussed some properties based on Arboricity and Span of G.

2. Preliminaries

Definition 2.1

Let $G = (\alpha, \beta)$ be a fuzzy graph. The set of all subsets of β in which they received the same color is called *Color Class of* G. It is denoted by $CC(G)$.

Example 2.2

Consider the fuzzy graph G given in Figure 2.1

In Fig 2.1, the edge set is $\beta = \{e_1(0.2), e_2(0.5), e_3(0.35), e_4(0.14)\}$ and all edges receive different colors. So all edges belong to the color class of G.

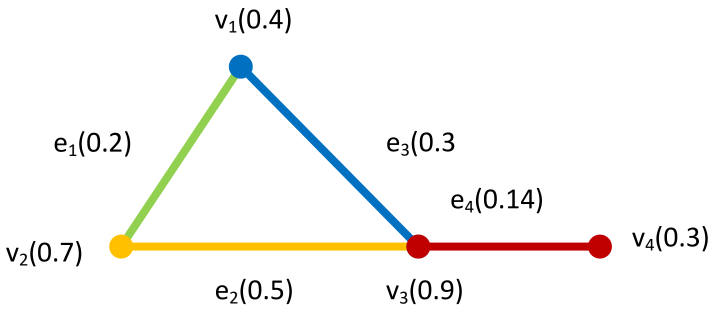

Figure 2.1: Color class of a fuzzy graph G

Definition 2.3

Let $G = (\alpha, \beta)$ be a fuzzy labeling graph. The number of elements in the color class is called as *coloring number* of G. It is denoted by $CN(G)$.

Definition 2.4

A family $\lambda = \{M_1, M_2, M_3 \dots \dots \dots . M_k\}$ of fuzzy matchings on a set β is called a $M -$ *fuzzy coloring* of $G = (\alpha, \beta)$ if

(i) $\vee \lambda = \beta$. It means single edge does not belong to two distinct color classes.

(ii) $\lambda_i \Lambda \lambda_j = 0$.

(iii) For every edge (x, y) of G $min\{\lambda_i(x), \lambda_j(y)\} = 0$ $(1 \leq i \leq k)$.

(This means any one of the edges does not receive different color).

Example 2.5

Consider the fuzzy graph G given in figure 2.2,

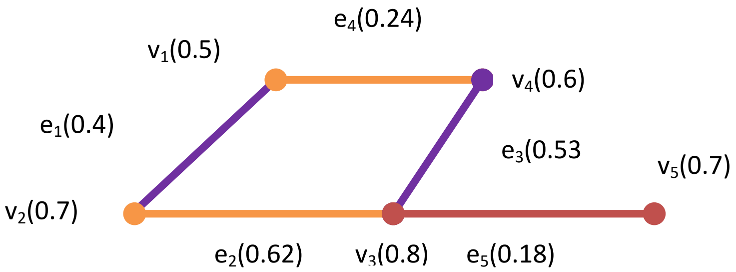

Figure 2.2: *M*-Fuzzy Coloring of G

In Fig 2.2, the fuzzy matchings are $M_1 = \{e_1(0.4), e_3(0.53\}$ and $M_2 = \{e_4(0.24), e_2(0.62)\}$. Here $\lambda = \{M_1, M_2\}$ is $M -$fuzzy coloring of G.

Definition 2.6

Let $G = (\alpha, \beta)$ be a fuzzy labeling graph .The minimum number k for which there exists a M −fuzzy coloring is called *fuzzy matching chromatic index* of G. It is denoted by $\chi^{\text{fmci}}(G)$.

Example 2.7

Consider the fuzzy graph given in Figure 2.3

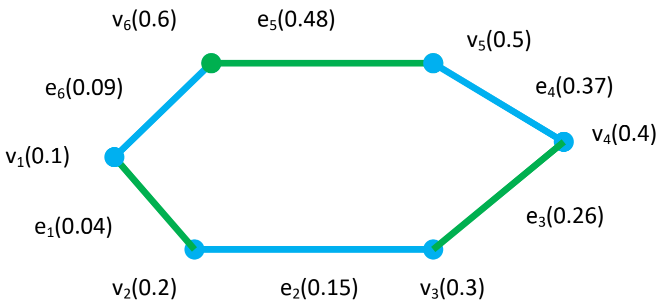

Figure 2.3: Fuzzy matching chromatic index of G

In Fig 2.3, the fuzzy matching of a given graph G are as follows $M_1 = \{e_1(0.04), e_3(0.26), e_5(0.48)\}, M_2 = \{e_2(0.15), e_4(0.37), e_6(0.09)\}$. Then the fuzzy matching chromatic index $\chi^{\text{fmci}}(G) = 2$ because $\lambda = \{M_1, M_2\}$ is M −fuzzy coloring of G.

3. Main Results

Definition 3.1

The minimum number of paths to cover all edges of a fuzzy graph G is called *Arboricity of* G. It is denoted by $A(G)$.

Example 3.2

Consider the following fuzzy graph G given in Figure 3.1

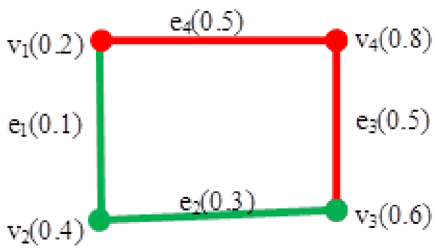

Figure 3.1: Arboricity of G

In fig 3.1 the path cover all edges of a given graph are $v_1e_1v_2e_2v_3$ and $v_3e_3v_4e_4v_1$. Here each colour represents one path and two distinct paths cover all edges of G. Hence, we obtain $A(G)=2$.

Theorem 3.3

For any fuzzy labeling graph G with n vertices has Arboricity $\left\lceil \frac{n}{2} \right\rceil$.

Proof

Consider a fuzzy labeling graph $G=(\alpha,\beta)$ with n vertices. Now we construct a path to cover all edges of *G*. We obtain many paths to cover all edges of *G*. But we need minimum number of path to cover all edges. Then we construct the path with maximum number of edges in G. This path occurs with maximum number of edges $(n-1)$. Hence, we can find all paths with $(n-1)$ edges and which is equal to $\left\lceil \frac{n}{2} \right\rceil$. Therefore, G has Arboricity $\left\lceil \frac{n}{2} \right\rceil$.

Example 3.4

Consider the fuzzy graph given in Figure 3.2,

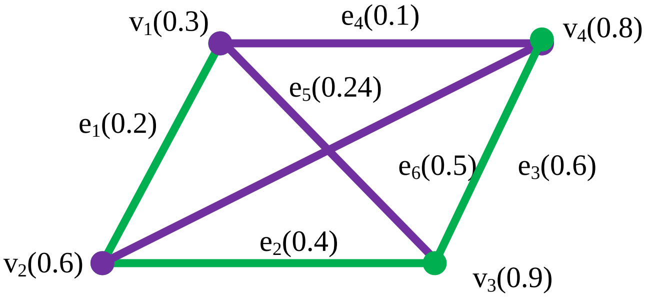

Figure 3.2: Fuzzy Labeling graph *G*

In fig 3.2 the path cover all edges of a given graph are $v_3e_5v_1e_4v_4e_6v_2$ and $v_1e_1v_2e_2v_3e_3v_4$. Here, each color represents one path and there are two paths exist to cover all edges of G. Hence, we get $A(G)=2=\left\lceil \frac{4}{2} \right\rceil$.

Theorem 3.5

For any fuzzy labeling graph $G, A(G) \leq \chi^{fmci}(G)$.

Proof

Consider a labeling fuzzy graph $G=(\alpha,\beta)$ and $M_1, M_2, M_3 ... M_k$ be the distinct fuzzy matching in G. Here $M_1, M_2, M_3 ... M_k$ contains the set of independent elements and each matching receive different colors. Now we find the path to cover all edges of G. Here we need more than one path to cover all edges of G. Already we have for any fuzzy graph with n vertices has arboricity $\left\lceil \frac{n}{2} \right\rceil$. Here we require two matching or more than two matching to cover all edges of G. Then the number of path to cover all edges is less than the number of matching in any color class of G. Hence, we obtain $A(G) \leq \chi^{fmci}(G)$.

Example 3.6

Consider the fuzzy graph given in Figure 3.3,

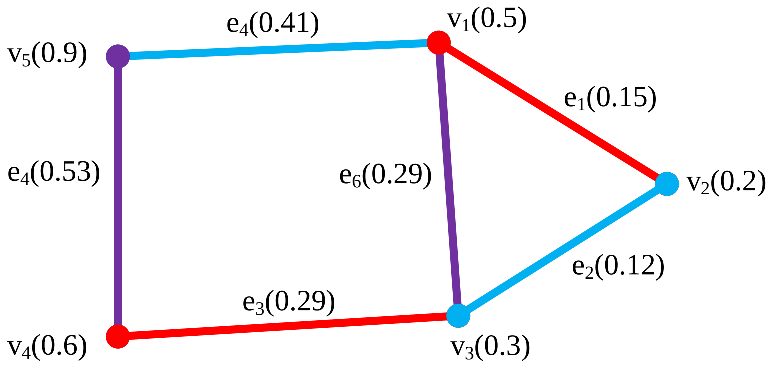

Figure 3.3: Fuzzy Graph *G*

In fig 3.3 the fuzzy matching of a given graph G is as follows

$M_1 = \{e_4(0.53), e_6(0.29)\}, M_2 = \{e_1(0.15), e_3(0.29)\}$ and

$M_3=\{e_2(0.12),e_4(0.41)\}$. Also the color class $CC(G)=\{M_1,M_2,M_3\}$. Here $\chi^{fmci}(G)=3$ and the paths to cover all edges of G are $v_5e_4v_1e_1v_2e_2v_3$ and $v_1e_6v_3e_3v_4e_4v_5$. Then $A(G)=2$. Hence, we obtain $A(G)\le\chi^{fmci}(G)$.

Definition 3.7

The minimum number of interchanging path to cover all vertices of a fuzzy labeling graph G is called *spanning arboricity of G*.

Example 3.8

Consider the following fuzzy graph G given in Figure 3.4

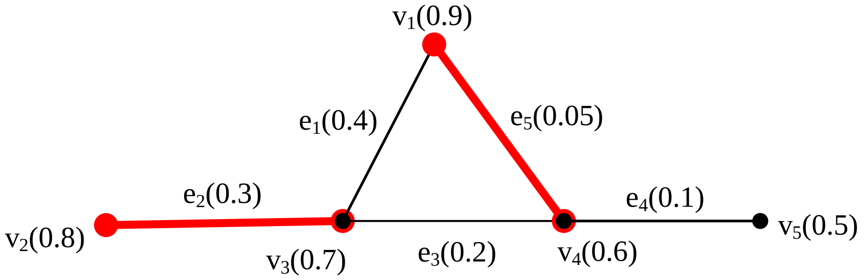

Figure 3.4: Spanning arboricity of G

In fig 3.4 one of the fuzzy matching of a given graph G is $M_1=\{e_2(0.3),e_5(0.05)\}$. Then the interchanging path $v_2e_2v_3e_1v_1e_5v_4e_4v_5$ is to cover all points of G. Here spanning arboricity of G is one.

Theorem 3.9

A fuzzy labeling graph $G=(\alpha,\beta)$ with n vertices has spanning arboricity as unity.

Proof

Let us consider a fuzzy labeling graph $G=(\alpha,\beta)$ with n vertices. Then we identify the fuzzy matching of G and an interchanging path with maximum lines to cover all vertices of G. Now we can find an interchanging path to cover all vertices of G. An interchanging path is a path in which the edges are alternatively in M and $(\beta-M)$. But we can able to construct a fuzzy matching with maximum number of edges in the boundary of G. Then an interchanging path in the boundary path of any graph is only enough to cover all vertices of G

Example 3.10

Consider the following fuzzy graph given in Figure 3.5

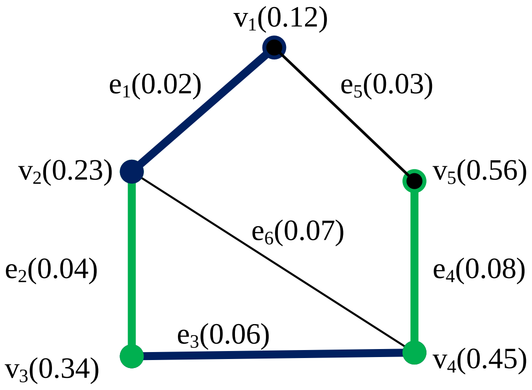

Figure 3.5: Spanning arboricity of *G*

In fig 3.5 the fuzzy matching of a given graph G is $M_1 = \{e_2(0.04), e_4(0.08)\}$ and $v_1e_1v_2e_2v_3e_3v_4e_4v_5$ is an interchanging path to cover all vertices of G. Hence, the spanning arboricity of G is one.

Definition 3.11

The number of edges in an interchanging path to cover all vertices of a fuzzy graph G is called *span of* ***G***. It is denoted by $\mathcal{S}(G)$.

Example 3.12

Consider the following fuzzy graph given in Figure 3.6,

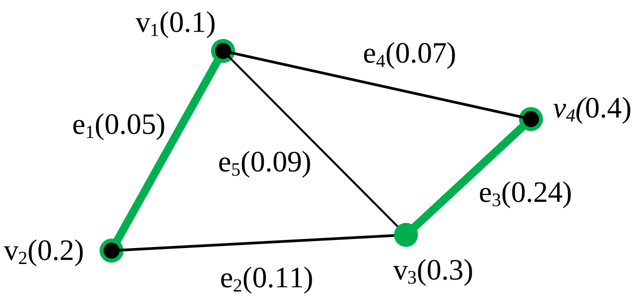

Figure 3.6 : Span of *G*

In fig 3.6 the fuzzy matching of a given graph G is $M_1 = \{e_1(0.05), e_3(0.24)\}$ and the number of edges in an interchanging path $v_1e_1v_2e_2v_3e_3v_4$ is 3. Hence, we have $\mathcal{S}(G) = 3.$

Theorem 3.13

For any fuzzy labeling graph $G = (\alpha, \beta)$ with n vertices, we have $\mathcal{S}(G) \geq \chi^{fmci}(G).$

Proof

Let us consider a fuzzy labeling graph $G = (\alpha, \beta)$ with n vertices. Now we can form fuzzy matching of G and we find an interchanging path with maximum edges to cover all vertices of $G.$ Then the interchanging path in the boundary of any graph is only enough to cover all vertices of $G.$ to cover all vertices of $G.$ But this path occurs with maximum number of edges $(n-1)$ because each fuzzy matching receives distinct colors. Therefore, span of G is $(n-1).$ Here the fuzzy matching chromatic index is less than $(n-1).$ and hence, we obtain $S(G) \geq \chi^{fmci}(G).$

Example 3.14

Consider the following fuzzy graph given in Figure 3.7,

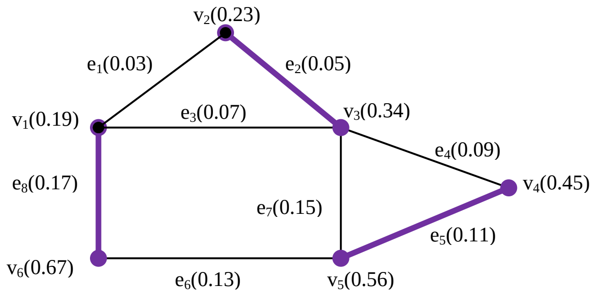

Figure 3.7: Span of *G*

In fig 3.7 the fuzzy matching is $M_1 = \{e_2(0.05), e_5(0.11), e_8(0.17)\}$ and $v_1e_1v_2e_2v_3e_4v_4e_5v_5e_6v_6$ is an interchanging path to cover all vertices of G. Then we have $S(G) = 5 \geq \chi^{fmci}(G) = 2.$

4. Conclusion

Fuzzy set theory proposes to be a useful tool for handling vagueness and degrees of certainty, and for giving a consistent representation of linguistically formulated knowledge which allows the use of precise properties and algorithms. In this paper, we introduced the new concept of Arboricity and span of fuzzy labeling graph. Also we compared these values with M-fuzzy Chromatic Index with related examples. In future, we plan to extend our research work to operations like union, intersection and Cartesian product of two fuzzy labeling graphs.

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