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Soft Semi*δ-continuity in Soft Topological Spaces

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Abstract

In this paper, we introduce the concept of soft semi*δ-continuous functions and soft semi*δ-irresolute functions in soft topological spaces. Also, we investigate its properties and study its relation with other soft continuous functions.

Keywords: soft semi*δ-open, soft semi*δ-closed, soft semi*δ-continuous, soft semi*δ-irresolute.

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1. Introduction

The concept of soft set theory was first introduced by Molotov [8] in 1999 to deal with uncertainty. According to him, a soft set over the universe is a parameterized family of subsets of the universe. In 2011, Muhammad Shabir and Munazza Naz [10] introduced soft topological spaces which are defined over an initial universe with a fixed set of parameters. Meanwhile, in 2010, Athar Kharal and B. Ahmad [4] defined the notion of soft mappings on soft classes. Later, in 2013, Aras and Sonmez[2] introduced and studied soft continuous mappings. Further, many authors defined and studied various forms of soft functions. Recently, the authors[12] of this paper introduced a new class of soft sets namely soft semi*δ-open sets and soft semi*δ-closed sets. In this paper, we introduce the concept of soft semi*δ-continuous functions and soft semi*δ-irresolute functions in soft topological spaces. We also investigate its properties and study its relation with other soft continuous functions.

2. Preliminaries

Throughout this work, $(X,\tilde{\tau},E),(Y,\tilde{\sigma},K)$ and $(Z,\tilde{\mu},L)$ are soft topological spaces. $S_tcl(F,A)$, $S_tint(F,A), S_tcl^*(F,A)$ and $S_tint^*(F,A)$ denote soft closure, soft interior, soft generalized closure and soft generalized interior of (F,A) respectively.

Definition 2.1. [10] Let X be an initial universe and E be a set of parameters. Let $P(X)$ denote the power set of X and A be a non – empty subset of E. A pair (F,A) is called a ***soft set*** over X where F is a mapping given by $F: A \to P(X)$.
The collection of all soft sets over X is called a soft class and denoted by $S_t(X,E)$.

Definition 2.2. [10] Let $\tilde{\tau}$ be the collection of soft set over X. Then $\tilde{\tau}$ is said to be a ***soft topology*** on X if

1) $\tilde{\phi}$, $\tilde{X}$ belongs to $\tilde{\tau}$
2) The union of any number of soft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$
3) The intersection of any two soft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$.

The triplet $(X,\tilde{\tau},E)$ is called a soft topological space. The members of $\tilde{\tau}$ are called soft open and its complements are called soft closed.

Definition 2.3. [4] Let $S_t(X,E)$ and $S_t(Y,K)$ be soft classes. Let $u: X \to Y$ and $p: E \to K$ be mappings. Then a mapping $\tilde{f}: S_t(X,E) \to S_t(Y,K)$ is defined as: for a soft set (F,A) in $S_t(X,E)$, $(\tilde{f}(F,A),B), B = p(A) \subseteq K$ is a soft set in $S_t(Y,K)$ given by

$$\tilde{f}(F,A)(\beta) = \begin{cases} u\left(\bigcup_{\alpha \in p^{-1}(\beta)\cap A} F(\alpha)\right), & \text{if } p^{-1}(\beta) \cap A \neq \phi \\ \phi & \text{otherwise} \end{cases}$$

for $\beta \in B \subseteq K$. $(\tilde{f}(F,A),B)$ is called ***soft image*** of a soft set (F,A). If $B = K$, then $(\tilde{f}(F,A),K)$ is written as $\tilde{f}(F,A)$.

Definition 2.4. [4] Let $\tilde{f}: S_t(X,E) \to S_t(Y,K)$ be a mapping from a soft class $S_t(X,E)$ to $S_t(Y,K)$ and (G,C) be a soft set in $S_t(Y,K)$ where $C \subseteq K$. Let $u: X \to Y$ and $p: E \to K$ be mappings. Then $(\tilde{f}^{-1}(G,C),D)$, $D = p^{-1}(C)$ is a soft set in $S_t(X,E)$ defined as

$$\tilde{f}^{-1}(G,C)(\alpha) = \begin{cases} u^{-1}(G(p(\alpha))), & \text{if } p(\alpha) \in C \\ \phi & \text{otherwise} \end{cases}$$

for $\alpha \in D \subseteq E$. $(\tilde{f}^{-1}(G,C),D)$ is called a ***soft inverse image*** of (G,C). We shall write $(\tilde{f}^{-1}(G,C),E)$ as $\tilde{f}^{-1}(G,C)$.

Definition 2.5. Let $(X,\tilde{\tau},E)$ and $(Y,\tilde{\sigma},K)$ be soft topological spaces. A soft function $\tilde{f}: (X,\tilde{\tau},E) \to (Y,\tilde{\sigma},K)$ is soft continuous[2] (respectively soft semi-continuous[5], soft pre-continuous[9], soft α-continuous[6], soft β-continuous [14], soft b-continuous[7], soft regular continuous[3], soft δ-continuous [11], soft generalized continuous[13], soft semi*-continuous, soft pre*-continuous, soft β*-continuous [1]) if $\tilde{f}^{-1}(G,B)$ is soft open (respectively soft semi-open, soft pre-open, soft α-open, soft β-open, soft b-open, soft regular open, soft δ-open, soft generalized open, soft semi*-open, soft pre*-open, soft β*-open) in $(X,\tilde{\tau},E)$ for every soft open set (G,B) in $(Y,\tilde{\sigma},K)$.

Definition 2.6.[12] A subset (F,A) of a soft topological space $(X,\tilde{\tau},E)$ is called ***soft semi*δ-open set*** if there exists a soft δ-open set (O,A) such that $(O,A) \tilde{\subseteq} (F,A) \tilde{\subseteq} S_t cl^*(O,A)$. The complement of soft semi*δ-open set is called soft semi*δ-closed. The class of soft semi*δ-open sets in $(X,\tilde{\tau},E)$ is denoted by $S_t S^* \delta O(X,\tilde{\tau},E)$.

Theorem 2.7.[12] In any soft topological space $(X,\tilde{\tau},E)$,

(i) Every soft δ-open set is soft semi*δ-open.
(ii) Every soft regular open set is soft semi*δ-open.
(iii) Every soft semi*δ-open set is soft semi-open.
(iv) Every soft semi*δ-open set is soft semi*-open.
(v) Every soft semi*δ-open set is soft β-open.
(vi) Every soft semi*δ-open set is soft β^*-open.
(vii) Every soft semi*δ-open set is soft b-open.

Remark 2.8:[12] The above theorem is also true for soft semi*δ-closed sets.

3. Soft semi*δ-continuous functions

Definition 3.1. Let $(X,\tilde{\tau},E)$ and $(Y,\tilde{\sigma},K)$ be soft topological spaces. Let $u: X \to Y$ and $p: E \to K$ be mappings. Then the soft function $\tilde{f}: (X,\tilde{\tau},E) \to (Y,\tilde{\sigma},K)$ is said to be soft semi*δ-

continuous if $\tilde{f}^{-1}(G,B)$ is soft semi*δ-open in $(X,\tilde{\tau},E)$ for every soft open set (G,B) in $(Y,\tilde{\sigma},K)$.
The following soft sets are used in all examples
Let $X=\{a,b\}$ and $E=\{e_1,e_2\}$. Then the soft sets are

$F_1=\{(e_1,\{\phi\}),(e_2,\{\phi\})\}=\tilde{\Phi}$
$F_2=\{(e_1,\{\phi\}),(e_2,\{a\})\}$
$F_3=\{(e_1,\{\phi\}),(e_2,\{b\})\}$
$F_4=\{(e_1,\{\phi\}),(e_2,\{a,b\})\}$
$F_5=\{(e_1,\{a\}),(e_2,\{\phi\})\}$
$F_6=\{(e_1,\{a\}),(e_2,\{a\})\}$
$F_7=\{(e_1,\{a\}),(e_2,\{b\})\}$
$F_8=\{(e_1,\{a\}),(e_2,\{a,b\})\}$
$F_9=\{(e_1,\{b\}),(e_2,\{\phi\})\}$
$F_{10}=\{(e_1,\{b\}),(e_2,\{a\})\}$
$F_{11}=\{(e_1,\{b\}),(e_2,\{b\})\}$
$F_{12}=\{(e_1,\{b\}),(e_2,\{a,b\})\}$
$F_{13}=\{(e_1,\{a,b\}),(e_2,\{\phi\})\}$
$F_{14}=\{(e_1,\{a,b\}),(e_2,\{a\})\}$
$F_{15}=\{(e_1,\{a,b\}),(e_2,\{b\})\}$
$F_{16}=\{(e_1,\{a,b\}),(e_2,\{a,b\})\}=\tilde{X}$

Similarly, let $Y=\{x,y\}$ and $K=\{k_1,k_2\}$. Then the soft sets $G_1,G_2,\ldots,G_{16}$ are obtained by replacing a,b,e_1 and e_2 by x,y,k_1 and k_2 respectively in the above sets.

Example 3.2. Let $X=\{a,b\},Y=\{x,y\},E=\{e_1,e_2\},K=\{k_1,k_2\}$. Define $u:X\to Y$ and $p:E\to K$ as $u(a)=x,u(b)=y,p(e_1)=k_2,p(e_2)=k_1$. Consider the soft topologies $\tilde{\tau}=\{\tilde{X},\tilde{\phi},F_4,F_5,F_8\}$ and $\tilde{\sigma}=\{\tilde{Y},\tilde{\phi},G_2,G_{13},G_{14}\}$. Let $\tilde{f}:(X,\tilde{\tau},E)\to(Y,\tilde{\sigma},K)$ be a soft mapping. Then, $\tilde{f}^{-1}(G_2)=F_5,\tilde{f}^{-1}(G_{13})=F_4$ and $\tilde{f}^{-1}(G_{14})=F_8$. Here, F_4,F_5,F_8 are soft semi*δ-open. Hence $\tilde{f}$ is soft semi*δ-continuous.

Theorem 3.3. Let $\tilde{f}:(X,\tilde{\tau},E)\to(Y,\tilde{\sigma},K)$ be a soft δ-continuous function. Then $\tilde{f}$ is soft semi*δ-continuous.
Proof: Let (G,B) be a soft open set in Y. Since $\tilde{f}$ is soft δ-continuous, $\tilde{f}^{-1}(G,B)$ is soft δ-open in X. Then by theorem 2.7(i), $\tilde{f}^{-1}(G,B)$ is soft semi*δ-open. Hence $\tilde{f}$ is soft semi*δ-continuous.

Remark 3.4. The converse of the above theorem need not be true.

Example 3.5. Let$X=\{a,b\},Y=\{x,y\},E=\{e_1,e_2\},K=\{k_1,k_2\}$. Define $u:X\to Y$ and $p:E\to K$ as $u(a)=x,u(b)=y,p(e_1)=k_1,p(e_2)=k_2$. Consider the soft topologies $\tilde{\tau}=\{\tilde{X},\tilde{\phi},F_2,F_{11},F_{12}\}$ and $\tilde{\sigma}=\{\tilde{Y},\tilde{\phi},G_6,G_{11}\}$. Let $\tilde{f}:(X,\tilde{\tau},E)\to(Y,\tilde{\sigma},K)$ be a soft mapping. Since $\tilde{f}^{-1}(G_6)=F_6$ is soft semi*δ-open but not soft δ-open, $\tilde{f}$ is soft semi*δ-continuous but not soft δ-continuous.

Theorem 3.6. Let $\tilde{f}:(X,\tilde{\tau},E)\to(Y,\tilde{\sigma},K)$ be a soft regular continuous function. Then $\tilde{f}$ is soft semi*δ-continuous.
Proof. Similar to theorem 3.3, the proof follows from theorem 2.7(ii).

Remark 3.7. The converse of the above theorem need not be true.

Example 3.8. Let $X=\{a,b\},Y=\{x,y\},E=\{e_1,e_2\},K=\{k_1,k_2\}$. Define $u:X\to Y$ and $p:E\to K$ as $u(a)=x,u(b)=y,p(e_1)=k_2,p(e_2)=k_1$. Consider the soft topologies $\tilde{\tau} = \{\tilde{X}, \tilde{\phi}, F_5, F_9, F_{13}\}$ and $\tilde{\sigma} = \{\tilde{Y}, \tilde{\phi}, G_2, G_3, G_4\}$.Let $\tilde{f}: (X, \tilde{\tau}, E) \to (Y, \tilde{\sigma}, K)$ be a soft mapping. Since $\tilde{f}^{-1}(G_4) = F_{13}$ is soft semi*δ-open but not soft regular open, $\tilde{f}$ is soft semi*δ-continuous but not soft regular continuous.

Theorem 3.9. Let $\tilde{f}: (X, \tilde{\tau}, E) \to (Y, \tilde{\sigma}, K)$ be a soft function.

(i) If $\tilde{f}$ is soft semi*δ-continuous, then $\tilde{f}$ is soft semi-continuous.
(ii) If $\tilde{f}$ is soft semi*δ-continuous, then $\tilde{f}$ is soft semi*-continuous.
(iii) If $\tilde{f}$ is soft semi*δ-continuous, then $\tilde{f}$ is soft β-continuous.
(iv) If $\tilde{f}$ is soft semi*δ-continuous, then $\tilde{f}$ is soft β^*-continuous.
(v) If $\tilde{f}$ is soft semi*δ-continuous, then $\tilde{f}$ is soft b-continuous.

Proof. (i) Let (G, B) be a soft open set in Y. Since $\tilde{f}$ is soft semi*δ-continuous, $\tilde{f}^{-1}(G, B)$ is soft semi*δ-open in X. Then by theorem 2.7(iii), $\tilde{f}^{-1}(G, B)$ is soft semi-open. Hence $\tilde{f}$ is soft semi-continuous.
The other proofs are similar.

Remark 3.10. The converse of each of the statements in above theorem need not be true.

Example 3.11. Let $X = \{a, b\}, Y = \{x, y\}, E = \{e_1, e_2\}, K = \{k_1, k_2\}$. Define $u: X \to Y$ and $p: E \to K$ as $u(a) = x, u(b) = y, p(e_1) = k_2, p(e_2) = k_1$. Consider the soft topologies $\tilde{\tau} = \{\tilde{X}, \tilde{\phi}, F_5, F_9, F_{13}\}$and $\tilde{\sigma} = \{\tilde{Y}, \tilde{\phi}, G_6, G_{14}\}$. Let $\tilde{f}: (X, \tilde{\tau}, E) \to (Y, \tilde{\sigma}, K)$ be a soft mapping. Since $\tilde{f}^{-1}(G_6) = F_6$ and $\tilde{f}^{-1}(G_{14}) = F_8$ are soft semi-open but not soft semi*δ-open, $\tilde{f}$ is soft semi-continuous but not soft semi*δ-continuous.

Example 3.12. Let $X = \{a, b\}, Y = \{x, y\}, E = \{e_1, e_2\}, K = \{k_1, k_2\}$. Define $u: X \to Y$ and $p: E \to K$ as $u(a) = y, u(b) = x, p(e_1) = k_1, p(e_2) = k_2$. Consider the soft topologies $\tilde{\tau} = \{\tilde{X}, \tilde{\phi}, F_6, F_{14}\}$and $\tilde{\sigma} = \{\tilde{Y}, \tilde{\phi}, G_{11}, G_{15}\}$. Let $\tilde{f}: (X, \tilde{\tau}, E) \to (Y, \tilde{\sigma}, K)$ be a soft mapping. Since $\tilde{f}^{-1}(G_{11}) = F_6$ and $\tilde{f}^{-1}(G_{15}) = F_{14}$ are soft semi*-open but not soft semi*δ-open, $\tilde{f}$ is soft semi*-continuous but not soft semi*δ-continuous.

Example 3.13. Let $X = \{a, b\}, Y = \{x, y\}, E = \{e_1, e_2\}, K = \{k_1, k_2\}$. Define $u: X \to Y$ and $p: E \to K$ as $u(a) = x, u(b) = y, p(e_1) = k_1, p(e_2) = k_2$. Consider the soft topologies $\tilde{\tau} = \{\tilde{X}, \tilde{\phi}, F_4, F_9, F_{12}\}$ and $\tilde{\sigma} = \{\tilde{Y}, \tilde{\phi}, G_2, G_6, G_{10}, G_{14}\}$. Let $\tilde{f}: (X, \tilde{\tau}, E) \to (Y, \tilde{\sigma}, K)$ be a soft mapping. Since $\tilde{f}^{-1}(G_2) = F_2, \tilde{f}^{-1}(G_6) = F_6, \tilde{f}^{-1}(G_{10}) = F_{10}$ and $\tilde{f}^{-1}(G_{14}) = F_{14}$ are both soft β-open and soft β^*-open but not soft semi*δ-open, $\tilde{f}$ is both soft β-continuous and soft β^*-continuous but not soft semi*δ-continuous.

Example 3.14. Let $X = \{a, b\}, Y = \{x, y\}, E = \{e_1, e_2\}, K = \{k_1, k_2\}$. Define $u: X \to Y$ and $p: E \to K$ as $u(a) = y, u(b) = x, p(e_1) = k_1, p(e_2) = k_2$. Consider the soft topologies $\tilde{\tau} = \{\tilde{X}, \tilde{\phi}, F_3, F_9, F_{11}\}$ and $\tilde{\sigma} = \{\tilde{Y}, \tilde{\phi}, G_2, G_4, G_6, G_8\}$. Let $\tilde{f}: (X, \tilde{\tau}, E) \to (Y, \tilde{\sigma}, K)$ be a soft mapping. Since $\tilde{f}^{-1}(G_4) = F_4$ and $\tilde{f}^{-1}(G_8) = F_{12}$ are soft b-open but not soft semi*δ-open, $\tilde{f}$ is soft b-continuous but not soft semi*δ-continuous.

Remark 3.15. The concept of soft semi*δ-continuity and soft continuity are independent as shown in the following example.

Example 3.16. Let $X = \{a,b\}, Y = \{x,y\}, E = \{e_1,e_2\}, K = \{k_1,k_2\}$. Define $u: X \to Y$ and $p: E \to K$ as $u(a) = x, u(b) = y, p(e_1) = k_2, p(e_2) = k_1$.Consider the soft topologies $\tilde{\tau} = \{\tilde{X}, \tilde{\phi}, F_4, F_5, F_8\}$ and $\tilde{\sigma} = \{\tilde{Y}, \tilde{\phi}, G_{15}\}$. Let $\tilde{f}: (X, \tilde{\tau}, E) \to (Y, \tilde{\sigma}, K)$ be a soft mapping. Since $\tilde{f}^{-1}(G_{15}) = F_{12}$ is soft semi*δ-open but not soft open, $\tilde{f}$ is soft semi*δ-continuous but not soft continuous. Now, consider the soft topology $\tilde{\tau} = \{\tilde{X}, \tilde{\phi}, F_4, F_8, F_{12}\}$ on X. Here, since $\tilde{f}^{-1}(G_{15}) = F_{12}$ is soft open but not soft semi*δ-open, $\tilde{f}$ is soft continuous but not soft semi*δ-continuous.

Remark 3.17. The concept of soft semi*δ-continuity and soft generalized continuity are independent as shown in the following example.

Example 3.18. Let $X = \{a,b\}, Y = \{x,y\}, E = \{e_1,e_2\}, K = \{k_1,k_2\}$. Define $u: X \to Y$ and $p: E \to K$ as $u(a) = x, u(b) = y,\ p(e_1) = k_1, p(e_2) = k_2$. Consider the soft topologies $\tilde{\tau} = \{\tilde{X}, \tilde{\phi}, F_6, F_9, F_{14}\}$ and $\tilde{\sigma} = \{\tilde{Y}, \tilde{\phi}, G_6, G_{11}\}$. Let $\tilde{f}: (X, \tilde{\tau}, E) \to (Y, \tilde{\sigma}, K)$ be a soft mapping. Since $\tilde{f}^{-1}(G_{11}) = F_{11}$ is soft semi*δ-open but not soft generalized open, $\tilde{f}$ is soft semi*δ-continuous but not soft generalized continuous. Now, consider the soft topology $\tilde{\sigma} = \{\tilde{Y}, \tilde{\phi}, G_{13}\}$ on Y. Here, since $\tilde{f}^{-1}(G_{13}) = F_{13}$ is soft generalized open but not soft semi*δ-open, $\tilde{f}$ is soft generalized continuous but not soft semi*δ-continuous.

Remark 3.19. The concept of soft semi*δ-continuity and soft pre-continuity are independent as shown in the following example.

Example 3.20. Let $X = \{a,b\}, Y = \{x,y\}, E = \{e_1,e_2\}, K = \{k_1,k_2\}$. Define $u: X \to Y$ and $p: E \to K$ as $u(a) = x, u(b) = y,\ p(e_1) = k_2, p(e_2) = k_1$. Consider the soft topologies $\tilde{\tau} = \{\tilde{X}, \tilde{\phi}, F_2, F_{11}, F_{12}\}$and $\tilde{\sigma} = \{\tilde{Y}, \tilde{\phi}, G_5, G_6, G_{15}\}$. Let $\tilde{f}: (X, \tilde{\tau}, E) \to (Y, \tilde{\sigma}, K)$ be a soft mapping. Since $\tilde{f}^{-1}(G_6) = F_6$ is soft semi*δ-open but not soft pre-open, $\tilde{f}$ is soft semi*δ-continuous but not soft pre-continuous. Now, consider the mapping $u(a) = y, u(b) = x, p(e_1) = k_2, p(e_2) = k_1$. Here, since $\tilde{f}^{-1}(G_5) = F_3$ and $\tilde{f}^{-1}(G_{15}) = F_8$ are soft pre-open but not soft semi*δ-open, $\tilde{f}$ is soft pre-continuous but not soft semi*δ-continuous.

Remark 3.21. The concept of soft semi*δ-continuity and soft pre*-continuity are independent as shown in the following example.

Example 3.22. Let $X = \{a,b\}, Y = \{x,y\}, E = \{e_1,e_2\}, K = \{k_1,k_2\}$. Define $u: X \to Y$ and $p: E \to K$ as $u(a) = y, u(b) = x,\ p(e_1) = k_1, p(e_2) = k_2$. Consider the soft topologies $\tilde{\tau} = \{\tilde{X}, \tilde{\phi}, F_6, F_9, F_{14}\}$ and $\tilde{\sigma} = \{\tilde{Y}, \tilde{\phi}, G_{11}, G_{12}\}$. Let $\tilde{f}: (X, \tilde{\tau}, E) \to (Y, \tilde{\sigma}, K)$

be a soft mapping. Since $\tilde{f}^{-1}(G_{12}) = F_8$ is soft semi*δ-open but not soft pre*-open, $\tilde{f}$ is soft semi*δ-continuous but not soft pre*-continuous. Now, consider the soft topology $\tilde{\sigma} = \{\tilde{Y}, \tilde{\phi}, G_3, G_9, G_{11}\}$. Here, since $\tilde{f}^{-1}(G_3) = F_2$ and $\tilde{f}^{-1}(G_9) = F_5$ are soft pre*-open but not soft semi*δ-open, $\tilde{f}$ is soft pre*-continuous but not soft semi*δ-continuous.

Remark 3.23. The concept of soft semi*δ-continuity and soft α-continuity are independent as shown in the following example.

Example 3.24. Let $X = \{a, b\}, Y = \{x, y\}, E = \{e_1, e_2\}, K = \{k_1, k_2\}$. Define $u: X \to Y$ and $p: E \to K$ as $u(a) = y, u(b) = x$, $p(e_1) = k_1, p(e_2) = k_2$. Consider the soft topologies $\tilde{\tau} = \{\tilde{X}, \tilde{\phi}, F_4, F_9, F_{12}\}$ and $\tilde{\sigma} = \{\tilde{Y}, \tilde{\phi}, G_4, G_8, G_{12}\}$. Let $\tilde{f}: (X, \tilde{\tau}, E) \to (Y, \tilde{\sigma}, K)$ be a soft mapping. Since $\tilde{f}^{-1}(G_{12}) = F_8$ is soft semi*δ-open but not soft α-open, $\tilde{f}$ is soft semi*δ-continuous but not soft α-continuous. Now, consider the soft topology $\tilde{\tau} = \{\tilde{X}, \tilde{\phi}, F_4, F_8, F_{12}\}$ on X. Here, since $\tilde{f}^{-1}(G_4) = F_4$, $\tilde{f}^{-1}(G_8) = F_{12}$ and $\tilde{f}^{-1}(G_{12}) = F_8$ are soft α-open but not soft semi*δ-open, $\tilde{f}$ is soft α-continuous but not soft semi*δ-continuous.

From the above discussions, we have the following diagram

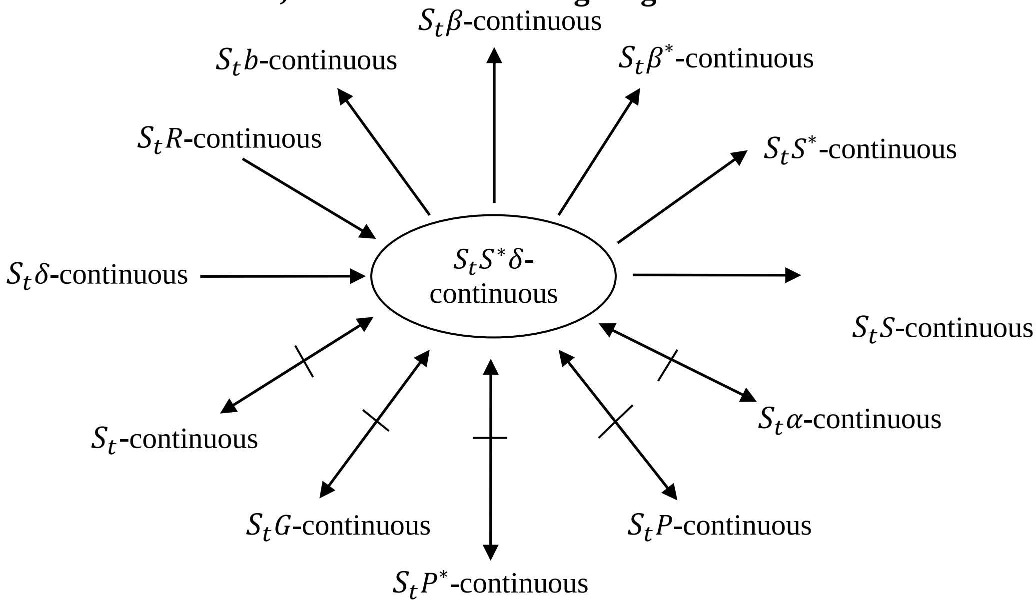

Figure 1

Theorem 3.25. Let $\tilde{f}: (X, \tilde{\tau}, E) \to (Y, \tilde{\sigma}, K)$ be a soft function. Then the following statements are equivalent:

(i) $\tilde{f}$ is soft semi*δ-continuous.

(ii) The inverse image of every soft closed set in $(Y, \tilde{\sigma}, K)$ is soft semi*δ-closed in $(X, \tilde{\tau}, E)$.

(iii) $\tilde{f}(S_ts^*\delta cl(F, A)) \tilde{\subseteq} S_tcl(\tilde{f}(F, A))$ for every soft set (F, A) over X.

(iv) $S_t s^* \delta cl(\tilde{f}^{-1}(G,B)) \tilde{\subseteq} \tilde{f}^{-1}(S_t cl(G,B))$ for every soft set (G,B) over Y.

Proof.

(i) ⟹(ii) Let $\tilde{f}$ be a soft semi*δ-continuous function and (H,B) be a soft closed set in Y. Then $(H,B)^{\tilde{c}}$ is soft open in Y. Since $\tilde{f}$ is soft semi*δ-continuous, $\tilde{f}^{-1}((H,B)^{\tilde{c}})$ is soft semi*δ-open in X. That is, $(\tilde{f}^{-1}(H,B))^{\tilde{c}}$ is soft semi*δ-open in $(X,\tilde{\tau},E)$. Hence $\tilde{f}^{-1}(H,B)$ is soft semi*δ-closed in $(X,\tilde{\tau},E)$.

(ii)⟹(i) Let (G,B) be soft open in Y. Then $(G,B)^{\tilde{c}}$ be soft closed in Y. By assumption, $\tilde{f}^{-1}((G,B)^{\tilde{c}})$ is soft semi*δ-closed in X. That is, $(\tilde{f}^{-1}(G,B))^{\tilde{c}}$ is soft semi*δ-closed in X. Hence, $\tilde{f}^{-1}(G,B)$ is soft semi*δ-open in X. Therefore, $\tilde{f}$ is soft semi*δ-continuous.

(ii)⟹(iii) Let (F,A) be a soft set over X. Now, $(F,A) \tilde{\subseteq} \tilde{f}^{-1}(\tilde{f}(F,A))$ implies $(F,A) \tilde{\subseteq} \tilde{f}^{-1}\left(S_t cl\left(\tilde{f}(F,A)\right)\right)$. Since $S_t cl\left(\tilde{f}(F,A)\right)$ is soft closed in Y, by assumption $\tilde{f}^{-1}\left(S_t cl\left(\tilde{f}(F,A)\right)\right)$ is a soft semi*δ-closed set containing (F,A). Also, $S_t s^* \delta cl(F,A)$ is the smallest soft semi*δ-closed set containing (F,A). Hence, $S_t s^* \delta cl(F,A) \tilde{\subseteq} \tilde{f}^{-1}\left(S_t cl\left(\tilde{f}(F,A)\right)\right)$. Therefore, $\tilde{f}(S_t s^* \delta cl(F,A)) \tilde{\subseteq} S_t cl\left(\tilde{f}(F,A)\right)$.

(iii)⟹(ii) Let (H,B) be a soft closed set in Y. Then, by assumption, $\tilde{f}(S_t s^* \delta cl(\tilde{f}^{-1}(H,B))) \tilde{\subseteq} S_t cl(\tilde{f}(\tilde{f}^{-1}(H,B))) \tilde{\subseteq} S_t cl(H,B) = (H,B)$ implies $S_t s^* \delta cl\left(\tilde{f}^{-1}(H,B)\right) \tilde{\subseteq} \tilde{f}^{-1}(H,B)$. Also, $\tilde{f}^{-1}(H,B) \tilde{\subseteq} S_t s^* \delta cl\left(\tilde{f}^{-1}(H,B)\right)$. Hence, $\tilde{f}^{-1}(H,B)$ is soft semi*δ-closed in X.

(iii)⟹(iv) Let (G,B) be a soft set over Y and let $(F,A) = \tilde{f}^{-1}(G,B)$. By assumption, $\tilde{f}(S_t s^* \delta cl(F,A)) \tilde{\subseteq} S_t cl\left(\tilde{f}(F,A)\right) = S_t cl(G,B)$. This implies $S_t s^* \delta cl\left(\tilde{f}^{-1}(G,B)\right) \tilde{\subseteq} \tilde{f}^{-1}(S_t cl(G,B))$.

(iv)⟹(iii) Let $(G,B) = \tilde{f}(F,A)$. Then, by assumption $S_t s^* \delta cl(F,A) = S_t s^* \delta cl\left(\tilde{f}^{-1}(G,B)\right) \tilde{\subseteq} \tilde{f}^{-1}(S_t cl(G,B)) = \tilde{f}^{-1}\left(S_t cl\left(\tilde{f}(F,A)\right)\right)$. This implies $\tilde{f}(S_t s^* \delta cl(F,A)) \tilde{\subseteq} S_t cl\left(\tilde{f}(F,A)\right)$

(iv)⟹(i) Let (G,B) be soft open in Y. Then, $(G,B)^{\tilde{c}}$ is soft closed in Y. By assumption, $\tilde{f}^{-1}((G,B)^{\tilde{c}}) = \tilde{f}^{-1}(S_t cl(G,B)^{\tilde{c}}) \tilde{\supseteq} S_t s^* \delta cl(\tilde{f}^{-1}(G,B)^{\tilde{c}})$. Also, we know that $\tilde{f}^{-1}((G,B)^{\tilde{c}}) \tilde{\subseteq} S_t s^* \delta cl(\tilde{f}^{-1}(G,B)^{\tilde{c}})$. Hence $\tilde{f}^{-1}((G,B)^{\tilde{c}}) = S_t s^* \delta cl(\tilde{f}^{-1}(G,B)^{\tilde{c}})$. Therefore, $\tilde{f}^{-1}((G,B)^{\tilde{c}})$ is soft semi*δ-closed. That is, $(\tilde{f}^{-1}(G,B))^{\tilde{c}}$ is soft semi*δ-closed in X. Hence, $\tilde{f}^{-1}(G,B)$ is soft semi*δ-open in X. Therefore, $\tilde{f}$ is soft semi*δ-continuous.

Theorem 3.26. Let $\tilde{f}: (X,\tilde{\tau},E) \to (Y,\tilde{\sigma},K)$ be a soft function. Then $\tilde{f}$ is soft semi*δ-continuous if and only if $\tilde{f}^{-1}(S_t int(G,B)) \tilde{\subseteq} S_t s^* \delta int\left(\tilde{f}^{-1}(G,B)\right)$ for every soft set (G,B) over Y.

Proof. Let $\tilde{f}$ be a soft semi*δ-continuous function and (G,B) be a soft set over Y. Then $S_t int(G,B)$ is soft open set in Y. By assumption, $\tilde{f}^{-1}(S_t int(G,B))$ is soft semi*δ-open in X. Now, $\tilde{f}^{-1}(S_t int(G,B)) \tilde{\subseteq} \tilde{f}^{-1}(G,B)$ and $S_t s^* \delta int\left(\tilde{f}^{-1}(G,B)\right)$ is the largest soft semi*δ-open set contained in $\tilde{f}^{-1}(G,B)$. Hence $\tilde{f}^{-1}(S_t int(G,B)) \tilde{\subseteq} S_t s^* \delta int\left(\tilde{f}^{-1}(G,B)\right)$. Conversely, Let (G,B) be soft open in Y. Then, $\tilde{f}^{-1}(G,B) = \tilde{f}^{-1}(S_t int(G,B)) \tilde{\subseteq} S_t s^* \delta int\left(\tilde{f}^{-1}(G,B)\right)$. Also, $S_t s^* \delta int\left(\tilde{f}^{-1}(G,B)\right) \tilde{\subseteq} \tilde{f}^{-1}(G,B)$. This implies $\tilde{f}^{-1}(G,B)$ is soft semi*δ-open in X. Hence $\tilde{f}$ is soft semi*δ-continuous.

Remark 3.27. The composition of two soft semi*δ-continuous functions need not be soft semi*δ-continuous.

Example 3.28. Let $X = \{a,b,c\}, Y = \{x,y\}, Z = \{m,n\}, E = \{e_1,e_2\}, K = \{k_1,k_2\}$, $L = \{l_1,l_2\}$. Consider the soft topologies $\tilde{\tau} = \{\tilde{X}, \tilde{\phi}, F_5, F_{12}, F_{16}\}$, $\tilde{\sigma} = \{\tilde{Y}, \tilde{\phi}, G_2, G_7, G_8\}$ and $\tilde{\mu} = \{\tilde{Z}, \tilde{\phi}, H_{10}\}$ where $F_5 = \{(e_1,\{\phi\}),(e_2,\{a,b\})\}$, $F_{12} = \{(e_1,\{a\}),(e_2,\{c\})\}$, $F_{16} = \{(e_1,\{a\}),(e_2,\{a,b,c\})\}$, $G_2 = \{(k_1,\{\phi\}),(k_2,\{x\})\}$, $G_7 = \{(k_1,\{x\}),(k_2,\{y\})\}$, $G_8 = \{(k_1,\{x\}),(k_2,\{x,y\})\}$ and $H_{10} = \{(l_1,\{n\}),(l_2,\{m\})\}$. Define $u_1 : X \longrightarrow Y$ and $p_1 : E \longrightarrow K$ as $u_1(a) = u_1(b) = x, u_1(c) = y$, $p_1(e_1) = k_1, p_1(e_2) = k_2$. Then the soft mapping $\tilde{f} : (X,\tilde{\tau},E) \longrightarrow (Y,\tilde{\sigma},K)$ is soft semi*δ-continuous. Also, define $u_2 : Y \longrightarrow Z$ and $p_2 : K \longrightarrow L$ as $u_2(x) = m, u_2(y) = n, p_2(k_1) = l_1$ and $p_2(k_2) = l_2$. Then the soft mapping $\tilde{g} : (Y,\tilde{\sigma},K) \longrightarrow (Z,\tilde{\mu},L)$ is soft semi*δ-continuous. Now, let $\tilde{g} \circ \tilde{f} : (X,\tilde{\tau},E) \longrightarrow (Z,\tilde{\mu},L)$ be the composition of two soft semi*δ-continuous functions. Then $\tilde{g} \circ \tilde{f}$ is not soft semi*δ-continuous since $\left(\tilde{g} \circ \tilde{f}\right)^{-1}(H_{10}) = \tilde{f}^{-1}(\tilde{g}^{-1}(H_{10})) = \tilde{f}^{-1}(G_{10}) = \{(e_1,\{c\}),(e_2,\{a,b\})\}$ is not soft semi*δ-open.

Theorem 3.29. Let $(X,\tilde{\tau},E)$, $(Y,\tilde{\sigma},K)$ and $(Z,\tilde{\mu},L)$ be soft topological spaces and let $(Y,\tilde{\sigma},K)$ be a space in which every soft semi*δ-open set is soft open. Then the composition $\tilde{g} \circ \tilde{f} : (X,\tilde{\tau},E) \longrightarrow (Z,\tilde{\mu},L)$ of two soft semi*δ-continuous functions $\tilde{f} : (X,\tilde{\tau},E) \longrightarrow (Y,\tilde{\sigma},K)$ and $\tilde{g} : (Y,\tilde{\sigma},K) \longrightarrow (Z,\tilde{\mu},L)$ is soft semi*δ-continuous.

Proof. Let (H,C) be any soft open set in Z. Since $\tilde{g}$ is soft semi*δ-continuous, $\tilde{g}^{-1}(H,C)$ is soft semi*δ-open in Y. Then, by assumption $\tilde{g}^{-1}(H,C)$ is soft open in Y. Also, since $\tilde{f}$ is soft semi*δ-continuous, $\tilde{f}^{-1}(\tilde{g}^{-1}(H,C)) = \left(\tilde{g} \circ \tilde{f}\right)^{-1}(H,C)$ is soft semi*δ-open in X. Hence $\tilde{g} \circ \tilde{f}$ is soft semi*δ-continuous.

Theorem 3.30. Let $\tilde{f} : (X,\tilde{\tau},E) \longrightarrow (Y,\tilde{\sigma},K)$ be a soft semi*δ-continuous function and $\tilde{g} : (Y,\tilde{\sigma},K) \longrightarrow (Z,\tilde{\mu},L)$ be a soft continuous function. Then their composition $\tilde{g} \circ \tilde{f} : (X,\tilde{\tau},E) \longrightarrow (Z,\tilde{\mu},L)$ is soft semi*δ-continuous.

Proof. Let (H, C) be any soft open set in Z. Since $\tilde{g}$ is soft continuous, $\tilde{g}^{-1}(H, C)$ is soft open in Y. Also, since $\tilde{f}$ is soft semi*δ-continuous, $\tilde{f}^{-1}(\tilde{g}^{-1}(H, C)) = (\tilde{g} \circ \tilde{f})^{-1}(H, C)$ is soft semi*δ-open in X. Hence $\tilde{g} \circ \tilde{f}$ is soft semi*δ-continuous.

4. Soft semi*δ-irresolute functions

Definition 4.1. Let $(X, \tilde{\tau}, E)$ and $(Y, \tilde{\sigma}, K)$ be soft topological spaces. Let $u: X \to Y$ and $p: E \to K$ be mappings. Then the soft function $\tilde{f}: (X, \tilde{\tau}, E) \to (Y, \tilde{\sigma}, K)$ is said to be soft semi*δ-irresolute if $\tilde{f}^{-1}(G, B)$ is soft semi*δ-open in $(X, \tilde{\tau}, E)$ for every soft semi*δ-open set (G, B) in $(Y, \tilde{\sigma}, K)$.

Example 4.2. Let $X = \{a, b\}, Y = \{x, y\}, E = \{e_1, e_2\}, K = \{k_1, k_2\}$. Define $u: X \to Y$ and $p: E \to K$ as $u(a) = y, u(b) = x$, $p(e_1) = k_1, p(e_2) = k_2$. Consider the soft topologies $\tilde{\tau} = \{\tilde{X}, \tilde{\phi}, F_4, F_9, F_{12}\}$ and $\tilde{\sigma} = \{\tilde{Y}, \tilde{\phi}, G_4, G_5, G_8\}$.Here, $S_tS^*\delta O(X, \tilde{\tau}, E) = \{\tilde{X}, \tilde{\phi}, F_4, F_8, F_9, F_{12}, F_{13}\}$ and $S_tS^*\delta O(Y, \tilde{\sigma}, K) = \{\tilde{Y}, \tilde{\phi}, G_4, G_5, G_8, G_{12}, G_{13}\}$. Let $\tilde{f}: (X, \tilde{\tau}, E) \to (Y, \tilde{\sigma}, K)$ be a soft mapping. Then, $\tilde{f}^{-1}(G_4) = F_4$, $\tilde{f}^{-1}(G_5) = F_9$ $\tilde{f}^{-1}(G_8) = F_{12}$, $\tilde{f}^{-1}(G_{12}) = F_8$and $\tilde{f}^{-1}(G_{13}) = F_{13}$. Hence, $\tilde{f}$ is soft semi*δ-irresolute.

Theorem 4.3. Let $(X, \tilde{\tau}, E)$ and $(Y, \tilde{\sigma}, K)$ be soft topological spaces and let $(Y, \tilde{\sigma}, K)$ be a space in which every soft semi*δ-open set is soft open. If $\tilde{f}: (X, \tilde{\tau}, E) \to (Y, \tilde{\sigma}, K)$ is soft semi*δ-continuous, then $\tilde{f}$ is soft semi*δ-irresolute.
Proof. Let (G, B) be soft semi*δ-open in Y. Then, by assumption (G, B) is soft open in Y. Since $\tilde{f}$ is soft semi*δ-continuous, $\tilde{f}^{-1}(G, B)$ is soft semi*δ-open in X. Hence $\tilde{f}$ is soft semi*δ-irresolute.

Theorem 4.4. Let $\tilde{f}: (X, \tilde{\tau}, E) \to (Y, \tilde{\sigma}, K)$ be a soft function. Then the following statements are equivalent:
(i) $\tilde{f}$ is soft semi*δ-irresolute.
(ii) The inverse image of every soft semi*δ-closed set in $(Y, \tilde{\sigma}, K)$ is soft semi*δ-closed in $(X, \tilde{\tau}, E)$.
(iii) $\tilde{f}(S_ts^*\delta cl(F, A)) \tilde{\subseteq} S_ts^*\delta cl(\tilde{f}(F, A))$ for every soft set (F, A) over X.
(iv) $S_ts^*\delta cl(\tilde{f}^{-1}(G, B)) \tilde{\subseteq} \tilde{f}^{-1}(S_ts^*\delta cl(G, B))$ for every soft set (G, B) over Y.
Proof. The proof is similar to theorem 3.25

Theorem 4.5. If $\tilde{f}: (X, \tilde{\tau}, E) \to (Y, \tilde{\sigma}, K)$ and $\tilde{g}: (Y, \tilde{\sigma}, K) \to (Z, \tilde{\mu}, L)$ are soft semi*δ-irresolute functions, then their composition $\tilde{g} \circ \tilde{f}: (X, \tilde{\tau}, E) \longrightarrow (Z, \tilde{\mu}, L)$ is also soft semi*δ-irresolute.
Proof. Let (H, C) be soft semi*δ-open in Z. Since $\tilde{g}$ is soft semi*δ-irresolute, $\tilde{g}^{-1}(H, C)$ is soft semi*δ-open in Y. Again, since $\tilde{f}$ is soft semi*δ-irresolute,

$\tilde{f}^{-1}(\tilde{g}^{-1}(H,C)) = (\tilde{g} \circ \tilde{f})^{-1}(H,C)$ is soft semi*δ-open in X. Hence $\tilde{g} \circ \tilde{f}: (X, \tilde{\tau}, E) \longrightarrow (Z, \tilde{\mu}, L)$ is soft semi*δ-irresolute.

Theorem 4.6. If $\tilde{f}: (X, \tilde{\tau}, E) \rightarrow (Y, \tilde{\sigma}, K)$ is soft semi*δ-irresolute and $\tilde{g}: (Y, \tilde{\sigma}, K) \rightarrow (Z, \tilde{\mu}, L)$ is soft semi*δ-continuous, then $\tilde{g} \circ \tilde{f}: (X, \tilde{\tau}, E) \longrightarrow (Z, \tilde{\mu}, L)$ is soft semi*δ-continuous.

Proof. Let (H,C) be a soft open set in Z. Since $\tilde{g}$ is soft semi*δ-continuous, $\tilde{g}^{-1}(H,C)$ is soft semi*δ-open in Y. Now, since $\tilde{f}$ is soft semi*δ-irresolute, $\tilde{f}^{-1}(\tilde{g}^{-1}(H,C)) = (\tilde{g} \circ \tilde{f})^{-1}(H,C)$ is soft semi*δ-open in X. Hence $\tilde{g} \circ \tilde{f}: (X, \tilde{\tau}, E) \longrightarrow (Z, \tilde{\mu}, L)$ is soft semi*δ-continuous.

5. Conclusions

We have studied the concept of continuity in soft topological spaces by means of soft semi*δ-open sets. We have also introduced the concept of soft semi*δ-irresolute functions. Further, we have compared it with other existing soft functions and we have also investigated the characterization of these functions.

References

[1] P. Anbarasi Rodrigo, S. Maheshwari, Functions Related to Soft β^*-closed sets in Soft Topological Spaces, Proceedings of International Conference on Recent Advances in Computational Mathematics and Engineering 2021.

[2] C.G. Aras, A. Sonmez, H. Cakalli, On Soft Mappings, arXiv: 1305.4545. 2013.

[3] I. Arockiarani, A. Selvi, Soft πg-continuous functions and irresolute functions, International Journal of Innovation and Applied Studies, Volume 7(2), 440-446. 2014.

[4] Athar Kharal and B. Ahmad, Mappings on Soft Classes, New Mathematics and Natural Computation, Volume 7, 471-481. 2010.

[5] J. Mahanta, P.K. Das, On Soft Topological Spaces via Semi open and Semi closed Soft Sets, Kyungpook Math. Journal, 54, 221-236. 2014.

[6] Metin Akdag, Alkan Ozkan, Soft alpha – open sets and soft alpha – continuous functions, Hindawi Publishing Corporation, Abstract and Applied Mathematics, 2014.

[7] Metin Akdag, Alkan Ozkan, Soft b – open sets and soft b – continuous functions, Math Sci 8, 124. 2014.

[8] D. Molostsov, Soft Set Theory – First Results, Computers and Mathematics with Applications, 37, 19 – 31. 1999.

[9] Mrudula Ravindran, G. Manju, Some results on soft pre-continuity, Malaya Journal of Mathematik, 10-17. 2015.

[10] Muhammud Shabir, Munazza Naz, On Soft Topological Spaces, Computers and Mathematics with Applications, 61, 1786 – 1799. 2011.

[11] Ramadhan A. Mohammed, O. R. Sayed, A. Eliow, Some Properties of Soft Delta – Topology, Academic Journal of Nawroz University, 2019.

[12] C. Reena, K.S. Yaamini, A New Class of Soft Sets Weaker than Soft δ-open sets, Proceedings of International Conference on Smart Technologies and Applications (ICSTA 2022).

[13] Sattar Hameed Hamzah, Samer Adnan Jubair, On Soft Generalized Continuous Mappings, Journal of Al-Qadisiyah for Computer Science and Mathematics, 8(1), 83-92. 2016

[14] Yenus Yumak, Aynur Keskin Kaymakci, Soft $\beta -$ open sets and their applications, Journal of New Theory, 80 – 89. 2015.