Ratio Mathematica Volume 42, 2022

An efficient block-based image compression and quality-wise decompression algorithm

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Abstract

In this paper, we propose a block-based lossy image compression algorithm that makes use of spatial redundancies of neighboring pixels in image data. Compression is achieved by replacing a block of pixels with their statistical mean. The algorithm helps in decompressing the image at different quality levels. Quality matrices constructed from the quantization table of the JPEG baseline algorithm are used to achieve different qualities of the reconstructed data. Experimental results show that the proposed method outperforms existing polynomial-based algorithms both in computation time and complexity.
Keywords: lossy compression;JPEG compression; polynomial-based compression.
2020 AMS subject classifications: 97M10 [1]

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[1]Received on April 23rd, 2022. Accepted on June 24th, 2022. Published on June 30th, 2022. doi: 10.23755/rm.v41i0.771. ISSN: 1592-7415. eISSN: 2282-8214.

1 Introduction

Image compression is a process that reduces the size of image data files while keeping necessary information. We can classify image compression schemes into four groups according to the process element as pixel-based, block-based, subband-based, and region-based. Image compression has been a hot topic of research for many years and several image compression standards have been developed ([Sonal, 2007], [Memon and Sayood, 1995]). Among these, the block-based compression scheme JPEG that uses discrete cosine transform (DCT) and the subband-based scheme JPEG 2000 has got much attention and popularity ([Wallace, 1992], [Rabbani, 2002]). Since these two methods involve transforms such as DCT and wavelet transform, their computational complexity is very high. Researches are still going on in developing simple and fast compression algorithms that can show better performance than the existing one. As an alternative to transform-based techniques, polynomial-based compression and statistical approach in compression are also developed([Shukla et al., 2005], [Ameer, 2009], [Sajikumar and Anilkumar, 2017], [Sajikumar et al., 2021]). Even though many algorithms have been reported in this field, research is still needed to cope with the continuous demand for efficient transmission or storage of image data.

If the information retained after decompression is 100%, the compression method is called lossless otherwise it is lossy. If we take a pixel in an image at random there is a good chance that its neighbours will have the same intensity or very similar intensity. Typically hence, image compression is based on the fact that the neighbouring pixels are highly correlated ([Salomon, 2007], [Sayood, 2012]). Most image compression methods exploit this feature to obtain efficient compression. Lossless compression can be achieved with the techniques like Run Length Encoding (RLE), Huffman coding, Arithmetic coding, etc.([Gallager, 1978], [Jain, 1989], [Taubman and Marcellin, 2012], [Witten et al., 1987]). Lossy techniques include transform coding methods such DCT/JPEG, JPEG2000, etc. ([Pennebaker and Mitchell, 1992], [Gonzalez and Woods, 2008], Goyal [2001]). Polynomial-based compression is another type of lossy compression method ([Sadeh, 1996], [Eden et al., 1986]). S. Sajikumar and A. K. Anilkumar [Sajikumar and Anilkumar, 2017] introduced a compression scheme using Chebyshev polynomials. Lossy compression techniques tested for their performance based on three commonly used measures, the Root Mean Square Error (RMSE), Peak Signal to Noise Ratio (PSNR) and the Compression Ratio (CR). The RMSE between original image $f(x, y)$ and reconstructed image $\hat{f}(x, y)$ of size $M \times N$ is defined by [Joshi, 2018]:

$$RMSE = \left[\frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \left[f(x, y) - \hat{f}(x, y) \right]^2 \right]^{\frac{1}{2}} \tag{1}$$

For an 8- bit gray level image,

$$PSNR = 10\log_{10}\Big(\frac{255^2}{MSE}\Big)(dB) \tag{2}$$

$$\text{CR} = \frac{\text{compressed image size}}{\text{uncompressed image size}}\,\% \tag{3}$$

In digital image compression, the basic data redundancies are due to coding redundancy, inter-pixel redundancy, and psycho-visual redundancy. Statistical approaches in image compression are the compression techniques that try to decorrelate this inter-pixel redundancy. Dimensionality reduction is another aspect of image compression. The Principal Component Analysis (PCA) is the significant one in this area ([Du and Fowler, 2007], [Sonal, 2007]). The PCA approach is implemented via the Statistical approach and the Neural Network approach [Dony and Haykin, 1995]. To reduce the storage space required, we can make use of statistical measures of central dispersion such as mean and variance of pixel values in an image [Sajikumar et al., 2021]. This paper presents a simple and efficient lossy image compression method using the mean value of a block of pixels. The input image is partitioned into non-overlapping blocks and the mean pixel value for each block is used to represent the entire block of pixels followed by a quality gradation at the decompression stage. We compare the proposed method with polynomial-based compression techniques such as plane fitting model and Chebyshev polynomial surface fit method ([Ameer and Basir, 2006], [Sajikumar and Anilkumar, 2017]). In comparison, it is found that the proposed method outperforms these algorithms.

2 Proposed method

Divide the input image matrix into non-overlapping blocks of size $n \times n$. Subtract 128 from each pixel in the image matrix to change the gray levels from $[0, 255]$ to values centered about zero. Thus the modified range becomes $[-128, 127]$. Compute the mean value of each block of pixels and store this mean for each block as the reconstruction parameter. At the reconstruction stage, replace all pixels in each block by the respective mean values. That is, n^2 pixel values in each block are replaced by a single parameter and hence high compression can be achieved as the block size increases.

To reduce the loss of information at the decompression stage, a quality matrix of dimension $n \times n$ is introduced. This matrix allows us to decompress output images at different quality levels. The quality determination process outputs images at different bit-rates of lower to a higher order. We have adopted this matrix from the JPEG's baseline compression algorithm where it is used as the quantization matrix ([Wallace, 1992], [Ahumada Jr and Peterson, 1992], [Watson, 1993]).

$$Q_{50} = \begin{pmatrix} 16 & 11 & 10 & 16 & 24 & 40 & 51 & 61 \\ 12 & 12 & 14 & 19 & 26 & 58 & 60 & 55 \\ 14 & 13 & 16 & 24 & 40 & 57 & 69 & 56 \\ 14 & 17 & 22 & 29 & 51 & 87 & 80 & 62 \\ 18 & 22 & 37 & 56 & 68 & 109 & 103 & 77 \\ 24 & 35 & 55 & 64 & 81 & 104 & 113 & 92 \\ 49 & 64 & 78 & 87 & 103 & 121 & 120 & 101 \\ 72 & 92 & 95 & 98 & 112 & 100 & 103 & 99 \end{pmatrix} \tag{4}$$

If N is the visual quality level of the decompressed image, then we can obtain different quality matrices Q_N using the following equation [Khedr and Abdelrazek, 2016]:

$$Q_N = \begin{cases} \left(\dfrac{100-N}{50}\right) Q_{50}, & N > 50 \\ \left(\dfrac{50}{N}\right) Q_{50}, & N < 50 \end{cases} \tag{5}$$

We have considered submatrices of size $n \times n$ with elements taken in order from the top left corner of the matrix Q_{50}. The quality matrix Q_{50} for different block sizes 2×2 , 3×3, 4×4 are:

$$\begin{pmatrix} 16 & 11 \\ 12 & 12 \end{pmatrix}, \begin{pmatrix} 16 & 11 & 10 \\ 12 & 12 & 14 \\ 14 & 13 & 16 \end{pmatrix}, \begin{pmatrix} 16 & 11 & 10 & 16 \\ 12 & 12 & 14 & 19 \\ 14 & 13 & 16 & 24 \\ 14 & 17 & 22 & 29 \end{pmatrix}$$

3 Experimental results

Test images of size 256×256 with gray levels in the range $[0, 256]$ are considered. Experimental results with block sizes 2×2, 3×3, 4×4 are given in Tables 1-3 and Figures 1-5. Compression qualities are analyzed at different levels $5, 10, 50, 90,$ and 95. Decompressed images at these levels are given in Figures 2-5.

In 2×2 blocks, four gray values are replaced by the mean and hence save 75% storage space with CR 25%. At this CR, all the test images show reasonable reconstructed image quality with low RMSE and exhibit superior performance to polynomial-based compression schemes. With the 25% CR and quality index 95, Rice image shows PSNR value 31.5585 (dB), Lena 27.5988 (dB), and Cameraman

25.4852(dB). These results are promising in comparison with the polynomial-based algorithms. Detailed comparison results with different block sizes are given in the next section. For 3×3 blocks, CR is 11.11% and it is 6.25% for 4×4 blocks. As the block size increases reconstruction quality becomes poor with a marginal increase in RMSE.

Test Image	$CR\,\%$	Performance	Quality Index				
			95	90	50	10	5
Rice	25	PSNR	31.5585	31.5340	30.4099	21.0875	16.0389
		RMSE	6.7393	6.7583	7.6921	22.4491	40.2340
Lena	25	PSNR	27.5988	27.5823	27.0974	21.7713	16.3263
		RMSE	10.6317	10.6518	11.2634	220.7958	38.9249
Cameraman	25	PSNR	25.4852	25.4779	25.1629	20.6225	17.0641
		RMSE	13.5604	13.5720	14.0733	23.7365	35.7547

Table 1: Compression performance at different reconstruction qualities in the case of 2×2 blocks.

Test Image	$CR\,\%$	Performance	Quality Index				
			95	90	50	10	5
Rice	11.11	PSNR	26.0410	26.0311	25.6843	20.7885	15.2735
		RMSE	12.7202	12.7346	13.2534	23.2871	43.9405
Lena	11.11	PSNR	23.7492	23.7424	23.5305	19.9055	15.5763
		RMSE	16.5607	16.5738	16.9831	25.77901	42.4353
Cameraman	11.11	PSNR	22.1119	22.1079	21.9610	19.4491	16.6085
		RMSE	19.9961	20.0053	20.3466	21.1698	37.6804

Table 2: Compression performance at different reconstruction qualities in the case of 3×3 blocks.

Test Image	$CR\,\%$	Performance	Quality Index				
			95	90	50	10	5
Rice	6.25	PSNR	25.3334	25.3145	24.8320	18.4780	15.4024
		RMSE	13.7997	13.8298	14.6197	30.3836	43.2934
Lena	6.25	PSNR	23.3583	23.3464	23.0116	18.6289	14.3644
		RMSE	17.3231	17.3468	18.0285	29.8604	48.7890
Cameraman	6.25	PSNR	21.7695	21.7629	21.5635	17.9073	14.8582
		RMSE	20.8001	20.8160	21.2994	32.4471	46.0924

Table 3: Compression performance at different reconstruction qualities in the case of 4×4 blocks.

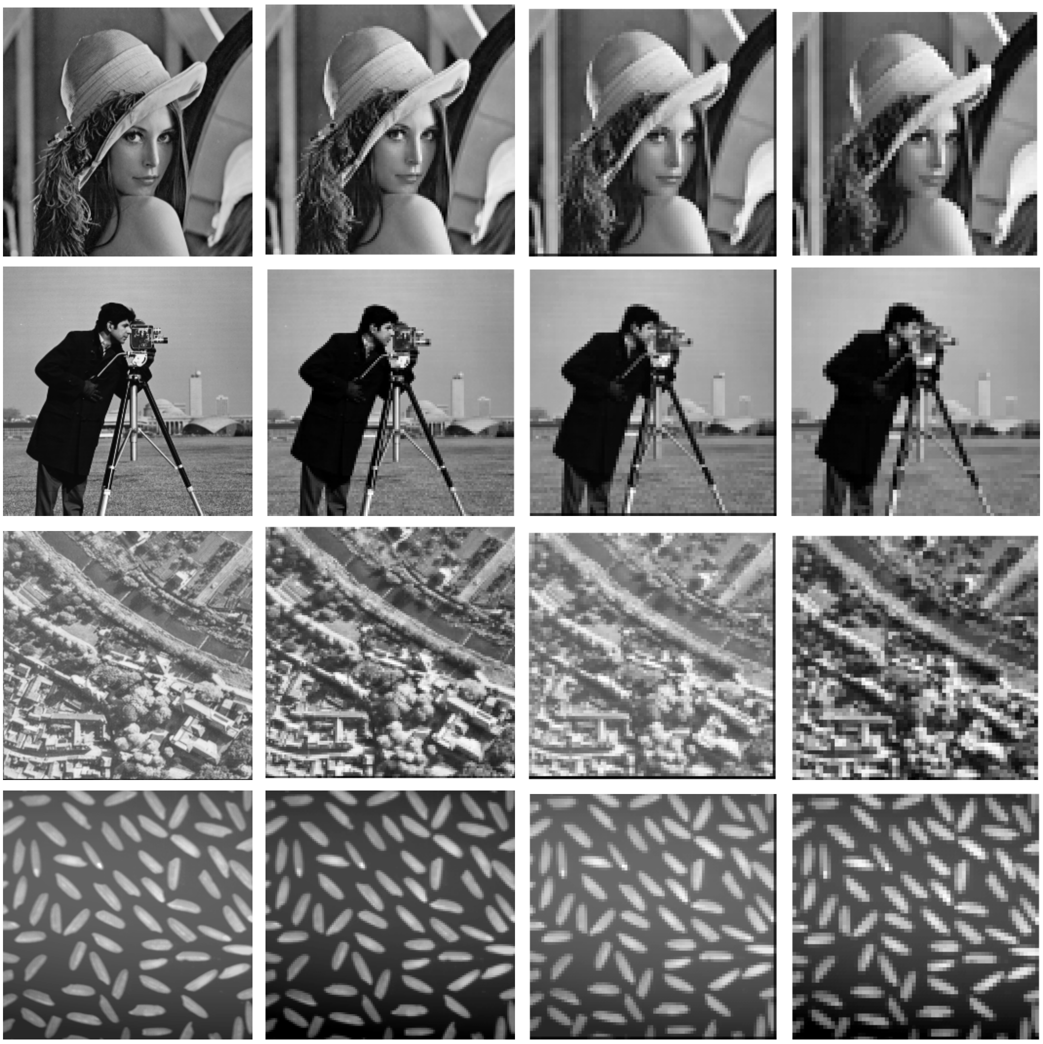

Figure 1: **The first column:** orinal images; **the second column:** reconstructed images using 2×2 blocks at quality index 95; **the third column:** reconstructed images using 3×3 blocks at quality index 95; **the fourth column:** reconstructed images using 4×4 blocks at quality index 95.

Figure 2: **Top left corner:** orinal image; **left to right:** reconstructed Lena image using 2×2 blocks at quality indices $5, 10, 50, 90, 95$.

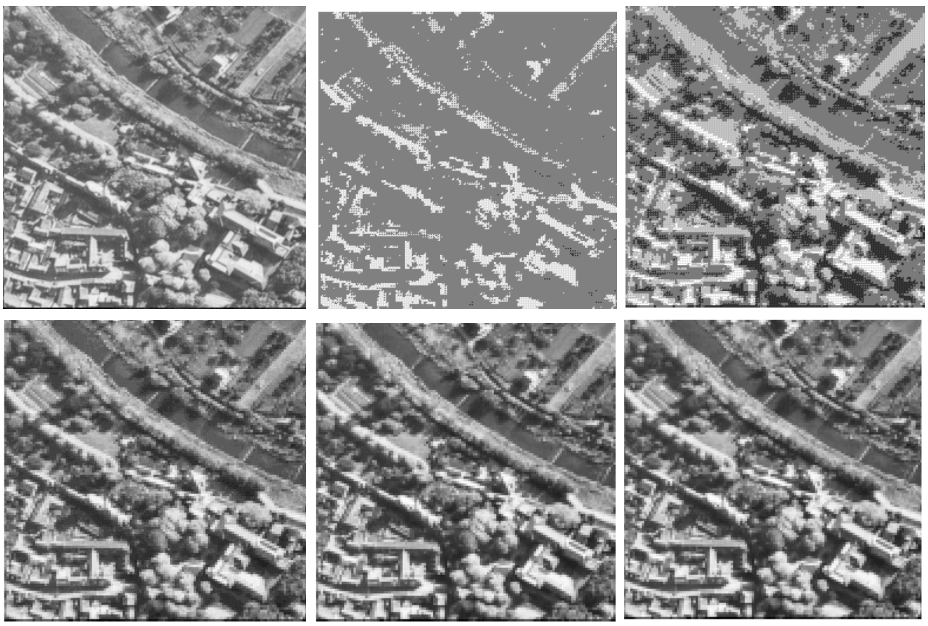

Figure 3: **Top left corner:** orinal image; **left to right:** reconstructed Aerial image using 2×2 blocks at quality indices $5, 10, 50, 90, 95$.

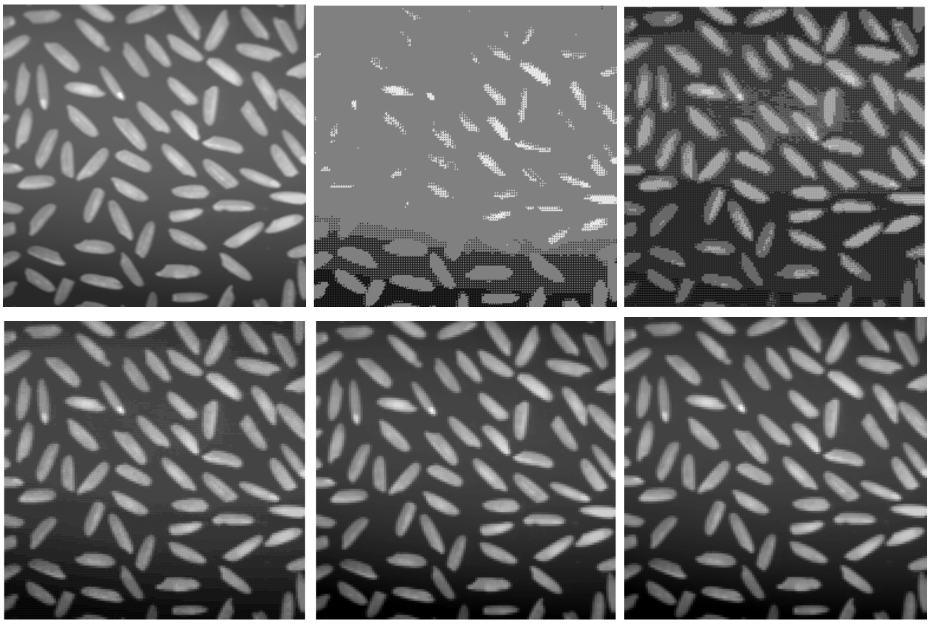

Figure 4: **Top left corner:** orinal image; **left to right:** reconstructed Rice image using 2×2 blocks at quality indices $5, 10, 50, 90, 95$.

Figure 5: **Top left corner:** orinal image; **left to right:** reconstructed Cameraman image using 2×2 blocks at quality indices $5, 10, 50, 90, 95$.

4 Comparison with Polynomial models

Experimental results are compared with the plane fitting model proposed by S. Ameer and O. Basir [Ameer and Basir, 2006] and the Chebyshev polynomial surface fit model proposed by S. Sajikumar and A. K. Anilkumar [Sajikumar and Anilkumar, 2017]. Both these methods are block-based algorithms and the proposed method outperforms these two for any block size. Comparison results for 2×2 blocks and 4×4 blocks are given in Tables 4-5. The plane fitting model and Chebyshev polynomial model have CR's 75% with 2×2 blocks and 18.75% with 4×4 blocks respectively. But the proposed method has a CR of only 25% with 2×2 blocks and it decreases as the block size increases. Even at 25% CR, the proposed method can give an improved result.

Test Image	$CR\,\%$	Performance	Plane Model	Chebyshev Poly. fit	Proposed Method
Rice	25	PSNR	28.9417	28.9417	31.5585
		RMSE	9.1104	9.1104	6.7393
Lena	25	PSNR	26.1790	26.1790	27.5988
		RMSE	12.5299	12.5299	10.6317
Cameraman	25	PSNR	23.8796	23.8796	25.4852
		RMSE	16.3095	16.3095	13.5607

Table 4: Performance comparison with Polynomial fitting model and Chebyshev polynomial surface fit model in the case of 2×2 blocks.

Test Image	$CR\,\%$	Performance	Plane Model	Chebyshev Poly. fit	Proposed Method
Rice	6.25	PSNR	24.4904	24.9876	25.3334
		RMSE	15.1987	14.3527	13.7997
Lena	6.25	PSNR	23.2617	23.2985	23.3583
		RMSE	17.5214	17.4356	17.3231
Cameraman	6.25	PSNR	21.3521	21.3374	21.7695
		RMSE	21.8174	21.8632	20.8001

Table 5: Performance comparison with Polynomial fitting model and Chebyshev polynomial surface fit model in the case of 4×4 blocks.

5 Conclusions

This paper presents a simple and efficient lossy image compression algorithm based on mean values of non-overlapping blocks of pixels. This mean value is taken as the parameter for reconstruction. A method for obtaining decompressed images at desired quality is also implemented. Using the quality matrix for different block sizes, the end-user or application has a choice for getting decompressed images according to the use. The proposed method outperforms existing polynomial-based methods in its speed of execution and computational complexity.

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