

Error Locating Codes Dealing with Repeated Low-Density Burst Errors

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Abstract. This paper presents a study of linear codes which are capable to detect and locate errors which are repeated low-density bursts of length b (fixed) with weight w or less. An illustration for such a kind of code has also been provided.

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1 Introduction

Burst errors are the type of errors that occur quite frequently in several communication channels. Codes developed to detect and correct such errors have been studied extensively by many authors. Abramson [1959] developed codes which dealt with the correction of single and double adjacent errors, which was extended by Fire [1959] as a more general concept called ‘burst errors’. A burst of length b is defined as follows:

Definition 1. A burst of length b is a vector whose only non-zero components are among some b consecutive components, the first and the last of which is non-zero.

The nature of burst errors differs from channel to channel depending upon the kind of channel. Chien and Tang [1965] proposed a modification in the definition of a burst and they defined a burst of length b , which shall be called as CT-burst of length b , as follows:

Definition 2. A CT-burst of length b is a vector whose only non-zero components are confined to some b consecutive positions, the first of which is non-zero.

Channels due to Alexander, Gryb and Nast [1960] fall in this category. This definition was further modified by Dass [1980] as follows:

Definition 3. A burst of length b (fixed) is an n -tuple whose only non-zero components are confined to b consecutive positions, the first of which is non-zero and the number of its starting positions is among the first $n - b + 1$ components.

This definition is useful for channels not producing errors near the end of a code word. In very busy communication channels errors repeat themselves. So is a situation when errors occur in the form of bursts. Dass, Garg and Zannetti [2008] studied this kind of repeated burst errors. They termed such errors as m -repeated burst errors of length b (fixed) which has been defined as follows:

Definition 4. An m -repeated bursts of length b (fixed) is an n -tuple whose only non-zero components are confined to m distinct sets of b consecutive digits, the first component of each set is non-zero and the number of its starting positions is among the first $n - mb + 1$ components.

In particular a 2-repeated bursts of length b (fixed) has been defined by Dass and Garg [2009(a)] as follows:

Definition 5. A 2-repeated bursts of length b (fixed) is an n -tuple whose only non-zero components are confined to 2 distinct sets of b consecutive digits, the first component of each set is non-zero and the number of its starting positions is among the first $n - 2b + 1$ components.

During the process of transmission some disturbances cause occurrence of burst errors in such a way that over a given length, some digits are received correctly while others get corrupted i.e. not all the digits inside a burst are in error. Such bursts are termed as low-density bursts [Wyner (1963)].

A low-density burst of length b (fixed) with weight w or less has been defined as follows:

Definition 6. A low-density burst of length b (fixed) with weight w or less is an n -tuple whose only non-zero components are confined to some b consecutive positions, the first of which is non-zero with at most w ($w \leq b$) non-zero components within such b consecutive digits and the number of starting positions of the burst is among the first $n - b + 1$ components.

Dass and Garg [2009(b)] studied codes which are capable to detect and/or correct m -repeated low-density bursts of length b (fixed) with weight w or less. They defined such codes as follows:

Definition 7. An m -repeated low-density burst of length b (fixed) with weight w or less is an n -tuple whose only non-zero components are confined to m distinct sets of b consecutive positions, the first component of each set is non-zero where each set can have at most w non-zero components ($w \leq b$), and the number of its starting positions in an n -tuple is among the first $n - mb + 1$ positions.

In particular, a 2-repeated low-density burst of length b (fixed) with weight w or less has been defined as follows:

Definition 8. A 2-repeated low-density burst of length b (fixed) with weight w or less is an n -tuple whose only non-zero components are confined to two distinct sets of b consecutive positions, the first component of each set is non-zero where each set can have at most w non-zero components ($w \leq b$), and the number of its starting positions in an n -tuple is among the first $n - 2b + 1$ positions.

As an illustration, (21010000102000) is a 2-repeated low-density burst of length up to 6(fixed) with weight 3 or less over $\text{GF}(3)$ whereas (0010000111110) is a 2-repeated low-density burst of length at most 5(fixed) with weight 4 or less over $\text{GF}(2)$.

In this paper we have presented a study of codes dealing with the location of such kind of errors occurring within a sub-block. The concept of error-locating codes, lying midway between error detection and error correction, was introduced by Wolf and Elspas [1963]. In this technique the block of received digits is to be regarded as subdivided into mutually exclusive sub-blocks and while decoding it is possible to detect the error and in addition the receiver is able to identify which particular sub-block contains error. Such codes are referred to as Error-Locating codes (EL-codes). Wolf and Elspas [1963] studied binary codes which are capable of detecting and locating a single sub-block containing random errors. A study of codes locating burst errors of length b (fixed) has been made by Dass and Kishanchand [1986]. Dass and Arora [2010] obtained bounds for codes capable of locating repeated burst errors of length b (fixed) occurring within a sub-block.

In this paper we have obtained bounds on the number of check digits required to locate 2-repeated low-density bursts of length b (fixed), and m -repeated low-density bursts of length b (fixed). An illustration of such a code has also been given. Development of such codes will economize in the number of parity-check digits required in comparison to the usual low-density burst error locating codes while considering such repeated bursts as single bursts.

The paper has been organized as follows. In section 2 the necessary condition for the detection and location of 2-repeated low-density burst of length b (fixed) with weight w or less has been derived. This is followed by a sufficient condition for the existence of such a code. An illustration of 2-repeated low-density burst of length b (fixed) with weight w or less over $\text{GF}(2)$ has also been given. In section 3 a necessary condition for the detection and location of m -repeated low-density burst of length b (fixed) with weight w or less has been given followed by a sufficient condition for the existence of such a code.

In what follows we shall consider a linear code to be a subspace of n -tuples over $\text{GF}(q)$. The block of n digits, consisting of r check digits and $k = n - r$ information digits, is considered to be divided into s mutually exclusive sub-blocks. Each sub-block contains $t = n/s$ digits.

2 2-Repeated Low-density Burst Error Locating Codes

In this section, we consider (n, k) linear codes over $\text{GF}(q)$ that are capable of detecting and locating all 2-repeated low-density burst of length b (fixed) with weight w or less within a single sub-block.

It may be noted that an EL-code capable of detecting and locating a single sub-block containing an error which is in the form of a 2-repeated low-density bursts of length b (fixed) with weight w or less must satisfy the following conditions:

- (a) The syndrome resulting from the occurrence of a 2-repeated low-density burst of length b (fixed) with weight w or less within any one

sub-block must be distinct from the all zero syndrome.

- (b) The syndrome resulting from the occurrence of any 2-repeated low-density burst of length b (fixed) with weight w or less within a single sub-block must be distinct from the syndrome resulting likewise from any 2-repeated low-density burst of length b (fixed) with weight w or less *within* any other sub-block.

In this section we shall derive two results. The first result derives a lower bound on the number of check digits required for the existence of a linear code over $\text{GF}(q)$ capable of detecting and locating a single sub-block containing errors that are 2-repeated low-density burst of length b (fixed) with weight w or less. In the second result, an upper bound on the number of check digits which ensures the existence of such a code has been derived.

As the code is divided into several blocks of length t each and we wish to detect a 2-repeated low-density burst of length b (fixed) with weight w or less, we may come across with a situation when the difference between $2b$ and t ($b+w$ and t) becomes narrow. We note that if $t - b + 1 < b + w$ and if we consider any two 2-repeated low-density bursts x_1 and x_2 of length b (fixed) with weight w or less such that their non-zero components are confined to first $t - b + 1$ positions with w components confining to some fixed w positions out of first b consecutive positions then their difference $x_1 - x_2$ is again a 2-repeated low-density burst of length b (fixed) with weight w or less. However if we do not restrict ourselves to first $t - b + 1$ positions then we may not get a 2-repeated burst of length b (fixed) with weight w or less. This may be better understood with the help of the following examples:

Example 1. Let $t = 9$, $b = 4$, $w = 3$ and $q = 2$. So that $t - b + 1 = 6 < b + w (= 7)$.

Let $x_1 = (101101001)$ and $x_2 = (100101011)$.

Then x_1 and x_2 are 2-repeated low-density burst of length 4(fixed) with weight 3 or less whereas $x_1 - x_2 = (001000010)$ is not a 2-repeated burst of length 4(fixed).

Example 2. Let $t = 11$, $b = 5$, $w = 3$ and $q = 2$.

Let $x_1 = (10101010010)$ and $x_2 = (10101010001)$

Then x_1 and x_2 are 2-repeated low-density burst of length 5(fixed) with weight 3 or less whereas $x_1 - x_2 = (00000000011)$ which is not even a 2-repeated burst of length 4(fixed) what to talk of its weight.

So, accordingly we discuss the following cases:

Case 1: When $t - b + 1 \geq 2b$.

Let X be the collection of all those vectors in which all the non-zero components are confined to some fixed w positions out of two sets of b consecutive positions each i.e. l -th to $(l + b)$ -th position and j -th to $(j + b)$ -th position where $j > l + b$.

We observe that the syndromes of all the elements of X should be different; else for any x_1, x_2 belonging to X having the same syndrome would imply that the syndrome of $x_1 - x_2$ which is also an element of X and hence a 2-repeated low density burst of length b (fixed) with weight w or less within the same sub-block becomes zero; in violation of condition (a). Also, since the error locates a single sub-block containing errors that are 2-repeated low-density bursts of length b (fixed) of weight w or less,

the syndromes produced by similar vectors in different sub-blocks must be distinct by condition (b).

Thus the syndromes of vectors which are 2-repeated low-density burst of length b (fixed) with weight w or less in fixed positions, whether in the same sub-block or in different sub-blocks, must be distinct. (It may be noted that the choice of different fixed components in different sub-blocks will also yield the same result).

As there are $(q^{2w} - 1)$ distinct non-zero syndromes corresponding to the vectors in any one sub-block and there are s sub-blocks in all, so we must have atleast $(1 + s(q^{2w} - 1))$ distinct syndromes counting the all zero syndrome.

As maximum number of distinct syndromes available using r check bits is q^r , so there are q^r distinct syndromes in all, therefore we must have

$$q^r \geq \{1 + s(q^{2w} - 1)\} \quad (1)$$

where $t - b + 1 \geq 2b$.

Case 2: When $b + w \leq t - b + 1 < 2b$.

Let X be the collection of all those vectors in which all the non-zero components are confined to some w fixed positions out of first b components i.e first and b -th position and another set of w fixed positions out of $(b + 1)$ -th to $(t - b + 1)$ -th positions.

As discussed in case 1 the syndromes of all the elements of X is different.

In this case also, there are $(q^{2w} - 1)$ distinct non-zero syndromes corresponding to the vectors in any one sub-block and there are s sub-

blocks in all, so we must have atleast $(1 + s(q^{2w} - 1))$ distinct syndromes counting the all zero syndrome.

So, in this case also, we must have

$$q^r \geq \{1 + s(q^{2w} - 1)\} \tag{2}$$

where $b + w \leq t - b + 1 < 2b$.

Case 3: When $t - b + 1 < b + w$.

In this case consider X to be collection of all those vectors in which all the non-zero components are confined to some w fixed positions out of first b positions and $t - 2b + 1$ components from $(b + 1)$ -th to $(t - b + 1)$ -th positions. In this case there are $(q^{w+(t-2b+1)} - 1)$ distinct non-zero syndromes corresponding to the vectors in any one sub-block. As and there are s sub-blocks in all, so we must have atleast $(1 + s(q^{w+(t-2b+1)} - 1))$ distinct syndromes counting the all zero syndrome.

Therefore in this case, we must have

$$q^r \geq \{1 + s(q^{w+(t-2b+1)} - 1)\} \tag{3}$$

where $t - b + 1 < b + w$.

From (1), (2), and (3) we have

$$r \geq \begin{cases} \log_q \{1 + s(q^{2w} - 1)\} & \text{where } t - b + 1 \geq 2b \\ & \text{and } b + w \leq t - b + 1 < 2b \\ \log_q \{1 + s(q^{w+(t-2b+1)} - 1)\} & \text{where } t - b + 1 < b + w. \end{cases}$$

Thus we have proved:

Theorem 1. *The number of parity check digits r in an (n, k) linear code subdivided into s sub-blocks of length t each, that locates a single corrupted*

sub-block containing errors that are 2-repeated low density burst of length b (fixed) with weight w or less is at least

$$\begin{cases} \log_q\{1 + s(q^{2w} - 1)\} & \text{where } t - b + 1 \geq 2b \\ & \text{and } b + w \leq t - b + 1 < 2b. \\ \log_q\{1 + s(q^{w+(t-2b+1)} - 1)\} & \text{where } t - b + 1 < b + w \end{cases}$$

Remark 1. For $w = b$, the weight consideration over the burst becomes redundant and the result coincides with Theorem 1 [Dass and Arora [2010]], when the bursts considered are 2-repeated bursts of length b (fixed).

In the following result we derive another bound on the number of check digits required for the existence of such a code. The proof is based on the technique used to establish Varshomov-Gilbert Sacks bound by constructing a parity check matrix for such a code [refer Sacks[1958], also Theorem 4.7 Peterson and Weldon[1972]]. This technique not only ensures the existence of such a code but also gives a method for the construction of such a code.

Theorem 2. An (n, k) linear EL-code over $\text{GF}(q)$ capable of detecting a 2-repeated low density burst of length b (fixed) with weight w or less ($w \leq b$) within a single sub-block and of locating that sub-block can always be constructed provided that

$$q^{n-k} > [1 + (q - 1)]^{(b-1, w-1)} \{1 + (q - 1)(t - 2b + 1)[1 + (q - 1)]^{(b-1, w-1)}\} \cdot \left\{ 1 + (s - 1) \sum_{i=1}^2 \binom{t - ib + i}{i} (q - 1)^i \{[1 + (q - 1)]^{(b-1, w-1)}\}^i \right\} \quad (4)$$

where $[1 + x]^{(m, r)}$ denotes the incomplete binomial expansion of $(1 + x)^m$ up to the term x^r in ascending power of x , viz.

$$[1 + x]^{(m, r)} = 1 + \binom{m}{1}x + \binom{m}{2}x^2 + \dots + \binom{m}{r}x^r.$$

Proof. The existence of such a code will be shown by constructing an appropriate $(n - k \times n)$ parity check matrix H by a synthesis procedure. For that we first construct a matrix H_1 from which the requisite parity check matrix H shall be obtained by reversing the order of the columns of each sub-block.

After adding $(s-1)t$ columns appropriately corresponding to the first $(s-1)$ sub-blocks, suppose that we have added the first $j-1$ columns h_1, h_2, \dots, h_{j-1} of the s -th sub-block also, out of which the first $b-1$ columns h_1, h_2, \dots, h_{b-1} may be chosen arbitrarily (non-zero). We now lay down the condition to add the j -th column h_j to H_1 as follows:

According to condition (a), for the detection of 2-repeated low-density burst of length b (fixed) with weight w or less in the s -th sub-block h_j should not be a linear combination of any $w-1$ or fewer columns among the immediately preceding $b-1$ columns $h_{j-b+1}, h_{j-b+2}, \dots, h_{j-1}$ together with any w or fewer columns from amongst some b consecutive columns from the first $j-b$ columns of the s -th sub-block.

i.e.

$$h_j \neq (\alpha_{j_1} h_{j_1} + \alpha_{j_2} h_{j_2} + \dots + \alpha_{j_{w-1}} h_{j_{w-1}}) + (\beta_{l_1} h_{l_1} + \beta_{l_2} h_{l_2} + \dots + \beta_{l_w} h_{l_w}) \quad (5)$$

where $h_{j_1}, h_{j_2}, \dots, h_{j_{w-1}}$ are any $w-1$ columns among $h_{j-b+1}, h_{j-b+2}, \dots, h_{j-1}$ and h_l 's are any w columns from a set of b consecutive columns among the first $j-b$ columns of the s -th sub-block such that either all the coefficients β_{l_i} 's are zero or if the p -th coefficient β_{l_p} is the last non-zero coefficients then $b \leq p \leq j-b$;

$$\alpha_{j_i}, \beta_{l_i} \text{'s} \in \text{GF}(q).$$

The number of ways in which the coefficients α_i 's can be selected is $[1 + (q - 1)]^{(b-1, w-1)}$. To enumerate the coefficients β_i 's is equivalent to enumerate the number of bursts of length b (fixed) with weight w or less in a vector of length $j - b$.

This number including the vector of all zeros [refer Theorem 1, Dass [1983]] is

$$1 + (j - 2b + 1)(q - 1)[1 + (q - 1)]^{(b-1, w-1)}$$

So, the number of linear combinations on the right hand side of (5) is

$$[1 + (q - 1)]^{(b-1, w-1)}[1 + (j - 2b + 1)(q - 1)[1 + (q - 1)]^{(b-1, w-1)}] \quad (6)$$

According to condition (b), for the location of 2-repeated low-density bursts of length b (fixed) with weight w or less, h_j should not be a linear combination of any $w - 1$ or fewer columns among the immediately preceding the $b - 1$ columns and any w columns from a set of b consecutive columns from the remaining $j - b$ columns of the s -th sub-block along with any w or less columns each from any of the two sets of b consecutive columns out of any one of the previously chosen $s - 1$ sub-blocks, the coefficient of the last column of either both or one of the sets being non-zero.

The number of 2-repeated low-density bursts of length b (fixed) with weight w or less in a sub-block of length t [refer Dass and Garg [2009(b)]] is

$$\sum_{i=1}^2 \binom{t - ib + i}{i} (q - 1)^i \{ [1 + (q - 1)]^{(b-1, w-1)} \}^i \quad (7)$$

Since there are $(s - 1)$ previous sub-blocks, therefore number of such linear

combinations becomes

$$(s-1) \sum_{i=1}^2 \binom{t-ib+i}{i} (q-1)^i \{ [1+(q-1)]^{(b-1, w-1)} \}^i \quad (8)$$

So, for the location of 2-repeated low-density burst of length b (fixed) with weight w or less the number of linear combinations to which h_j can not be equal to is the product of expr.(6) and expr.(8)

$$\text{i.e. expr.(6)} \times \text{expr.(8)} \quad (9)$$

Thus the total number of linear combinations to which h_i can not be equal to is the sum of exp.(6) and exp.(9) At worst all these combinations might yield distinct sum.

Therefore h_i can be added to the s -th sub-block provided that

$$q^{n-k} > [1+(q-1)]^{(b-1, w-1)} \{ 1+(q-1)(j-2b+1)[1+(q-1)]^{(b-1, w-1)} \} \\ \cdot \left\{ 1+(s-1) \sum_{i=1}^2 \binom{t-ib+i}{i} (q-1)^i \{ [1+(q-1)]^{(b-1, w-1)} \}^i \right\}$$

To obtain the length of the block as t we replace j by t in the above expression.

The required parity-check matrix H can be obtained from H_1 by reversing the order of the columns in each sub-block.

Remark 2. For $w = b$, the weight consideration over the burst becomes redundant and the inequality in Theorem 2 reduces to

$$q^{n-k} > q^{b-1} \{ 1+(q-1)(t-2b+1)q^{b-1} \} \\ \times \left\{ 1+(s-1) \sum_{i=1}^2 \binom{t-ib+i}{i} (q-1)^i q^{i(b-1)} \right\}$$

which coincides with the condition for the location of 2-repeated burst of length b (fixed) [refer Theorem 2, Dass and Arora [2010]].

We conclude this section with the following example:

Example 3. For an (27,15) linear code over GF(2) consider the following 12×27 matrix H which has been constructed by the synthesis procedure given in the proof of theorem 2 by taking $s = 3, t = 9, b = 3, w = 2$.

$$H = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 \end{bmatrix}$$

The null space of this matrix can be used as a code to locate a sub-block of length $t = 9$ containing 2-repeated burst of length 3(fixed). From the error pattern syndrome Table 1 we observe that:

The syndromes of 2-repeated burst of length 3(fixed) within any sub-block are all non-zero showing thereby that the code detects all 2-repeated low-density bursts of length 3(fixed) with weight 2 or less occurring within a sub-block.

It has been verified through MS-Excel program that the syndromes of the 2-repeated bursts of length 3(fixed) with weight 2 or less in any sub-block is different from the syndrome of a 2-repeated burst of length 3(fixed) with weight 2 or less within any other sub-block.

Table 1

Low density 2-repeated bursts of length 3(fixed)				Syndromes
Sub-block - 1				
1	10010000	00000000	00000000	0000 0100 1000
2	100101000	00000000	00000000	0001 0100 1000
3	100110000	00000000	00000000	0000 1100 1000
4	101100000	00000000	00000000	0000 0110 1000
5	101101000	00000000	00000000	0001 0110 1000
6	101110000	00000000	00000000	0000 1110 1000
7	110100000	00000000	00000000	0000 0101 1000
8	110101000	00000000	00000000	0001 0101 1000
9	110110000	00000000	00000000	0000 1101 1000
10	100010000	00000000	00000000	0000 1000 1000
11	100010100	00000000	00000000	0010 1000 1000
12	100011000	00000000	00000000	0001 1000 1000
13	101010000	00000000	00000000	0000 1010 1000
14	101010100	00000000	00000000	0010 1010 1000
15	101011000	00000000	00000000	0001 1010 1000
16	110010000	00000000	00000000	0000 1001 1000
17	110010100	00000000	00000000	0010 1001 1000
18	110011000	00000000	00000000	0001 1001 1000
19	100001000	00000000	00000000	0001 0000 1000
20	100001010	00000000	00000000	0101 0000 1000
21	100001100	00000000	00000000	0011 0000 1000
22	101001000	00000000	00000000	0001 0010 1000
23	101001010	00000000	00000000	0101 0010 1000
24	101001100	00000000	00000000	0011 0010 1000
25	110001000	00000000	00000000	0001 0001 1000
26	110001010	00000000	00000000	0101 0001 1000
27	110001100	00000000	00000000	0011 0001 1000
28	100000100	00000000	00000000	0010 0000 1000
29	100000101	00000000	00000000	1010 0000 1000

Low density 2-repeated bursts of length 3(fixed)				Syndromes
Sub-block - 1				
30	10000110	00000000	00000000	0110 0000 1000
31	101000100	00000000	00000000	0010 0010 1000
32	101000101	00000000	00000000	1010 0010 1000
33	101000110	00000000	00000000	0110 0010 1000
34	110000100	00000000	00000000	0010 0001 1000
35	110000101	00000000	00000000	1010 0001 1000
36	110000110	00000000	00000000	0110 0001 1000
37	010010000	00000000	00000000	0000 1001 0000
38	010010100	00000000	00000000	0010 1001 0000
39	010011000	00000000	00000000	0001 1001 0000
40	010110000	00000000	00000000	0000 1101 0000
41	010110100	00000000	00000000	0010 1101 0000
42	010111000	00000000	00000000	0001 1101 0000
43	011010000	00000000	00000000	0000 1011 0000
44	011010100	00000000	00000000	0010 1011 0000
45	011011000	00000000	00000000	0001 1011 0000
46	010001000	00000000	00000000	0001 0001 0000
47	010001010	00000000	00000000	0101 0001 0000
48	010001100	00000000	00000000	0011 0001 0000
49	010101000	00000000	00000000	0001 0101 0000
50	010101010	00000000	00000000	0101 0101 0000
51	010101100	00000000	00000000	0011 0101 0000
52	011001000	00000000	00000000	0001 0011 0000
53	011001010	00000000	00000000	0101 0011 0000
54	011001100	00000000	00000000	0011 0011 0000
55	010000100	00000000	00000000	0010 0001 0000
56	010000101	00000000	00000000	1010 0001 0000
57	010000110	00000000	00000000	0110 0001 0000
58	010100100	00000000	00000000	0010 0101 0000
59	010100101	00000000	00000000	1010 0101 0000
60	010100110	00000000	00000000	0110 0101 0000
61	011000100	00000000	00000000	0010 0011 0000
62	011000101	00000000	00000000	1010 0011 0000
63	011000110	00000000	00000000	0110 0011 0000
64	001001000	00000000	00000000	0001 0010 0000
65	001001010	00000000	00000000	0101 0010 0000
66	001001100	00000000	00000000	0011 0010 0000

Low density 2-repeated bursts of length 3(fixed)				Syndromes
Sub-block - 1				
67	001011000	000000000	000000000	0001 1010 0000
68	001011010	000000000	000000000	0101 1010 0000
69	001011100	000000000	000000000	0011 1010 0000
70	001101000	000000000	000000000	0001 0110 0000
71	001101010	000000000	000000000	0101 0110 0000
72	001101100	000000000	000000000	0011 0110 0000
73	001000100	000000000	000000000	0010 0010 0000
74	001000101	000000000	000000000	1010 0010 0000
75	001000110	000000000	000000000	0110 0010 0000
76	001010100	000000000	000000000	0010 1010 0000
77	001010101	000000000	000000000	1010 1010 0000
78	001010110	000000000	000000000	0110 1010 0000
79	001100100	000000000	000000000	0010 0110 0000
80	001100101	000000000	000000000	1010 0110 0000
81	001100110	000000000	000000000	0110 0110 0000
82	000100100	000000000	000000000	0010 0100 0000
83	000100101	000000000	000000000	1010 0100 0000
84	000100110	000000000	000000000	0110 0100 0000
85	000101100	000000000	000000000	0011 0100 0000
86	000101101	000000000	000000000	1011 0100 0000
87	000101110	000000000	000000000	0111 0100 0000
88	000110100	000000000	000000000	0010 1100 0000
89	000110101	000000000	000000000	1010 1100 0000
90	000110110	000000000	000000000	0110 1100 0000
91	100000000	000000000	000000000	0000 0000 1000
92	101000000	000000000	000000000	0000 0010 1000
93	110000000	000000000	000000000	0000 0001 1000
94	010000000	000000000	000000000	0000 0001 0000
95	010100000	000000000	000000000	0000 0101 0000
96	011000000	000000000	000000000	0000 0011 0000
97	001000000	000000000	000000000	0000 0010 0000
98	001010000	000000000	000000000	0000 1010 0000
99	001100000	000000000	000000000	0000 0110 0000
100	000100000	000000000	000000000	0000 0100 0000
101	000101000	000000000	000000000	0001 0100 0000
102	000110000	000000000	000000000	0000 1100 0000

Low density 2-repeated bursts of length 3(fixed)				Syndromes
Sub-block - 1				
103	000010000	000000000	000000000	0000 1000 0000
104	000010100	000000000	000000000	0010 1000 0000
105	000011000	000000000	000000000	0001 1000 0000
106	000001000	000000000	000000000	0001 0000 0000
107	000001010	000000000	000000000	0101 0000 0000
108	000001100	000000000	000000000	0011 0000 0000
109	000000100	000000000	000000000	0010 0000 0000
110	000000101	000000000	000000000	1010 0000 0000
111	000000110	000000000	000000000	0110 0000 0000

Low density 2-repeated bursts of length 3(fixed)				Syndromes
Sub-block - 2				
112	000000000	100100000	000000000	0011 0000 1100
113	000000000	100101000	000000000	1111 0000 1100
114	000000000	100110000	000000000	1111 0000 1111
115	000000000	101100000	000000000	1001 1010 0110
116	000000000	101101000	000000000	0101 1010 0110
117	000000000	101110000	000000000	0101 1010 0101
118	000000000	110100000	000000000	1101 1110 0010
119	000000000	110101000	000000000	0001 1110 0010
120	000000000	110110000	000000000	0001 1110 0001
121	000000000	100010000	000000000	0011 1100 0011
122	000000000	100010100	000000000	0011 1100 0010
123	000000000	100011000	000000000	1111 1100 0011
124	000000000	101010000	000000000	1001 0110 1001
125	000000000	101010100	000000000	1001 0110 1000
126	000000000	101011000	000000000	0101 0110 1001
127	000000000	110010000	000000000	1101 0010 1101
128	000000000	110010100	000000000	1101 0010 1100
129	000000000	110011000	000000000	0001 0010 1101
130	000000000	100001000	000000000	0011 1100 0000
131	000000000	100001010	000000000	0011 1100 0010
132	000000000	100001100	000000000	0011 1100 0001
133	000000000	101001000	000000000	1001 0110 1010
134	000000000	101001010	000000000	1001 0110 1000
135	000000000	101001100	000000000	1001 0110 1011
136	000000000	110001000	000000000	1101 0010 1110

Low density 2-repeated bursts of length 3(fixed)				Syndromes
Sub-block - 2				
137	00000000	110001010	000000000	1101 0010 1100
138	00000000	110001100	000000000	1101 0010 1111
139	00000000	100000100	000000000	1111 1100 0001
140	00000000	100000101	000000000	1111 1100 0101
141	00000000	100000110	000000000	1111 1100 0011
142	00000000	101000100	000000000	0101 0110 1011
143	00000000	101000101	000000000	0101 0110 1111
144	00000000	101000110	000000000	0101 0110 1001
145	00000000	110000100	000000000	0001 0010 1111
146	00000000	110000101	000000000	0001 0010 1011
147	00000000	110000110	000000000	0001 0010 1101
148	00000000	010010000	000000000	0010 1110 1101
149	00000000	010010100	000000000	0010 1110 1100
150	00000000	010011000	000000000	1110 1110 1101
151	00000000	010110000	000000000	1110 0010 0001
152	00000000	010110100	000000000	1110 0010 0000
153	00000000	010111000	000000000	0010 0010 0001
154	00000000	011010000	000000000	1000 0100 0111
155	00000000	011010100	000000000	1000 0100 0110
156	00000000	011011000	000000000	0100 0100 0111
157	00000000	010001000	000000000	0010 1110 1110
158	00000000	010001010	000000000	0010 1110 1100
159	00000000	010001100	000000000	0010 1110 1111
160	00000000	010101000	000000000	1110 0010 0010
161	00000000	010101010	000000000	1110 0010 0000
162	00000000	010101100	000000000	1110 0010 0011
163	00000000	011001000	000000000	1000 0100 0100
164	00000000	011001010	000000000	1000 0100 0110
165	00000000	011001100	000000000	1000 0100 0101
166	00000000	010000100	000000000	1110 1110 1111
167	00000000	010000101	000000000	1110 1110 1011
168	00000000	010000110	000000000	1110 1110 1101
169	00000000	010100100	000000000	0010 0010 0011
170	00000000	010100101	000000000	0010 0010 0111
171	00000000	010100110	000000000	0010 0010 0001
172	00000000	011000100	000000000	0100 0100 0101
173	00000000	011000101	000000000	0100 0100 0001

Low density 2-repeated bursts of length 3(fixed)				Syndromes
Sub-block - 2				
174	00000000	011000110	00000000	0100 0100 0111
175	00000000	001001000	00000000	0110 1010 1010
176	00000000	001001010	00000000	0110 1010 1000
177	00000000	001001100	00000000	0110 1010 1011
178	00000000	001011000	00000000	1010 1010 1001
179	00000000	001011010	00000000	1010 1010 1011
180	00000000	001011100	00000000	1010 1010 1000
181	00000000	001101000	00000000	1010 0110 0110
182	00000000	001101010	00000000	1010 0110 0100
183	00000000	001101100	00000000	1010 0110 0111
184	00000000	001000100	00000000	1010 1010 1011
185	00000000	001000101	00000000	1010 1010 1111
186	00000000	001000110	00000000	1010 1010 1001
187	00000000	001010100	00000000	0110 1010 1000
188	00000000	001010101	00000000	0110 1010 1100
189	00000000	001010110	00000000	0110 1010 1010
190	00000000	001100100	00000000	0110 0110 0111
191	00000000	001100101	00000000	0110 0110 0011
192	00000000	001100110	00000000	0110 0110 0101
193	00000000	000100100	00000000	1100 1100 1101
194	00000000	000100101	00000000	1100 1100 1001
195	00000000	000100110	00000000	1100 1100 1111
196	00000000	000101100	00000000	0000 1100 1101
197	00000000	000101101	00000000	0000 1100 1001
198	00000000	000101110	00000000	0000 1100 1111
199	00000000	000110100	00000000	0000 1100 1110
200	00000000	000110101	00000000	0000 1100 1010
201	00000000	000110110	00000000	0000 1100 1100
202	00000000	100000000	00000000	1111 1100 0000
203	00000000	101000000	00000000	0101 0110 1010
204	00000000	110000000	00000000	0001 0010 1110
205	00000000	010000000	00000000	1110 1110 1110
206	00000000	010100000	00000000	0010 0010 0010
207	00000000	011000000	00000000	0100 0100 0100
208	00000000	001000000	00000000	1010 1010 1010
209	00000000	001010000	00000000	0110 1010 1001
210	00000000	001100000	00000000	0110 0110 0110
211	00000000	000100000	00000000	1100 1100 1100

Low density 2-repeated bursts of length 3(fixed)				Syndromes
Sub-block - 2				
212	00000000	000101000	000000000	0000 1100 1100
213	00000000	000110000	000000000	0000 1100 1111
214	00000000	000010000	000000000	1100 0000 0011
215	00000000	000010100	000000000	1100 0000 0010
216	00000000	000011000	000000000	0000 0000 0011
217	00000000	000001000	000000000	1100 0000 0000
218	00000000	000001010	000000000	1100 0000 0010
219	00000000	000001100	000000000	1100 0000 0001
220	00000000	000000100	000000000	0000 0000 0001
221	00000000	000000101	000000000	0000 0000 0101
222	00000000	000000110	000000000	0000 0000 0011

Low density 2-repeated bursts of length 3(fixed)				Syndromes
Sub-block - 3				
223	00000000	000000000	100100000	1111 1110 0111
224	00000000	000000000	100101000	1101 0111 0011
225	00000000	000000000	100110000	1000 1110 1001
226	00000000	000000000	101100000	0111 0111 1000
227	00000000	000000000	101101000	0101 1110 1100
228	00000000	000000000	101110000	0000 0111 0110
229	00000000	000000000	110100000	1111 1110 1101
230	00000000	000000000	110101000	1101 0111 1001
231	00000000	000000000	110110000	1000 1110 0011
232	00000000	000000000	100010000	1000 1111 0110
233	00000000	000000000	100010100	0110 0000 0001
234	00000000	000000000	100011000	1010 0110 0010
235	00000000	000000000	101010000	0000 0110 1001
236	00000000	000000000	101010100	1110 1001 1110
237	00000000	000000000	101011000	0010 1111 1101
238	00000000	000000000	110010000	1000 1111 1100
239	00000000	000000000	110010100	0110 0000 1011
240	00000000	000000000	110011000	1010 0110 1000
241	00000000	000000000	100001000	1101 0110 1100
242	00000000	000000000	100001010	0011 0110 1011
243	00000000	000000000	100001100	0011 1001 1011
244	00000000	000000000	101001000	0101 1111 0011

Low density 2-repeated bursts of length 3(fixed)				Syndromes
Sub-block - 3				
245	00000000	00000000	101001010	1011 1111 0100
246	00000000	00000000	101001100	1011 0000 0100
247	00000000	00000000	110001000	1101 0110 0110
248	00000000	00000000	110001010	0011 0110 0001
249	00000000	00000000	110001100	0011 1001 0001
250	00000000	00000000	100000100	0001 0000 1111
251	00000000	00000000	100000101	1001 0000 1110
252	00000000	00000000	100000110	1111 0000 1000
253	00000000	00000000	101000100	1001 1001 0000
254	00000000	00000000	101000101	0001 1001 0001
255	00000000	00000000	101000110	0111 1001 0111
256	00000000	00000000	110000100	0001 0000 0101
257	00000000	00000000	110000101	1001 0000 0100
258	00000000	00000000	110000110	1111 0000 0010
259	00000000	00000000	010010000	0111 0000 0100
260	00000000	00000000	010010100	1001 1111 0011
261	00000000	00000000	010011000	0101 1001 0000
262	00000000	00000000	010110000	0111 0001 1011
263	00000000	00000000	010110100	1001 1110 1100
264	00000000	00000000	010111000	0101 1000 1111
265	00000000	00000000	011010000	1111 1001 1011
266	00000000	00000000	011010100	0001 0110 1100
267	00000000	00000000	011011000	1101 0000 1111
268	00000000	00000000	010001000	0010 1001 1110
269	00000000	00000000	010001010	1100 1001 1001
270	00000000	00000000	010001100	1100 0110 1001
271	00000000	00000000	010101000	0010 1000 0001
272	00000000	00000000	010101010	1100 1000 0110
273	00000000	00000000	010101100	1100 0111 0110
274	00000000	00000000	011001000	1010 0000 0001
275	00000000	00000000	011001010	0100 0000 0110
276	00000000	00000000	011001100	0100 1111 0110
277	00000000	00000000	010000100	1110 1111 1101
278	00000000	00000000	010000101	0110 1111 1100
279	00000000	00000000	010000110	0000 1111 1010
280	00000000	00000000	010100100	1110 1110 0010
281	00000000	00000000	010100101	0110 1110 0011

Low density 2-repeated bursts of length 3(fixed)				Syndromes
Sub-block - 3				
282	00000000	00000000	010100110	0000 1110 0101
283	00000000	00000000	011000100	0110 0110 0010
284	00000000	00000000	011000101	1110 0110 0011
285	00000000	00000000	011000110	1000 0110 0101
286	00000000	00000000	001001000	1010 0000 1011
287	00000000	00000000	001001010	0100 0000 1100
288	00000000	00000000	001001100	0100 1111 1100
289	00000000	00000000	001011000	1101 0000 0101
290	00000000	00000000	001011010	0011 0000 0010
291	00000000	00000000	001011100	0011 1111 0010
292	00000000	00000000	001101000	1010 0001 0100
293	00000000	00000000	001101010	0100 0001 0011
294	00000000	00000000	001101100	0100 1110 0011
295	00000000	00000000	001000100	0110 0110 1000
296	00000000	00000000	001000101	1110 0110 1001
297	00000000	00000000	001000110	1000 0110 1111
298	00000000	00000000	001010100	0001 0110 0110
299	00000000	00000000	001010101	1001 0110 0111
300	00000000	00000000	001010110	1111 0110 0001
301	00000000	00000000	001100100	0110 0111 0111
302	00000000	00000000	001100101	1110 0111 0110
303	00000000	00000000	001100110	1000 0111 0000
304	00000000	00000000	000100100	1110 1110 1000
305	00000000	00000000	000100101	0110 1110 1001
306	00000000	00000000	000100110	0000 1110 1111
307	00000000	00000000	000101100	1100 0111 1100
308	00000000	00000000	000101101	0100 0111 1101
309	00000000	00000000	000101110	0010 0111 1011
310	00000000	00000000	000110100	1001 1110 0110
311	00000000	00000000	000110101	0001 1110 0111
312	00000000	00000000	000110110	0111 1110 0001
313	00000000	00000000	100000000	1111 1111 1000
314	00000000	00000000	101000000	0111 0110 0111
315	00000000	00000000	110000000	1111 1111 0010
316	00000000	00000000	010000000	0000 0000 1010
317	00000000	00000000	010100000	0000 0001 0101
318	00000000	00000000	011000000	1000 1001 0101

Low density 2-repeated bursts of length 3(fixed)				Syndromes
Sub-block - 3				
319	00000000	00000000	00100000	1000 1001 1111
320	00000000	00000000	00101000	1111 1001 0001
321	00000000	00000000	00110000	1000 1000 0000
322	00000000	00000000	00010000	0000 0001 1111
323	00000000	00000000	00010100	0010 1000 1011
324	00000000	00000000	00011000	0111 0001 0001
325	00000000	00000000	00001000	0111 0000 1110
326	00000000	00000000	00001010	1001 1111 1001
327	00000000	00000000	00001100	0101 1001 1010
328	00000000	00000000	00000100	0010 1001 0100
329	00000000	00000000	00000101	1100 1001 0011
330	00000000	00000000	00000110	1100 0110 0011
331	00000000	00000000	00000010	1110 1111 0111
332	00000000	00000000	000000101	0110 1111 0110
333	00000000	00000000	000000110	0000 1111 0000

Remark 3. The space visible between vectors in the first column in Table 1 has been given to distinguish between different sub-blocks whereas the space given in the syndrome vector is for convenience.

Observation. Syndromes of some of the 2-repeated bursts of length 3(fixed) occurring within the second sub-block are same. For coding efficiency it is desired that the syndromes of the error patterns within any sub-block is identical whenever possible.

3 Location of m -Repeated Low-density burst of length b (fixed)

In this section a necessary and sufficient condition for the location of an m -repeated low-density burst of length b (fixed) with weight w or less has been given.

It may be noted that an EL-code capable of detecting and locating a single sub-block containing an error which is in the form of an m -repeated low-density burst of length b (fixed) with weight w or less ($w \leq b$) must satisfy the following conditions:

- (c) The syndrome resulting from the occurrence of an m -repeated low-density burst of length b (fixed) with weight w or less within any one sub-block must be distinct from the all zero syndrome.
- (d) The syndrome resulting from the occurrence of any m -repeated low-density burst of length b (fixed) with weight w or less within a single sub-block must be distinct from the syndrome resulting likewise from any m -repeated low-density burst of length b (fixed) with weight w or less *within* any other sub-block.

In this section we shall derive two results. The first result gives a lower bound on the number of check digits required for the existence of a linear code over $GF(q)$ capable of detecting and locating a single sub-block containing errors that are m -repeated low-density bursts of length b (fixed) with weight w or less. In the second result, we derive an upper bound on the number of check digits which ensures the existence of such a code.

Theorem 3. *The number of parity check digits r in an (n, k) linear code subdivided into s sub-blocks of length t each, that locates a single corrupted sub-block containing errors that are 2-repeated low density bursts of length b (fixed) with weight w or less is at least*

$$\begin{cases} \log_q\{1 + s(q^{mw} - 1)\} & \text{where } t - b + 1 \geq mb \\ \log_q\{1 + s(q^{(m-1)w+(t-mb+1)} - 1)\} & \text{and } (m-1)b+w \leq t-b+1 < mb \end{cases} \quad (10)$$

where $t - b + 1 < (m - 1)b + w$.

The proof of this result is on the similar lines as that of proof of Theorem 1 so we omit the proof.

Remark 4. For $m = 2$ the result coincides with that of Theorem 1 when 2-repeated low-density bursts of length b (fixed) with weight w or less are considered.

Remark 5. For $m = 1$, the result obtained in (10) reduces to

$$\begin{cases} \log_q\{1 + s(q^w - 1)\} & \text{where } t - b + 1 \geq b \\ & \text{and } w \leq t - b + 1 < b . \\ \log_q\{1 + s(q^{(t-b+1)} - 1)\} & \text{where } t - b + 1 < w \end{cases}$$

which is a case of detecting and locating a sub-block containing errors which are usual low-density bursts of length b (fixed) with weight w or less.

Remark 6. For $w = b$, the result obtained in (10) reduces to

$$r \geq \begin{cases} \log_q\{1 + s(q^{mb} - 1)\} & \text{where } t - b + 1 \geq mb \\ \log_q\{1 + s(q^{(t-b+1)} - 1)\} & \text{where } t - b + 1 < mb \end{cases}$$

which coincides with the result due to Dass and Arora [Theorem 3, 2010].

In the following result we derive another bound on the number of check digits required for the existence of such a code. As earlier the proof is based on the technique used to establish Varshomov-Gilbert Sacks bound by constructing a parity check matrix for such a code (refer Sacks, Theorem 4.7 Peterson and Weldon(1972)).

Theorem 4. *An (n, k) linear EL-code over $\text{GF}(q)$ capable of detecting an m -repeated low density burst of length b (fixed) with weight w or less*

($w \leq b$) within a single sub-block and of locating that sub-block can always be constructed provided that

$$q^{n-k} > [1 + (q - 1)]^{(b-1, w-1)} \cdot \left\{ \sum_{i=0}^{m-1} \binom{t - (i+1)b + i}{i} (q - 1)^i [1 + (q - 1)]^{(b-1, w-1)} \right\} \cdot \left\{ 1 + (s - 1) \sum_{i=1}^m \binom{t - ib + i}{i} (q - 1)^i \{ [1 + (q - 1)]^{(b-1, w-1)} \}^i \right\} \quad (11)$$

where $[1 + x]^{(m, r)}$ denotes the incomplete binomial expansion of $(1 + x)^m$ up to the term x^r in ascending power of x , viz.

$$[1 + x]^{(m, r)} = 1 + \binom{m}{1}x + \binom{m}{2}x^2 + \dots + \binom{m}{r}x^r.$$

As in theorem 3 we omit the proof because proof of this result is on the similar lines as that of proof of Theorem 2.

Remark 7. For $m = 2$ the result coincides with that of Theorem 2 when 2-repeated low-density bursts of length b (fixed) with weight w or less are considered.

Remark 8. For $m = 1$, the result obtained in (11) reduces to

$$q^{n-k} > [1 + (q - 1)]^{(b-1, w-1)} \{ 1 + (s - 1)(t - b + 1)(q - 1)[1 + (q - 1)]^{(b-1, w-1)} \}$$

which is a necessary condition for detecting and locating a sub-block containing errors which are usual low-density bursts of length b (fixed) with weight w or less.

Remark 9. For $w = b$, the result obtained in (11) reduces to

$$q^{n-k} > q^{b-1} \left\{ \sum_{i=0}^{m-1} \binom{j - (i+1)b + i}{i} (q-1)^i q^{i(b-1)} \right\} \\ \cdot \left\{ 1 + (s-1) \sum_{i=1}^m \binom{j - (i+1)b + i}{i} (q-1)^i q^{i(b-1)} \right\}$$

which coincides with the result due to Dass and Arora [Theorem 4, 2010].

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