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Soft $I_{g\delta s}$-Closed Functions

Y. Rosemathy[1]
Dr. K. Alli[2]

Abstract

In this paper, we have introduced a new class of open and closed functions called soft $I_{g\delta s}$-closed and soft $I_{g\delta s}$-open functions in ideal topological spaces and also investigated some of its characterizations and properties with the existing sets.

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[1] Research Scholar (Reg.No-18111072092002) The M.D.T Hindu College, Tirunelveli-627010, TamilNadu, India. (Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli-627012, Tamil Nadu, India). Email: ravimathy18@gmail.com

[2]Assistant Professor, Department of Mathematics, The M.D.T Hindu College, Tirunelveli-627010, Tamil Nadu, India. (Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli-627012, Tamil Nadu, India)

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1. Introduction

The concept of soft sets was first introduced by Molodtsov [12] in 1999 as a general mathematical tool for dealing with uncertain objects. In [12, 13], Molodtsov successfully applied the soft theory in several directions, such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration, probability, theory of measurement, and so on. After presentation of the operations of soft sets [11], the properties and applications of soft set theory have been studied increasingly [3, 8, 13]. In [14] O.Ravi et all Decompositions of Ï g-Continuity via Idealization and In recent years, many interesting applications of soft set theory have been expanded by embedding the ideas of fuzzy sets [1, 2, 4, 9, 10, 11, 13]. To develop soft set theory, the operations of the soft sets are redefined and a uni-int decision making method was constructed by using these new operations [5]. Recently, in 2011, Shabir and Naz [15] initiated the study of soft topological spaces. They defined soft topology on the collection τ of soft sets over X. Consequently, they defined basic notions of soft topological spaces such as soft open and soft closed sets, soft subspace, soft interior, soft closure, soft neighborhood of a point, soft separation axioms, soft regular spaces and soft normal spaces and established their several properties. Hussain and Ahmad [6] investigated the properties of soft open, soft closed, soft interior, soft closure, soft neighborhood of a point. They also defined and discussed the properties of soft interior, soft exterior and soft boundary which are fundamental for further research on soft topology and will strengthen the foundations of the theory of soft topological spaces. In [16] S. Tharmar and R. Senthilkumar Introduced Soft Locally Closed Sets in Soft Ideal Topological Spaces.

In this paper, we have introduced a new class of open and closed functions called soft $I_{g\delta s}$-closed and soft $I_{g\delta s}$-open functions in ideal topological spaces and also investigated some of its characterizations and properties with the existing sets.

2. Preliminaries

In this section, we present some basic definitions and results which are needed in further study of this paper which may found in earlier studies. Throughout this paper, X refers to an initial universe, E is a set of parameters, $\wp(X)$ is the power set of X, and $A\subset E$

Definition 2.1. [12] A soft set F_A over the universe X is defined by the set of ordered pairs $F_A=\{(e, F_A(e)) : e \in E, F_A(e)\in\wp(X)\}$ where $F_A : E\to\wp(X)$, such that $F_A(e)\neq\emptyset$, if $e\in A\subset E$ and $F_A(e)=\emptyset$ if $e\notin A$. The family of all soft sets over X is denoted by SS(X).

Definition 2.2. [11] The soft set $F\emptyset$ over a common universe set X is said to be null soft set, denoted by $\emptyset$. Here $F\emptyset(e)=\emptyset$, $\forall e\in E$.

Definition 2.3. [11] A soft set F_A over X is called an absolute soft set, denoted by $\tilde{A}$, if $e\in A$, $F_A(e)=X$.

Definition 2.4. [11] Let F_A, G_B be soft sets over a common universe set X. Then F_A is a soft subset of G_B, denoted $F_A\subset G_B$ if $F_A(e)\subset G_B(e)$, $\forall e\in E$.

Definition 2.5. [11] Let F_A, G_B be soft sets over a common universe set X. The union of F_A and G_B, is a soft set H_C defined by $H_C(e)=F_A(e)\cup G_B(e)$, $\forall e\in E$, where $C=A\cup B$.
That is, $H_C=F_A\cup G_B$.

Definition 2.6. [11] Let F_A, G_B be soft sets over a common universe set X. The intersection of F_A and G_B, is a soft set H_C defined by $H_C(e)=F_A(e)\cap G_B(e)$, $\forall e\in E$, where $C=A\cap B$.
That is, $H_C=F_A\cap G_B$.

Definition 2.7. *[16] The complement of the soft set F_A over X_A denoted by F^c is defined by* $A^C(e)=X-F_A(e)$, $\forall e\in E$.

Definition 2.8. [16] Let F_A be a soft set over X and $x\in X$. We say that $x\in F_A$ if $x\in F_A(e)$, $\forall e\in A$. For any $x\in X$, $x\notin F_A$ if $x\notin F_A(e)$ for some $e\in A$.

Definition 2.9. [20] The soft set $F_A\in SS(X)$ is called a soft point in SS(X) if there exist $x\in X$ and $e\in E$ such that $F(e)=\{x\}$ and $F(e^c)=\emptyset$ for each $e^c\in E-\{e\}$ and the soft point F_A is denoted by x_ε.

Definition 2.10. [16] A soft topology τ is a family of soft sets over X satisfying the following properties.

(1) $\emptyset$, $\tilde{X}$ belong to τ.

(2) The union of any number of soft sets in τ belongs to τ.

(3) The intersection of any two soft sets in τ belongs to τ.

The triplet (X, τ, E) is called a soft topological space.

Definition 2.11. [15] Let (X, τ, E) be a soft topological space over X. Then

(1) The members of τ are called soft open sets in X.

(2) A soft set F_A over X is said to be a soft closed set in X if $F^c \in\tau$. A

(3) A soft set F_A is said to be a soft neighborhood of a point $x\in X$ if $x\in F_A$ and F_A is soft open in (X, τ, E)

(4) The soft interior of a soft set F_A is the union of all soft open subsets of F_A. The

soft
interior of F_A is denoted by int(F_A).

(5) The soft closure of F_A is the intersection of all soft closed super sets of F_A. The soft closure of F_A is denoted by cl(F_A) or $\overline{F_A}$.

Definition 2.12. [18] A soft set F_A in a soft topological space (X, τ, E) is said to be a soft regular open (resp. soft regular closed) if F_A=int(cl(F_A)) (resp. F_A=cl(int(F_A))).

Definition 2.13. Let I be a non-null collection of soft sets over a universe X with the same set of parameters E. Then I⊂SS(X) is called a soft ideal on X with the same set E if

(1) $F_A \in I$ and $G_A \in I \Rightarrow F_A \cup G_A \in I$.
(2) $F_A \in I$ and $G_A \subset F_A \Rightarrow G_A \in I$.

Definition 2.14. Let (X, τ , E) be a soft topological space and I be a soft ideal over X with the same set of parameters E. Then $F^*_A = \cup\{x_e \in X : O_{x_e} \cap F_A \notin I$, for all $O_{x_e} \in \tau\}$ is called the soft local function of F_A with respect to I and τ, where O_{x_e} is a τ-open set containing x_e.

Theorem 2.15. Let I and J be any two soft ideals with the same set of parameters E on a soft topological space (X, τ, E). Let F_A, $G_A \in$SS(X). Then

(1) $(\emptyset)^* = \emptyset$.
(2) $F_A \subset G_A \Rightarrow F^*_A \subset G^*_A$.
(3) $I \subset J \Rightarrow F^*_A(J) \subset FA^*(I)$.
(4) $F^*_A \subset cl(F_A)$, where cl is the soft closure w.r.t τ.
(5) F^*_A is τ-closed soft set.
(6) $(F^*_A)^* \subset F^*_A$.
(7) $(F_A \cup G_A)^* = F^*_A \cup GA^*$.

Definition 2.16. Let (X, τ , E) be a soft topological space, I be a soft ideal over X with the same set of parameters E and cl* : SS(X) →SS(X) be the soft closure operator. Then there exists a unique soft topology over X with the same set of parameters E, finer than τ , called the ⋆-soft topology, defined by τ*, given by $\tau^* = \{ F_A \in SS(X) : cl^*(X - F_A) = X - F_A\}$.

Definition 2.17. [7] Let F_A be a soft subset of soft topological space (X, τ, E). Then

(1) x_ε is called a soft δ-cluster point of F_A if $F_A \cap int(cl(U_A)) \neq \emptyset$ for every soft open set U_A containing x_ε.

(2) The family of all soft δ-cluster point of F_A is called the soft δ-closure of F_A and is denoted by $cl_\delta(F_A)$.

(3) A soft subset F_A is said to be soft δ-closed if $cl_\delta(F_A)=F_A$. The complement of a soft
δ-closed set of X is said to be soft δ-open.

Lemma 2.18. [7] Let F_A be a soft subset of soft topological space (X, τ, E). Then, the following properties hold:

(1) Int (cl (F_A)) is soft regular open,

(2) Every soft regular open set is soft δ-open,

(3) Every soft δ-open set is the union of a family of soft regular open sets.

(4) Every soft δ-open set is soft open.

Proposition 2.19. [7] Intersection of two soft regular open sets is soft regular open.

Lemma 2.20. [7] Let F_A and G_A be soft subsets of soft topological space (X, τ, E). Then, the following properties hold.

(1) $F_A \subset cl_\delta(F_A)$,

(2) If $F_A \subset G_A$, then $cl_\delta(F_A) \subset cl_\delta(G_A)$,

(3) $cl_\delta(F_A)=\cap\{G_A \in SS(X): F_A \subset G_A$ and G_A is soft δ-closed},

(4) If $(F_A)_\alpha$ is a soft δ-closed set of X for each $\alpha \in \triangle$, then $\cap\{(F_A)_\alpha: \alpha \in \triangle\}$ is soft δ-closed,

(5) $cl_\delta(F_A)$ is soft δ-closed.

Theorem 2.21. [7] Let (X, τ, E) be a soft topological space and $\tau_\delta=\{F_A \in SS(X) : F_A$ is a soft δ-open set}. Then τ_δ is a soft topology weaker than τ.

Definition 2.22. A soft subset F_A of a soft ideal topological space (X, τ, E, I) is said to be

(1) soft pre-I-open if $F_A \subset int (cl (F_A))$,

(2) soft semi-I-open if $F_A A \subset cl (int (F_A))$,

(3) soft α-I-open if $F_A \subset int (cl (int (F_A)))$.

The complement of soft pre-I-open (resp. soft semi-I-open, soft α-I-open) set is called a soft pre-I-closed (resp. soft semi-I-closed, soft α-I-closed).

Definition 2.23. The soft semi-I-closure of F_A is defined by the intersection of all soft semi-I- closed sets containing F_A and is denoted by SI_{scl}-(F_A)

Definition 2.24. A soft set F_A of soft ideal topological space X is called soft generalized δ semi-closed (briefly soft $I_{g\delta s}$-closed) set if $SI_{scl}(F_A) \subset G_A$ whenever $F_A \subset G_A$ and G_A are soft δ-open over X.

A soft set F_A of X is called soft generalized δ semi-open (briefly soft $SI_{g\delta s}$-open) set if F_A^c is soft $SI_{g\delta s}$-closed.

The family of all soft $SI_{g\delta s}$-closed subsets of the space X is denoted by $SI_{g\delta s}$-C(X) and soft $SI_{g\delta s}$-open subsets of the space X is denoted by $SI_{g\delta s}$-O(X).

3. soft SIgδs-closed and soft SIgδs-open functions

Definition 3.1. A function f: (X, τ, E) → (Y, σ, K, I) is said to be soft $SI_{g\delta s}$-closed (resp. soft

$SI_{g\delta s}$-open) if f (V_A) is soft $SI_{g\delta s}$-closed (resp. sost $SI_{g\delta s}$-open) over Y for every soft closed (resp. soft open) set V_A over X.

Definition 3.2. (1) A function f: $(X, \tau, E, I) \rightarrow (Y, \sigma, K, J)$ is soft $I_{g\delta s}$-irresolute if $f^{-1}(V_A)$ is soft $SI_{g\delta s}$-closed over X for every soft $SI_{g\delta s}$-closed set V_A over Y.
(2) A function f: $(X,\tau,E,I) \rightarrow (Y,K,\sigma)$ is soft $SI_{g\delta s}$-continuous if $f^{-1}(V_A)$ is soft $SI_{g\delta s}$-closed over X for every soft closed set V_A over Y.

Theorem 3.3. A function $f:(X, \tau, E) \rightarrow (Y, \sigma, K, I)$ is soft $SI_{g\delta s}$-closed if and only if f (V_A) is soft $SI_{g\delta s}$-open over Y for every soft open set V_A over X.
Proof: Suppose f: $(X,\tau,E) \rightarrow (Y,\sigma,K,I)$ is soft $SI_{g\delta s}$-closed function and V_A is a soft open set over X. Then $\tilde{X} - V_A$ is soft closed over X. By hypothesis $f(\tilde{X} - V_A) = \tilde{Y} - f(V_A)$ is a soft $SI_{g\delta s}$-closed set over Y and hence $f(V_A)$ is soft $SI_{g\delta s}$-open set over Y. On the other hand, if F_A is soft closed set over X, then $\tilde{X} - F_A$ is a soft open set over X. By hypothesis $f(\tilde{X} - F_A) = \tilde{Y} - f(F_A)$ is soft $SI_{g\delta s}$-open set over Y, implies f (F_A) is soft $SI_{g\delta s}$-closed set over Y. Therefore, f is soft $SI_{g\delta s}$-closed function.

Definition 3.4. A soft ideal topological space X is said to be soft $TI_{g\delta s}$-space if every soft $SI_{g\delta s}$-closed set is soft closed over X.

Definition 3.5. A soft ideal topological space X is said to be soft $SI_{g\delta s}$-T_2 space if every soft $SI_{g\delta s}$-closed set is soft semi-closed over X.

Theorem 3.6. If f: $(X, \tau, E) \rightarrow (Y, \sigma, K, I)$ is soft $SI_{g\delta s}$-closed function and Y is soft $TI_{g\delta s}$- space, then f is a soft closed function.
Proof: Let V_A be a soft closed set over X. Since f is a soft $SI_{g\delta s}$-closed function, implies f (V_A) is soft $SI_{g\delta s}$-closed over Y. Now Y is soft $TI_{g\delta s}$-space, implies f (V_A) is a soft closed set over Y. Therefore, f is a soft closed function.

Theorem 3.7. If $f:(X,\tau,E) \rightarrow (Y,\sigma,K,I)$ is soft $SI_{g\delta s}$-closed function and Y is soft $SI_{g\delta s}$-T_2 space, then f is soft semi-closed function.
Proof: Let V_A be a soft closed set over X. Since f is a soft $SI_{g\delta s}$-closed function, f (V_A) soft is $SI_{g\delta s}$-closed set over Y. Now Y is soft $SI_{g\delta s}$-T_2 space, implies f (V_A) is a soft semi-closed set over Y. Therefore, f is a soft semi-closed function.
Theorem 3.8. For the function f: $(X, \tau, E) \rightarrow (Y, \sigma, K, I)$, the following statements are equivalent.

(1) f is a soft $SI_{g\delta s}$-open function.

(2) For each soft subset F_A of X, $f(int(F_A)) \subset SI_{g\delta s} - int(f(F_A))$

(3) For each $x_e \in \tilde{X}$, the image of every soft nhd of x_e is soft $SI_{g\delta s}$-nhd of $f(x_e)$.

Proof: (1) → (2) Suppose (1) holds and $F_A \subset X$. Then int (F_A) is a soft open set over X. By (1), f (int (F_A)) is a soft $SI_{g\delta s}$-open set over Y.

Therefore $SI_{g\delta s}$ – int (f (int (F_A))) = f (int (F_A)). Since f (int (F_A)) ⊂ f (F_A), implies $SI_{g\delta s}$ – int (f (int (F_A))) ⊂ $SI_{g\delta s}$ – int (f (F_A)). That is f(int (F_A)) ⊂ $SI_{g\delta s}$ – int (f(F_A)).

(2) → (3) Suppose (2) holds. Let $x_e \in \tilde{X}$ and F_A be an arbitrary soft nhd of x_e over X. Then there exists a soft open set H_A in X such that $x_e \in H_A \subset F_A$. By (2), f(H_A) = f(int (H_A)) ⊂ $SI_{g\delta s}$–int (f(H_A)). But $SI_{g\delta s}$–int (f(H_A)) ⊂ f(H_A) is always true. Therefore, f(H_A) = $SI_{g\delta s}$–int (f (H_A)) and hence f (H_A) is soft $SI_{g\delta s}$-open set over Y. Further f (x_e) ∈ f (H_A) ⊂ f (F_A), this implies, f(F_A) is a soft $SI_{g\delta s}$-nhd of f(x_e) over Y. Hence (3) holds.

(3) → (1) Suppose (3) holds. Let V_A be any soft open set over X and $x_e \in V_A$. Then y_e = f (x_e) ∈ f (V_A). By (3), for each y_e ∈ f (V_A), there exists a soft $SI_{g\delta s}$-nhd $(Z_A)_{ye}$ of y_e over Y. Since $(Z_A)_{ye}$ is a soft $SI_{g\delta s}$-nhd of y_e, there exists a soft $SI_{g\delta s}$-open set $(V_A)_{ye}$ in V_A such that $y_e \in (V_A)_{ye} \subset (Z_A)_{ye}$. Therefore f ($V_A$) = ∪{$(V_A)_{ye}$: y_e ∈ f (V_A)}, which is union of soft $SI_{g\delta s}$-open sets and hence soft $SI_{g\delta s}$-open set over Y. Therefore, f is soft $SI_{g\delta s}$-open function.

Theorem 3.9. A function f: (X, τ, E) → (Y, σ, K, I) is soft $SI_{g\delta s}$-closed if and only if for each soft subset H_A over Y and for each soft open set U_A over X containing $f^{-1}(H_A)$, there exists a soft $SI_{g\delta s}$-open set V_A over Y such that $H_A \subset V_A$ and $f^{-1}(V_A) \subset U_A$.

Proof: Assume that f is soft $SI_{g\delta s}$-closed function. Let H_A ⊂ Y and U_A be a soft open set over X containing $f^{-1}(H_A)$. Since f is a soft $SI_{g\delta s}$-closed function and $\tilde{X} - U_A$ is soft closed over X, implies f ($\tilde{X} - U_A$) is soft $SI_{g\delta s}$-closed set over Y. Then $V_A = \tilde{Y}$ − f ($\tilde{X} - U_A$) is soft $SI_{g\delta s}$-open set over Y such that $H_A \subset V_A$ and $f^{-1}(V_A) \subset U_A$.

Conversely, let F_A be a soft closed set over X, then $\tilde{X} - F_A$ is a soft open set over X and

$f^{-1}(\tilde{Y} - f(F_A)) \subset \tilde{X} - F_A$. By hypothesis, there is a soft $SI_{g\delta s}$-open set V_A over Y such that $\tilde{Y} - f(F_A) \subset V_A$ and $f^{-1}(V_A) \subset \tilde{X} - F_A$. Therefore, $\tilde{Y} - V_A \subset f(F_A) \subset f(\tilde{X} - F^{-1}(V_A)) \subset \tilde{Y} - V_A$, this implies $f(F_A) = \tilde{Y} - V_A$. Since V_A is a soft $SI_{g\delta s}$-open set over Y and so f (F_A) is soft $SI_{g\delta s}$- closed over Y. Hence f is soft $SI_{g\delta s}$-closed function.

Theorem 3.10. If f: (X, τ, E) → (Y, σ, K, I) is soft $SI_{g\delta s}$-closed, then for each soft $SI_{g\delta s}$-closed set H_A over Y and each soft open set G_A over X containing $f^{-1}(H_A)$, there exists soft $SI_{g\delta s}$-open set V_A containing H_A such that $f^{-1}(V_A) \subset U_A$.

Proof: Suppose f: (X, τ, E) → (Y, σ, K, I) is soft $SI_{g\delta s}$-closed function. Let H_A be any soft $SI_{g\delta s}$-closed set over Y and U_A is a soft open set over X containing $f^{-1}(H_A)$, by theorem 3.9, there exists a soft $SI_{g\delta s}$-open set G_A over Y such that $H_A \subset G_A$ and $f^{-1}(G_A) \subset U_A$. Since H_A is soft $SI_{g\delta s}$-closed set and G_A is soft $SI_{g\delta s}$-open set containing H_A implies $H_A \subset I_{g\delta s}$-int ($G_A$). Put $V_A = I_{g\delta s}$-int (G_A), then $H_A \subset V_A$ and V_A are soft $SI_{g\delta s}$-open set over Y and $f^{-1}(V_A) \subset U_A$.

Theorem 3.11. A function f: (X, τ, E) → (Y, σ, K, I) is soft $SI_{g\delta s}$-closed, if and only if $SI_{g\delta s}$- cl(f(F_A)) ⊂ f(cl (F_A)), for every soft subset F_A over X.

Proof: Suppose f: (X, τ, E) → (Y, σ, K, I) is a soft $SI_{g\delta s}$-closed and F_A ⊂ X. Then f (cl (F_A)) is soft $SI_{g\delta s}$-closed over Y. Since f(F_A) ⊂ f(cl (F_A)), implies $SI_{g\delta s}$-cl(f(F_A)) ⊂ $SI_{g\delta s}$-cl (f (cl (F_A))) = f(cl (F_A)). Hence $SI_{g\delta s}$-cl(f(F_A)) ⊂ f(cl (F_A)).

Conversely, let F_A be any soft closed set over X. Then cl (F_A) = F_A. Therefore, f (F_A) = f (cl (F_A)). By hypothesis, $SI_{g\delta s}$-cl(f(F_A)) ⊂ f (cl (F_A)) = f (F_A) implies $SI_{g\delta s}$-cl(f(F_A)) ⊂ f (F_A). But f (F_A) ⊂ $SI_{g\delta s}$-cl(f(F_A)) is always true. This shows, f (F_A) = $SI_{g\delta s}$-cl(f(F_A)). Therefore f(F_A) is soft $SI_{g\delta s}$-closed set over Y and hence f is soft $SI_{g\delta s}$-closed.

Theorem 3.12. Let f : (X, τ, E) → (Y, σ, K, I) and g : (Y, σ, K, I) → (Z, μ, L, J) be any two functions. Then (g ∘ f) : (X, τ, E) → (Z, μ, L, J) is soft $SI_{g\delta s}$-closed function if f and g satisfy one of the following conditions

(1) f, g are soft $SI_{g\delta s}$-closed functions and Y is soft $TI_{g\delta s}$-space.

(2) f is soft closed and g is soft $SI_{g\delta s}$-closed function.

Proof: (1) Suppose F_A is soft closed set over X. Since f is soft $SI_{g\delta s}$-closed function f (F_A) is soft $SI_{g\delta s}$-closed set over Y. Now Y is soft $TI_{g\delta s}$-space, implies f (F_A) is soft closed set over Y. Also, g is soft $SI_{g\delta s}$-closed function, implies g (f (F_A)) = (g ∘ f) (F_A) is soft $SI_{g\delta s}$-closed set over Z. Hence (g ∘ f) is soft $SI_{g\delta s}$-closed function.

(2) Suppose F_A is soft closed set over X. Since f is soft closed function f (F_A) is soft closed set over Y. Now g is soft $SI_{g\delta s}$-closed function, implies g(f (F_A)) = (g ∘ f)(F_A) is soft $SI_{g\delta s}$-closed set over Z. Hence (g ∘ f) is soft $SI_{g\delta s}$-closed function.

Theorem 3.13. Let f: (X, τ, E) → (Y, σ, K, I) and g : (Y, σ, K, I) → (Z, μ, L, J) be any two functions such that (g ∘ f) : X → Z be soft $SI_{g\delta s}$-closed function. Then following results hold

(1) If f is soft continuous surjection, then g is soft $SI_{g\delta s}$-closed function.

(2) If g is soft $SI_{g\delta s}$-irresolute and injective, then f is soft $SI_{g\delta s}$-closed function.

Proof: (1) Suppose F_A is a soft closed set over Y. Since f is soft continuous and surjective, f^{-1}(F_A) is a soft closed set over X. Therefore, (g ∘ f) (f^{-1}(F_A)) = g(F_A) is soft $SI_{g\delta s}$-closed set over Z and hence g is soft $SI_{g\delta s}$-closed function.

(2) Suppose H_A is soft closed set over X. Then (g ∘ f) (H_A) is soft $SI_{g\delta s}$-closed set over Z. Since g is soft $SI_{g\delta s}$-irresolute, g^{-1}((g ∘ f) (H_A)) = f (H_A) is soft $SI_{g\delta s}$-closed set over Y. Hence f is soft $SI_{g\delta s}$-closed function.

Theorem 3.14. For any bijection f: (X, τ, E) → (Y, σ, K, I), the following statements are equivalent:

(1) f^{-1} is soft $SI_{g\delta s}$-continuous.

(2) f is a soft $SI_{g\delta s}$-open function.

(3) f is a soft $SI_{g\delta s}$-closed function.

Proof: (1) → (2) Suppose F_A is a soft open set over X, then by (1), $(f^{-1})^{-1}(F_A) = f(F_A)$ is soft $SI_{g\delta s}$-open set over Y and hence f is soft $SI_{g\delta s}$-open function.

(2) → (3) Suppose F_A is a soft closed set over X, then $\tilde{X} - F_A$ is a soft open set over X. By (2), $f(\tilde{X} - F_A) = \tilde{Y} - f(F_A)$ is soft $SI_{g\delta s}$-open over Y, implies $f(F_A)$ is soft $SI_{g\delta s}$-closed over Y. Therefore, f is soft $SI_{g\delta s}$-closed function.

(3) → (1) Let F_A be a soft closed set over X. By (3), $f(F_A) = (f^{-1})^{-1}(F_A)$ is soft $SI_{g\delta s}$-closed over Y. Therefore f^{-1} is soft $SI_{g\delta s}$ continuous function.

4. soft SIpgδs-closed functions

Definition 4.1. A function f: $(X, \tau, E) \to (Y, \sigma, K, I)$ is said to be soft $SI_{pg\delta s}$ closed (resp. soft $SI_{pg\delta s}$ open) if $f(V_A)$ is soft $SI_{g\delta s}$-closed (resp. soft $SI_{g\delta s}$-open) over Y for every soft semi-closed (resp. soft semi-open) set V_A over X.

Definition 4.2. (1) A function f: $(X, \tau, E) \to (Y, \sigma, K, I)$ is said to be soft semi-closed if $f(V_A)$ is soft semi-closed over Y for every soft semi-closed set V_A over X.

(2) A function f: $(X, \tau, E) \to (Y, \sigma, K)$ is said to be soft pre-closed if $f(V_A)$ is soft closed over Y for every soft semi-closed set V_A over X.

(3) A function f: $(X, \tau, E) \to (Y, \sigma, K)$ is said to be soft δ-continuous if $f^{-1}(V_A)$ is soft δ-closed over X for every soft δ-closed set V_A over Y.

Theorem 4.3. A function f: $(X, \tau, E) \to (Y, \sigma, K, I)$ is soft $SI_{pg\delta s}$-closed if and only if $f(V_A)$ is soft $SI_{g\delta s}$-open over Y for every soft semi-open set V_A over X.

Proof: Similar to the proof of theorem 3.3.

Remark 4.4. Every semi-open function is $I_{g\delta s}$-open function.

Theorem 4.5. If f: $(X, \tau, E) \to (Y, \sigma, K, I)$ is soft $SI_{pg\delta s}$-closed function and Y is soft $I_{g\delta s}$-$T_{1/2}$ space, then f is soft semi closed function.

Proof: Suppose V_A is a soft semi-closed set over X. Since f is a soft $SI_{pg\delta s}$-closed function $f(V_A)$ is soft $SI_{g\delta s}$-closed set over Y. Now Y is soft $I_{g\delta s}$-$T_{1/2}$ space $f(V_A)$ is a soft semi-closed set over Y. Therefore, f is a soft semi-closed function.

Theorem 4.6. A function f: $(X, \tau, E) \to (Y, \sigma, K, I)$ is soft $SI_{pg\delta s}$-closed if and only if for each soft subset H_A over Y and for each soft semi-open set U_A over X containing $f^{-1}(H_A)$, there exists

a soft $SI_{g\delta s}$-open set V_A over Y such that $H_A \subset V_A$ and $f^{-1}(V_A) \subset U_A$.
Proof: Similar to the proof of theorem 3.9.

Theorem 4.7. If f: $(X, \tau, E) \rightarrow (Y, \sigma, K, I)$ is soft $SI_{pg\delta s}$-closed, then for each soft $SI_{g\delta s}$-closed set H_A over Y and each soft semi-open set G_A over X containing $f^{-1}(H_A)$, there exists soft $SI_{g\delta s}$-open set V_A over Y containing H_A such that $f^{-1}(V_A) \subset U_A$.
Proof: Similar to the proof of theorem 3.10.

Theorem 4.8. If f is soft δ-continuous, soft $SI_{pg\delta s}$-closed, then f (H_A) is soft $SI_{g\delta s}$-closed over Y for each soft $SI_{g\delta s}$-closed H_A over X, with X is soft $I_{g\delta s}$-$T_{1/2}$ space.
Proof: Suppose H_A is any soft $SI_{g\delta s}$-closed set over X and V_A is a soft δ-open set over Y containing f (H_A). This implies $H_A \subset f^{-1}(V_A)$. Since f is soft δ-continuous, $f^{-1}(V_A)$ is a soft δ- open set containing H_A, therefore, $SI_{g\delta s}$-cl $(H_A) \subset f^{-1}(V_A)$ and hence f $(I_{g\delta s}$-cl $(H_A)) \subset V_A$. Since f is soft $SI_{pg\delta s}$-closed, implies f $(I_{g\delta s}$-cl $(H_A))$ is soft $SI_{g\delta s}$-closed set contained over Y, implies $SI_{g\delta s}$-cl $(f(SI_{g\delta s}$-cl $(H_A))) \subset V_A$. Thus, $SI_{g\delta s}$-cl$(f(H_A)) \subset SI_{g\delta s}$-cl (f $(SI_{g\delta s}$-cl $(H_A))) \subset V_A$. That is, $SI_{g\delta s}$-cl$(f(H_A)) \subset V_A$. This shows that $f(H_A)$ is soft $SI_{g\delta s}$-closed over Y.

Theorem 4.9. Let f: $(X, \tau, E) \rightarrow (Y, \sigma, K, I)$ and g: $(Y, \sigma, K, I) \rightarrow (Z, \mu, L, J)$ be any two functions. Then $(g \circ f)$: $X \rightarrow Z$ is soft $SI_{pg\delta s}$-closed function if f and g satisfy one of the following conditions:

(1) f, g are soft $SI_{pg\delta s}$-closed functions and Y is soft $I_{g\delta s}$-$T_{1/2}$ space.

(2) f is soft pre-closed and g is soft $SI_{g\delta s}$-closed function.

(3) f is soft semi-closed and g is soft $SI_{pg\delta s}$-closed function.

(4) f is soft $SI_{pg\delta s}$-closed function and g is soft δ-continuous, soft $SI_{pg\delta s}$-closed function and Y is soft $I_{g\delta s}$-$T_{1/2}$-space.

Proof: (1) Suppose F_A is soft semi-closed set over X. Since f is soft $SI_{pg\delta s}$-closed function f (F_A) is soft $SI_{pg\delta s}$-closed set over Y. Now Y is soft $I_{g\delta s}$-$T_{1/2}$-space, therefore f (F_A) is soft semi closed set over Y. Also g is soft $SI_{pg\delta s}$-closed function, implies $g(f(F_A)) = (g \circ f)(F_A)$ is soft $SI_{g\delta s}$-closed set over Z. Hence $(g \circ f)$ is soft $SI_{pg\delta s}$-closed function.

(2) Suppose F_A is soft semi-closed set over X. Since f is soft pre-closed, f (F_A) is soft closed set over Y. Now g is soft $SI_{g\delta s}$-closed function, implies $g(f(F_A)) = (g \circ f)(F_A)$ is soft $SI_{g\delta s}$-closed set over Z. Hence $(g \circ f)$ is soft $SI_{pg\delta s}$-closed function.

(3) Suppose F_A is soft semi-closed set over X. Since f is soft semi-closed function, f (F_A) is soft semi-closed set over Y. Now g is soft $SI_{pg\delta s}$-closed function, implies $g(f(F_A)) = (g \circ f)(F_A)$ is soft $SI_{g\delta s}$-closed set over Z. Hence $(g \circ f)$ is soft $SI_{pg\delta s}$-closed function.

(4) Suppose H_A is a soft semi-closed set over X. Since f is soft $SI_{pg\delta s}$-closed function f (H_A) is soft $SI_{g\delta s}$-closed set over Y. Since g is soft δ-continuous, soft $SI_{pg\delta s}$-closed function by Theorem 4.8, $g(f(H_A)) = (g \circ f)(H_A)$ is soft $SI_{g\delta s}$-closed set over Z. Hence $(g \circ f)$ is soft $SI_{pg\delta s}$-closed function.

5. Strongly soft SIgδs-closed and soft quasi SIgδs-closed functions

Definition 5.1. A function f: (X, τ, E) → (Y, σ, K, I) is said to be strongly soft $SI_{g\delta s}$-closed (resp. strongly soft $SI_{g\delta s}$-open), if f (F_A) is soft $SI_{g\delta s}$-closed (resp. soft $SI_{g\delta s}$-open) set over Y for every soft $SI_{g\delta s}$-closed (resp. soft $SI_{g\delta s}$-open) set F_A over X.

Remark 5.2. Every strongly soft $SI_{g\delta s}$-closed function is soft $SI_{g\delta s}$-closed function.

Theorem 5.3. A surjective function f: (X, τ, E) → (Y, σ, K, I) is strongly soft $SI_{g\delta s}$-closed (resp. strongly soft $SI_{g\delta s}$-open), if and only if for any soft subset G_A over V_A and each soft $SI_{g\delta s}$- open (resp. soft $SI_{g\delta s}$-closed) set U_A over X containing $f^{-1}(G_A)$, there exists a soft $SI_{g\delta s}$-open (resp. soft $SI_{g\delta s}$-closed) set V_A over Y containing G_A and $f^{-1}(V_A) \subset U_A$.
Proof: Similar to the proof of theorem 3.9.

Theorem 5.4. If a function f: (X, τ, E) → (Y, σ, K, I) is a strongly soft $SI_{g\delta s}$ closed function, then for each soft $SI_{g\delta s}$-closed set H_A over Y and each soft $SI_{g\delta s}$-open set U_A over X containing $f^{-1}(H_A)$, there exists soft $SI_{g\delta s}$-open set V_A over Y containing H_A such that $f^{-1}(V_A) \subset U_A$.
Proof: Similar to the proof of theorem 3.10.

Theorem 5.5. A function f: (X, τ, E) → (Y, σ, K, I) is strongly soft $SI_{g\delta s}$-closed, if and only if $SI_{g\delta s}-cl$ ($f(F_A)) \subset f(SI_{g\delta s}$-cl $(F_A))$ for every soft subset F_A over X.
Proof: Let f be strongly soft $SI_{g\delta s}$-closed function and $F_A \subset X$. Then f ($SI_{g\delta s}$-cl (F_A)) is soft $SI_{g\delta s}$-closed over Y. Since f (F_A) ⊂ f ($SI_{g\delta s}$-cl (F_A)), implies $SI_{g\delta s}$-cl(f(F_A)) ⊂ $SI_{g\delta s}$-cl (f ($SI_{g\delta s}$- cl (F_A))) = f ($SI_{g\delta s}$-cl (F_A)). Therefore, $SI_{g\delta s}$-cl(f(F_A)) ⊂ f($SI_{g\delta s}$-cl (F_A)).
Conversely, F_A is any soft $SI_{g\delta s}$-closed set over X. Then $SI_{g\delta s}$-cl (F_A) = F_A, implies, f (F_A) = f ($SI_{g\delta s}$-cl (F_A)). By hypothesis, $SI_{g\delta s}$-cl(f(F_A)) ⊂ f ($SI_{g\delta s}$-cl (F_A)) = f (F_A). Hence $SI_{g\delta s}$- cl(f(F_A)) ⊂ f (F_A). But f (F_A) ⊂ $SI_{g\delta s}$-cl(f(F_A)) is always true. This shows, f (F_A) = $SI_{g\delta s}$- cl(f(F_A)). Therefore, f (F_A) is soft $SI_{g\delta s}$-closed set over Y. Hence f is strongly soft $SI_{g\delta s}$-closed- closed function.

Theorem 5.6. Let f: (X, τ, E) → (Y, σ, K, I) and g: (Y, σ, K, I) → (Z, μ, L, J) be two functions, such that (g ∘ f) : X → Z is strongly soft $SI_{g\delta s}$-closed function. Then

(1) f is soft $SI_{g\delta s}$-irresolute and surjective implies g is strongly soft $SI_{g\delta s}$-closed.

(2) g is soft $SI_{g\delta s}$-irresolute and injective implies f is strongly soft $SI_{g\delta s}$-closed.

Proof: (1) Let F_A be soft $SI_{g\delta s}$-closed set over Y. Since f is soft $SI_{g\delta s}$ irresolute and surjective, $f^{-1}(F_A)$ is soft $SI_{g\delta s}$-closed set over X. Also since (g ∘ f) is strongly soft $SI_{g\delta s}$-closed function, implies (g ∘ f)($f^{-1}(F_A)$) = g(F_A) is soft $SI_{g\delta s}$-closed over Z. Therefore, g is strongly soft $SI_{g\delta s}$- closed.

(2) Let F_A be soft $SI_{g\delta s}$-closed set over X. Since (g ∘ f) is strongly soft $SI_{g\delta s}$-

closed function $(g \circ f)(F_A)$ is soft $SI_{g\delta s}$-closed over Z. Also, since g is soft $SI_{g\delta s}$-irresolute and injective, $g^{-1}(g \circ f)(F_A) = f(F_A)$ is soft $SI_{g\delta s}$-closed set over Y. Therefore, f is strongly soft $SI_{g\delta s}$ closed.

Theorem 5.7. For any bijection, f: (X, τ, E) → (Y, σ, K, I) the following statements are equivalent.

(1) f^{-1} is soft $SI_{g\delta s}$-irresolute.

(2) f is a strongly soft $SI_{g\delta s}$-open function.

(3) f is a strongly soft $SI_{g\delta s}$-closed function.

Proof: Similar to the proof of theorem 3.14.

Definition 5.8. A function f: (X, τ, E) → (Y, σ, K, I) is said to be soft quasi $SI_{g\delta s}$-closed (resp. soft quasi $SI_{g\delta s}$-open), if for each soft $SI_{g\delta s}$-closed (resp. soft $SI_{g\delta s}$-open) set F_A over X, $f(F_A)$ is soft closed (resp. open) set over Y.

Remark 5.9. Every soft quasi $SI_{g\delta s}$-closed function is soft closed, strongly soft $SI_{g\delta s}$ closed and soft $SI_{g\delta s}$-closed function.

Remark 5.10. Every soft quasi $SI_{g\delta s}$-closed function is soft $SI_{pg\delta s}$-closed.

Remark 5.11. Following diagram is obtained from the Definitions.

soft $SI_{pg\delta s}$-closed

↗ ↑ ↘

soft quasi $SI_{g\delta s}$-closed → Strongly soft $SI_{g\delta s}$-closed → soft $SI_{g\delta s}$-closed

↘ ↗

sot closed

Theorem 5.12. A surjective function f: (X, τ, E, I) → (Y, σ, K) is soft quasi $SI_{g\delta s}$-closed (resp. soft quasi $SI_{g\delta s}$-open), if and only if for any soft subset G_A over Y and each soft $SI_{g\delta s}$-open (resp. soft $SI_{g\delta s}$-closed) set U_A over X containing $f^{-1}(G_A)$, there exists a soft open (resp. soft closed) set V_A over Y containing G_A and $f^{-1}(V_A) \subset U_A$.

Proof: Similar to the proof of theorem 3.9.

Theorem 5.13. A function f: (X,τ,E,I) → (Y,σ,K) is soft quasi $SI_{g\delta s}$-closed if and only if

$Cl(f(F_A)) \subset f(SI_{g\delta s}\text{-}cl(F_A))$ for every soft subset F_A over X.

Proof: Suppose that f is soft quasi $SI_{g\delta s}$-closed function and $F_A \subset X$. Then $SI_{g\delta s} - cl(F_A)$ is soft $SI_{g\delta s}$-closed set over X. Therefore $f(SI_{g\delta s}\text{-}cl(F_A))$ is soft closed over Y. Since $f(F_A) \subset f(SI_{g\delta s}\text{-}cl(F_A))$, implies $cl(f(F_A)) \subset cl(f(SI_{g\delta s}\text{-}cl(F_A))) = f(SI_{g\delta s}\text{-}cl(F_A))$. This im- plies, $cl(f(F_A)) \subset f(SI_{g\delta s}\text{-}cl(F_A))$.

Conversely, F_A is any soft $SI_{g\delta s}$-closed set over X. Then $SI_{g\delta s}\text{-}cl(F_A) = F_A$. Therefore, $f(F_A) = f(SI_{g\delta s}\text{-}cl(F_A))$. By hypothesis, $cl(f(F_A)) \subset f(SI_{g\delta s}\text{-}cl(F_A)) = f(F_A)$. Hence $cl(f(F_A)) \subset f(F_A)$. But $f(F_A) \subset cl(f(F_A))$ is always true. This shows, $f(F_A) = cl(f(F_A))$. This implies $f(F_A)$ is soft closed set over Y. Therefore, f is soft quasi $SI_{g\delta s}$-closed function.

Theorem 5.14. Let $f: (X, \tau, E, I) \rightarrow (Y, \sigma, K, J)$ be a function from a space X to a soft
$TI_{g\delta s}$-space Y. Then following are equivalent

(1) f is strongly soft S-$I_{g\delta s}$-closed function.

(2) f is soft quasi-S-$I_{g\delta s}$-closed function.

Proof: (1) ⇒ (2) Suppose (1) holds. Let F_A be a soft $SI_{g\delta s}$-closed set over X. Then $f(F_A)$ is soft S-$I_{g\delta s}$-closed over Y. Since Y is soft $TI_{g\delta s}$-space, $f(F_A)$ is soft closed over Y. Therefore, f is soft quasi $SI_{g\delta s}$-closed function.
(2) ⇒ (1) Suppose (2) holds. Let F_A be a soft S-$I_{g\delta s}$-closed set over X. Then f (F_A) is soft closed and hence soft $SI_{g\delta s}$-closed over Y. Therefore, f is strongly soft S-$I_{g\delta s}$-closed. function.

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