Nano Semi*α-open sets

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Abstract

In this paper, we introduce a new class of sets called nano semi*α-open sets and discuss some of its properties in nano topological space. We also, present nano semi*α-interior, nano semi*α-closure and study some of its fundamental properties.

Keywords: nano semi*α-open, nano semi*α-closed, nano semi*α-interior, nano semi*α–closure.

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1. Introduction

The notion of Nano topology was introduced by Lellis Thivagar [8] which was defined in terms of approximations and boundary region of a subset of a universe using an equivalence relation on it and also defined Nano closed sets, Nano-interior and Nano-closure. He also introduced the weak forms of Nano open sets namely Nano α-open sets, Nano semi-open sets and Nano pre-open sets. In 2017, the concept of nano semi α open sets was introduced by Qays Hatem Imran [3]. In 2014, A. Robert and S. Pious Missier [7] have introduced and studied semi*α-open sets in general topology. In this paper we introduce nano semi*α-open sets and nano semi *α-closed sets in nano topological spaces. We investigate its fundamental properties and find its relation with other Nano sets and study some of its properties.

2. Preliminaries

Throughout this chapter (U, τ_R(X)) is a nano topological space with respect to X where X ⊆ U, R is an equivalence relation on U, U/R denotes the family of equivalence classes of U by R.

Definition 2.1 [8]: Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be discernible with one another. The pair (U, R) is said to be the approximation space. Let X⊆U

1. The **lower approximation** of X with respect to R is the set of all objects which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is L_R(X) = $\bigcup_{x\in U}\{R(x) / R(x) \subseteq X\}$ where R(x) denotes the equivalence class determined by X.
2. The **upper approximation** of X with respect to R is the set of all objects which can be possibly defined as X with respect to R and it is denoted by $U_R(X)$. That is $U_R(X)= \bigcup_{x\in U}\{R(x) / R(x) \cap X \neq \phi\}$
3. The **boundary region** of X with respect to R is the set of all objects which can be classified neither as X nor as not X with respect to R and is denoted by $B_R(X)$. That is $B_R(X) = U_R(X) – L_R(X)$.

Proposition 2.2 [8] If (U, R) is an approximation space and X, Y ⊆ U, then

1. $L_R(X) \subseteq X \subseteq U_R(X)$
2. $L_R(\emptyset) = U_R(\emptyset) = \emptyset$ and $L_R(U) = U_R(U) = U$
3. $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$
4. $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$
5. $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$
6. $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$
7. $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$
8. $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$
9. $U_RU_R(X) = L_R U_R(X) = U_R(X)$
10. $L_RL_R(X) = U_R L_R(X) = L_R(X)$

Definition 2.3 [8]: Let U be the universe, R be an equivalence relation on U and τ_R(X) = {U, ∅, L_R(X), U_R(X), B_R(X)} where X ⊆ U. Then by the proposition 2.2, R(X) satisfies the following axioms:
1. U and ∅ ∈ τ_R(X)
2. The union of the elements of any subcollection of τ_R(X) is in τ_R(X).
3.The intersection of the elements of any finite subcollection of τ_R(X) is in τ_R(X).
That is τ_R(X) is a topology on U called the nano topology on U with respect to X. We call (U, τ_R(X)) as the **nano topological space**. The elements of τ_R(X) are called as **nano-open sets**.

Definition 2.4 [8]: If (U,τ_R(X)) is a nano topological space with respect to X and if A ⊆ U, then (i) ***nano interior*** of A is defined as the union of all nano-open sets contained in A and is denoted by NInt (A). That is, NInt(A) is the largest nano-open subset of A.
(ii) n***ano closure*** of A is defined as the intersection of all nano-closed sets containing A and it is denoted by NCl(A). That is, NCl(A) is the smallest nano-closed set containing A.

Definition 2.5 [2]: Let (U, τ_R(X)) be a nano topological space. A subset A of (U, τ_R(X)) is called ***nano generalized-closed*** (briefly Ng- closed) if NCl(A) ⊆V where A ⊆V and V is Nano-open.
The complement of nano generalized -closed set is called as ***nano generalized-open***.

Definition 2.6 [2]: For every set A ⊆U, the ***nano generalized closure of A*** is defined as the intersection of all Ng- closed sets containing A and is denoted by NCl*(A).

Definition 2.7 [2]: For every set A ⊆U, the ***nano generalized interior of A*** is defined as the union of all Ng- open sets contained in A and is denoted by NInt*(A).

Definition 2.8: Let (U,τ_R(X)) be a nano topological space and A ⊆U. Then A is said to be
(i) **nano α-open [8]** if A ⊆ N Int (NCl (NInt (A)))
(ii) **nano semi*-open [1]** if A ⊆ NCl*(NInt(A))
(iii) **nano semi α -open [3]** if A ⊆ NCl (NInt (NCl (NInt A)))
(iv) **nano semi pre-open [6]** if A ⊆ NCl (NInt (NCl (A)))
(v) **nano regular-open** [**8**] if A = NInt (NCl (A))
(vi) **nano regular *-open [5]** if $A = \text{NInt}(\text{NCl}^*(A))$
(vii) **nano pre *-open [4]** if $A \subseteq \text{NInt}^*(\text{NCl }(A))$
(viii) **nano pre-open [8]** If A ⊆ NInt (NCl (A))
(ix) **nano θ-open [9]**, if for each x ∈ A, there exists a nano open set G such that x ∈ G ⊆ Ncl (A) ⊆ A.
The complements of the above-mentioned sets are called their respective ***nano-closed*** sets.

3. Nano Semi*α-open sets

Definition 3.1: A subset A of a nano topological space $(U,\tau_R(X))$ is called **nano semi*α- open** if there is a nano α-open set G in U such that $G \subseteq A \subseteq N_oCl^*(G)$.
The collection of all nano semi*α-open sets is denoted by $\boldsymbol{N_oS^*\alpha\, O(U,\tau_R(X))}$.

Example 3.2: Let $U = \{a,b,c,d\}$ with U/R= {{a},{b,c,d}}. Let $X = \{b,c,d\}$. Then $\tau_R(X) = \{U,\emptyset,\{b,c,d\}\}$.The nano-closed sets are $\{U,\emptyset,\{a\}\}$.The nano generalized-closed sets are$\{U,\emptyset,\{a\},\{a,b\},\{a,c\},\{a,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\}\}$**.**
The nano generalized-open sets are $\{U,\emptyset,\{b\},\{c\},\{d\},\{b,c\},\{b,d\},\{c,d\},\{b,c,d\}\}$.
$N_oS^*\alpha\, O(U,\tau_R(X)) = \{U,\phi,\{b,c,d\}\}$.

Theorem 3.3: For a subset A of a nano topological space $(U,\tau_R(X))$ the following statements are equivalent:
(i)A is nano semi$^*\alpha$-open.
(ii) $A \subseteq N_oCl^*(\alpha N_o\text{Int}\,(A))$
(iii) $N_oCl^*(\alpha N_o\text{Int}\,(A)) = N_oCl^*(A)$
Proof: (i)⟹(ii) If A is a nano semi$^*\alpha$-open, then there is a nano α-open set G in U such that $G \subseteq A \subseteq N_ocl^*(G)$. Now $G \subseteq A \Rightarrow G=\alpha N_o\text{Int}\,(G) \subseteq \alpha N_o\text{Int}\,(A) \Rightarrow A \subseteq N_oCl^*(G) \subseteq N_oCl^*(\alpha N_o\text{Int}\,(A))$.
(ii)⟹(iii)By assumption, $A\subseteq N_OCl^*(N_o\alpha Int\,(A))$. we have $N_OCl^*(A)\subseteq N_oCl^*(N_oCl^*(\alpha N_o\text{Int}\,(A)))= N_oCl^*(\alpha N_o\text{Int}\,(A))$. Now $\alpha N_o\text{Int}\,(A) \subseteq A$ implies that $N_oCl^*(\alpha N_o\text{Int}\,(A)) \subseteq N_oCl^*(A)$.Therefore $N_oCl^*(\alpha N_o\text{Int}\,(A)) = N_oCl^*(A)$
(iii) ⟹(i) Take $G=\alpha N_o\text{Int}\,(A)$.Then G is a nano α-open set in U such that
$G\subseteq A \subseteq N_OCl^*(G) = N_OCl^*(\alpha N_o\text{Int}\,(A)) = N_OCl^*(G)$.Therefore by definition 3.1, A is nano semi$^*\alpha$- open.

Theorem 3.4: Arbitrary union of nano semi $^*\alpha$ -open set is nano semi$^*\alpha$-open.
Proof: Let $\{A_\alpha\}$ be a collection of nano semi$^*\alpha$-open sets in nano topological space U. Then there exists a nano α-open set G_α such that $G_\alpha \subseteq A_\alpha \subseteq N_OCl^*(G_\alpha)$ for each α. Hence $\cup\, G_\alpha \subseteq \bigcup A_\alpha \subseteq \bigcup N_OCl^*(G_\alpha) \subseteq N_OCl^*(\cup\, G_\alpha)$.Since $\cup\, G_\alpha$is nano α-open , by definition 3.1 $\cup\, A_\alpha$ is nano semi *α -open.

Remark 3.5: The intersection of two nano semi$^*\alpha$- open sets need not be a nano semi$^*\alpha$-open as seen from the following example.

Example 3.6: Let $U = \{a,b,c,d\}$,$U/R = \{\{a\},\{c\},\{b,d\}\}$.Let $X = \{a,b\}$.Then $\tau_R(X) = \{U,\emptyset,\{a\},\{b,d\},\{a,b,d\}\}$ and $N_o\,S^*\alpha\, O(U,\tau_R(X)) = \{U,\emptyset,\{a\},\{a,c\},\{b,d\},$ $\{a,b,d\},\{b,c,d\}\}$. Here the subsets $A = \{a,c\}\ and\ B = \{b,c,d\}$ are nano semi$^*\alpha$-open sets, but $A\cap B = \{c\}$ is not nano semi$^*\alpha$- open.

Theorem 3.7: If A is nano semi$^*\alpha$-open in U and B is nano open in X ,then $A\cap B$ is nano semi$^*\alpha$-open in U.
Proof: Since A is nano semi$^*\alpha$-open in U,there is an nano α-open sets G such that $G \subseteq A \subseteq N_O\, Cl^*(G)$.Since B is nano open ,$G \cap B \subseteq A\cap B \subseteq N_O\, Cl^*(G)\cap B \subseteq N_O\, Cl^*(G\cap B)$.Since $G\cap B$is nano α-open,by definition 3.1,$A\cap B$ is nano semi$^*\alpha$-open.

Theorem 3.8: Every nano α-open set is nano semi$^*\alpha$-open.
Proof: Let A be a nano α-open set in U. Then $N_o\, \alpha\, intA = A$ and hence $A \subseteq N_O Cl^*(A) = N_O Cl^*(N_o \alpha Int\, (A))$.Hence A is nano semi$^*\alpha$-open.

Remark 3.9: The converse of the above theorem is not true as shown in the following example.

Example 3.10: Let $U = \{a,b,c,d\}$,$U/R = \{\{a\},\{c\},\{b,d\}\}$.Let $X = \{a,b\}$. Then $\tau_R(X) = \{U,\phi,\{a\},\{b,d\},\{a,b,d\}\}$ and $N_O \alpha O\big(U,\tau_R(X)\big) = \{U,\emptyset,\{a\},\{b,d\},$ $\{a,b,d\}\}$. Clearly the sets$\{a,c\}$and $\{b,c,d\}$ are nano semi* α-open but not nano α-open.

Theorem 3.11: Every nano open set is nano semi$^*\alpha$-open.
Proof: Let A be any nano open set. Since every nano open set is nano α-open and hence by theorem 3.8, A is nano semi$^*\alpha$-open.

Remark 3.12: The converse of the above theorem is not true as shown in the following example.

Example 3.13: Let $U = \{a,b,c,d\}$,$U/R = \{\{a\},\{c\},\{b,d\}\}$.Let $X = \{a,b\}$.
Then $\tau_R(X) = \{U,\phi,\{a\},\{b,d\},\{a,b,d\}\}$ and $N_o\, S^*\alpha\, O(U,\tau_R(X)) = \{U,\emptyset,\{a\},\{a,c\},$ $\{b,d\},\{a,b,d\},\{b,c,d\}\}$.Clearly the sets $\{a,c\}$and $\{b,c,d\}$ is nano semi$^*\alpha$- open but not nano open.

Theorem 3.14: Every nano semi*-open set is nano semi$^*\alpha$-open.
Proof: Let A be any nano semi* open set. Then there is a nano open set G in U such that $G \subseteq A \subseteq N_o Cl^*(G)$.Since every nano open set is nano α-open ,A is nano semi$^*\alpha$-open.

Remark 3.15: The converse of the above theorem is not true as shown in the following example.

Example 3.16: Let $U = \{a,b,c,d\}$ $U/R = \{\{a\},\{b\}\}$. Let X = $\{a,c,d\}$.Then $\tau_R(X) = \{U,\emptyset,\{a\}\}$. $N_o\, S^*\alpha\, O(U,\tau_R(X)) = \{U,\emptyset,\{a\},\{a,b\}\,,\{a,c\},\{a,d\},\{a,b,c\},\{a,b,d\},$ $\{a,c,d\}\}$. $N_o\, S^*\, O\big(U,\tau_R(X)\big) = \big\{U,\emptyset,\{a\}\big\}$. Clearly the sets $\{a,b\}$,$\{a,c\},\{a,d\},\{a,b,c\},$ $\{a,b,d\},\{a,c,d\}$ are nano semi$^*\alpha$-open but not nano semi*-open.

Theorem 3.17: Every nano semi$^*\alpha$ open set is nano semi α-open.

Proof: Let A be any nano semi* α open set. Then there is a nano α-open set G in U such that $G \subseteq A \subseteq N_oCl^*(G)$.Since $N_oCl^*(G) \subseteq N_oCl(G)$,we have $G \subseteq A \subseteq N_oCl(G)$.Hence A is nano semi α-open.

Remark 3.18: The converse of the above theorem is not true as shown in the following example.

Example3.19: Let $U = \{a, b, c, d\}$ $U/R = \{\{a\}, \{d\}, \{b, c\}\}$. Let $X = \{a, c\}$ $\tau_R(X) = \{U, \emptyset, \{a\}, \{b, c\}, \{a, b, c\}\}$.$N_o\, S^*\alpha\, O(U, \tau_R(X)) = \{U, \emptyset, \{a\}, \{b, c\}, \{a, b, c\}\}$.
$N_o\, S\alpha(U, \tau_R(X)) = \{U, \emptyset, \{a\}, \{a, d\}, \{b, c\}, \{a, b, c\}, \{b, c, d\}\}$.Clearly the subset $\{a, d\}$ and $\{b, c, d\}$are semi nano α-open but not nano semi*α-open.

Theorem 3.20: Every nano semi*α open set is nano semi pre-open.
Proof: Let A be any nano semi*α-open set. Then there is a nano α-open set G such that $G \subseteq A \subseteq N_oCl^*(G)$.Since every nano α-open set is nano pre-open and $N_oCl^*(G) \subseteq N_oCl(G)$, A is nano semi pre-open.

Remark 3.21: The converse of the above theorem is not true as shown in the following example.

Example 3.22: Let U={ $a, b, c, d\}$, U/R= $\{\{a\}, \{d\}, \{b, c\}\}$.Let X= $\{a, c\}$. $\tau_R(X) = \{U, \emptyset, \{a\}, \{b, c\}, \{a, b, c\}\}$. $N_oS^*\alpha\, O(U, \tau_R(X)) = \{U, \emptyset, \{a\}, \{b, c\}, \{a, b, c\}\}$ and $N_oSPO(U, \tau_R(X))$={ $U, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$. Clearly the subsets $\{b\}$,$\{c\}$, $\{a, b\}$, $\{a, c\}$, $\{a, d\}$, $\{b, d\}$, $\{c, d\}, \{a, b, d\}, \{a, c, d\}$ are nano semi pre-open but not nano semi*α-open.

Theorem 3.23: Every nano regular open set is nano semi*α-open.
Proof: Let A be any nano regular open set. Since every nano regular open set is nano open and by theorem 3.11, we have A is nano semi*α-open.

Remark 3.24: The converse of the above theorem is not true as shown in the following example.

Example 3.25: Let $U = \{a, b, c, d\}$ $U/R = \{\{a\}, \{d\}, \{b, c\}\}$. Let $X = \{a, c\}$. $\tau_R(X) = \{U, \emptyset, \{a\}, \{b, c\}, \{a, b, c\}\}$.$N_o\, S^*\, \alpha\, O(U, \tau_R(X)) = \{U, \emptyset, \{a\}, \{b, c\}, \{a, b, c\}\}$.
$N_o\, R\, O(U, \tau_R(X))$ = $\{U, \emptyset, \{a\}, \{b, c\}\}$.Clearly the subset $\{a, b, c\}$ is nano semi*α-open but not nano regular open.

Theorem 3.26: Every nano regular*-open set is nano semi*α-open.
Proof: Let A be any nano regular*-open set. Since every nano regular*-open set is nano open and by theorem 3.11, we have A is nano semi*α-open.

Remark 3.27: The converse of the above theorem is not true as shown in the following example.

Example 3.28: Let $U = \{a,b,c,d\}$ $U/R = \{\{a\},\{b\}\}$. Let X = $\{a,c,d\}$.Then $\tau_R(X) = \{U,\emptyset,\{a\}\}$, $N_o\,S^*\,\alpha\,O(U,\tau_R(X)) = \{\,U,\emptyset,\{a\},\{a,b\},\{a,c\}\,,\{a,d\},\{a,b,c\},\{a,b,d\},$ $\{a,c,d\}\}$. $N_o\,R^*O(U,\tau_R(X)) = \{U,\emptyset,\{a\}\}$.Clearly the subsets$\{a,b\},\{a,c\},\{a,d\}$, $\{a,b,c\}$, $\{a,b,d\}$, $\{a,c,d\}$ are nano semi$^*\alpha$-open but not nano regular*-open.

Theorem 3.29: Every nano semi$^*\alpha$-open set is nano pre *-open.
Proof: Let A be any nano semi$^*\alpha$-open set. Then there is a nano α-open set G in U such that $G \subseteq A \subseteq N_oCl^*(G)$ .Since every nano α-open set is nano pre*-open, we have A is nano pre*- open.
Remark 3.30: The converse of the above theorem is not true as shown in the following example.

Example3.31: Let $U = \{a,b,c,d\}$ $U/R = \{\{a\},\{b\}\}$. Let X = $\{a,c,d\}$.Then $\tau_R(X) = \{U,\emptyset,\{a\}\}$. $N_o\,S^*\alpha\,O(U,\tau_R(X)) = \{U,\emptyset,\{a\},\{a,b\}\,,\{a,c\},\{a,d\},\{a,b,c\},\{a,b,d\}$ $\{\,a,c,d\,\}$ and $N_o\,P^*O(U,\tau_R(X)) = \{U,\emptyset,\{a\},\{b\},\{c\},\{d\},\{a,b\},\{a,c\}\,,\{a,d\},\{b,c\},$ $\{b,d\},\{c,d\}$, $\{\,a,b,c\}$, $\{a,b,d\}$,$\{b,c,d\},\{a,c,d\}\}$.Clearly the subsets $\{b\},\{c\},\{d\}$, $\{b,c\},\{b,d\}$, $\{c,d\},\{b,c,d\}$are nano pre*-open but not nano semi$^*\alpha$-open.

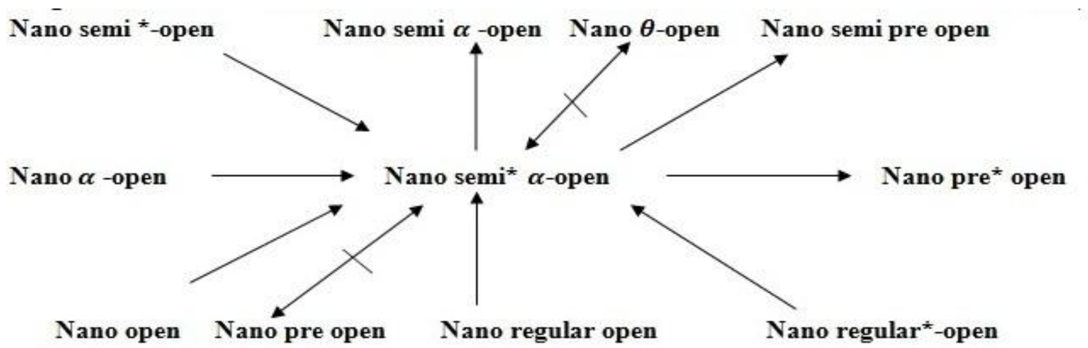
Remark 3.32: The concept of nano semi$^*\alpha$-open sets and nano pre-open sets are independent as shown in the following example.

Example 3.33: Let $U = \{a,b,c,d,e\}U/R = \{\{a\},\{d\}\{b,c\}\}$. Let X = $\{a,c\}$.Then $\tau_R(X) = \{U,\emptyset,\{a\},\{b,c\},\{a,b,c\}\}$. $N_o\,S^*\alpha\,O(U,\tau_R(X)) = \{U,\emptyset,\{a\},\{a,e\}\,,\{b,c\},$ $\{a,b,c\},\{a,b,c,d\},\{a,b,c,e\}\}$ and $N_o\,PO(U,\tau_R(X))$=$\{\,U,\emptyset,\{a\},\{b\},\{c\},\{a,b\},\{a,c\},$ $\{a,d\},\{b,c\},\{a,b,c\},\{a,b,d\}\,,\{a,b,e\}\,,\{a,c,d\},\{a,c,e\}\,,\{a,b,c,d\},\{a,b,c,e\},$ $\{a,b,d,e\},\{a,c,d,e\}\}$. Clearly the subset $\{a,e\}$ is nano semi$^*\alpha$-open but not nano pre-open and the subsets $\{b\},\{c\},\{a,b\},\{a,c\},\{a,d\}$, $\{a,b,d\},\{a,b,e\}$, $\{a,c,d\}\,,\{a,c,e\}$, $\{a,b,d,e\},\{a,c,d,e\}$ are nano pre-open but not nano semi *α-open.

Remark 3.34: The concept nano semi$^*\alpha$-open and nano θ-open sets are independent as shown in the following example.

Example 3.35: Let $U = \{a,b,c,d\}$ $U/R = \{\{a\},\{d\},\{b,c\}\}$. Let $X = \{a,c\}$. $\tau_R(X) = \{U,\emptyset,\{a\},\{b,c\},\{a,b,c\}\}$.$N_o\,S^*\alpha\,O(U,\tau_R(X)) = \{U,\emptyset,\{a\},\{b,c\},\{a,b,c\}\}$ and $N_o\,\theta O(U,\tau_R(X)) = \{U,\emptyset,\{d\},\{a,d\},\{b,c,d\}\}$.Clearly the subsets $\{a\},\{b,c\},\{a,b,c\}$ are nano semi$^*\alpha$-open but not nano θ-open and the subsets $\{d\},\{a,d\},\{b,c,d\}$ are nano θ-open but not semi$^*\alpha$-open.

Diagram 3.36: From the above discussions we have the following diagram.



4. Nano semi* α - closed sets

Definition 4.1: The complement of nano semi*α-open set is called as **nano semi*α-closed**. The collection of all nano semi*α-open sets is denoted by $\boldsymbol{N_o\, S^*\alpha\, C(U, \tau_R(X))}$.

Example 4.2: Let $U = \{a, b, c\}$ with $U/R = \{\{a\}, \{b\}, \{c\}\}$.Let $X = \{b, c\}$. Then $\tau_R(X) = \{U, \emptyset, \{b, c\}\}$. The nano-closed sets are {U,$\emptyset$, $\{a\}$}. The nano generalized – closed sets are $\{U, \emptyset, \{a\}, \{a, b\}, \{a, c\}\}$. The nano generalized open sets are $\{U, \emptyset, \{b\}, \{c\}, \{b, c\}\}$. $N_o\, S^*\alpha\, C(U, \tau_R(X)) = \{U, \emptyset, \{a\}\}$.

Theorem 4.3: Arbitrary intersection of nano semi*α-closed sets is nano semi*α- closed.
Proof: Let $\{A_\alpha\}$ be a collection of nano semi*α-closed sets in U. Since each A_α is nano semi*α- closed,$U\backslash A_\alpha$ is a nano semi*α-open. Since $U\backslash(\cap A_\alpha) = \cup (U\backslash A_\alpha)$ and hence by thm 3.4, $U\backslash(\cap A_\alpha)$is nano semi*$\alpha$-open. Hence $\cap A_\alpha$ is nano semi*α-closed.

Remark 4.4: Union of two nano semi*α-closed sets need not be nano semi*α-closed as shown in the following example.

Example 4.5: Consider $U = \{a, b, c, d\}$, U/R = $\{\{a\}, \{c\}, \{b, d\}\}$. $X = \{a, b\}$. Then $\tau_R^c(X) = \{U, \emptyset, \{c\}, \{a, c\}, \{b, c, d\}\}$ and $N_o\, S^*\alpha\, C(U, \tau_R(X)) = \{ U, \emptyset, \{a\}, \{c\}, \{a, c\}, \{b, d\}, \{b, c, d\}\}$. The sets $\{a\}$ and $\{b, d\}$ are nano semi*α-closed but their union $\{a\} \cup \{b, d\} = \{a, b, d\}$ is not nano semi*α-closed.

Theorem 4.6: In any nano topological space.
(i)Every nano α-closed set is nano semi*α-closed.
(ii) Every nano-closed set is nano semi*α-closed.
(iii) Every nano semi*-closed set is nano semi*α-closed.
(iv) Every nano semi*α-closed set is nano semi α-closed.
(v) Every nano semi*α- closed set is nano semi pre-closed.
(vi) Every nano regular closed set is nano semi*α-closed.

(vii) Every nano regular*- closed set is nano semi*α-closed.
(viii) Every nano semi*α- closed set is nano pre *-closed.
Proof: (i)Let A be any nano α-closed set in U, then U\A is nano α-open. By theorem 3.8, U\A is nano semi*α-open. Hence A is nano semi*α-closed. (ii) Let A be any nano-closed set in U. Then U\A is nano open. By theorem 3.11, U\A is nano semi*α-open. Hence A is nano semi*α-closed. (iii)Let A be any nano semi*-closed set in U, then U\A is nano semi*-open. By theorem 3.14, U\A is nano semi*α-open. Hence A is nano semi*α-closed.(iv) Let A be a nano semi*α-closed set in U. Then U\A is nano semi*α-open. By theorem 3.16, U\A is nano semi α-open. Hence A is nano semi α-closed. (v) Let A be a nano semi*α-closed set in U, then U\A is nano semi*α-open. By theorem 3.20, U\A is nano semi pre-open. Hence A is nano semi pre-closed. (vi) Let A be a nano regular closed set in U. Then U\A is nano regular open. By theorem 3.23, U\A is nano semi*α-open. Hence A is nano semi*α-closed. (vii) Let A be a nano regular*-closed set in U. Then U\A is nano regular*-open. By theorem 3.26, U\A is nano semi*α- open .Hence A is nano semi*α-closed .(viii) Let A be a nano semi*α-closed set in U. Then U\A is nano semi*α-open set. By theorem 3.28, U\A is nano pre*-open. Hence A is nano pre*-closed.

Remark 4.7: The converse of each of the statements in theorem 4.6 is not true as shown in the following examples.

Example 4.8: Let $U = \{a, b, c, d\}$ with U/R={$\{a\}, \{c\}, \{b, d\}$}.Let $X = \{a, b\}$.Then $\tau_R^c(X) = \{U, \emptyset, \{c\}, \{a, c\}, \{b, c, d\}\}$ and $N_o\, S^*\alpha\, C\big(U, \tau_R(X)\big) = \{U, \emptyset, \{a\}, \{c\}, \{a, c\}, \{b, d\}, \{b, c, d\}\}$ and $N_o\, \alpha\, C\big(U, \tau_R(X)\big) = \{U, \emptyset, \{c\}, \{a, c\}, \{b, c, d\}\}$.Clearly the subsets $\{a\}$and$\{b, d\}$ are nano semi*α-closed but not nano α closed .

Example 4.9: Let $U = \{a, b, c\}$ with U/R = $\{\{a\}, \{b\}\}$. Let X = $\{a, c\}$.Then $\tau_R{}^c(X) = \{U, \emptyset, \{b, c\}\}$and $N_o\, S^*\alpha\, C\big(U, \tau_R(X)\big) = \{U, \emptyset, \{b\}, \{c\}, \{b, c\}\}$.Clearly the subsets $\{b\}, \{c\}$ nano semi*α-closed but not nano-closed.

Example 4.10: Let $U = \{a, b, c, d\}$ with U/R = $\{\{a\}, \{b\}\}$. Let X =$\{a, c, d\}$. Then $\tau_R{}^c(X) = \{U, \emptyset, \{b, c, d\}\}$ and $N_o\, S^*\alpha\, C\big(U, \tau_R(X)\big) = \{U, \emptyset, \{b\}, \{c\}, \{d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{b, c, d\}\}$. $N_o S^* C(U, \tau_R(X)$={ $U, \emptyset, \{b, c, d\}$}. Clearly the subsets $\{b\}, \{c\}, \{d\}, \{b, c\}, \{b, d\}, \{c, d\}$ are nano semi*α-closed but not nano semi*-closed.

Example 4.11: Let $U = \{a, b, c, d\}$, $U/R = \{\{a\}, \{d\}, \{b, c\}\}$ $X = \{a, c\}$.Then $\tau_R{}^c(X) = \{U, \emptyset, \{a, d\}, \{b, c, d\}\}$ and $N_o\, S^*\alpha C\big(U, \tau_R(X)\big) = \big\{U, \emptyset, \{d\}, \{a, d\}, \{b, c, d\}\big\}$ $N_o\, S\, \alpha\, C\big(U, \tau_R(X)\big) = \{U, \emptyset, \{a\}, \{d\}, \{a, d\}, \{b, c\}, \{b, c, d\}\}$.Clearly the subsets $\{a\}, \{b, c\}$ are nano semi α- closed but not nano semi*α-closed .

Example 4.12: Let $U = \{a, b, c\}$ with $U/R = \{\{a\}, \{b\}, \{c\}\}$.Let $X = \{b, c\}$. Then $\tau_R^c(X) = \big\{U, \emptyset, \{a\}\big\}$ and $N_o\, S^*\alpha\, C\big(U, \tau_R(X)\big) = \{U, \emptyset, \{a\}\}$. $N_o SPC\big(U, \tau_R(X)\big) = \{U, \emptyset,$

$\{a\},\{b\},\{c\},\{a,c\},\{a,b\}\}$. Clearly the subsets {b}, {c} $\{a,c\},\{a,b\}$are nano semi pre-closed but not nano semi* α-closed.

Example 4.13: Let $U=\{a,b,c\}$ with $U/R=\{\{a\},\{b\},\{c\}\}$.Let $X = \{b,c\}$. Then $\tau_R^c(X)=\{U,\emptyset,\{a\}\}$, $N_o\,S^*\alpha\,C(U,\tau_R(X)) = \{U,\emptyset,\{a\}\}$.$N_o\,R\,C(U,\tau_R(X))=\{U,\emptyset\}$. Clearly the subset $\{a\}$ is nano semi* α-closed but not nano regular-closed.

Example 4.14: Let $U=\{a,b,c\}$ withU/R= $\{\{a\},\{b\},\{c\}\}$.Let $X = \{b,c\}$. Then $\tau_R^c(X)=\{U,\emptyset,\{a\}\}$, $N_o\,S^*\alpha\,C(U,\tau_R(X))=\{U,\emptyset,\{a\}\}$.$N_o\,R^*\,C(U,\tau_R(X))=\{U,\emptyset\}$ Clearly the subset $\{a\}$is nano semi* α- closed but not nano regular*- closed.

Example 4.15: Let $U=\{a,b,c\}$ with $U/R=\{\{a\},\{b\},\{c\}\}$.Let $X = \{b,c\}$. Then $\tau_R^c(X)=\{U,\emptyset,\{a\}\}$, $N_o\,S^*\alpha\,C(U,\tau_R(X))=\{U,\emptyset,\{a\}\}$. $N_o\,P^*C(U,\tau_R(X))=\{U,\emptyset,$ $\{a\},\{b\},\{c\},\{a,c\},\{a,b\}\}$. Clearly the subsets $\{b\},\{c\},\{a,c\},\{a,b\}$ are nano pre* -closed but not nano semi* α-closed.

Remark 4.16: The concept of nano semi* α-closed sets and nano pre-closed sets are independent as shown in the following example.

Example 4.17: Let $U=\{a,b,c,d,e\}U/R=\{\{a\},\{d\}\{b,c\}\}$. Let X = $\{a,c\}$. Then $\tau_R{}^c(X)$=$\{U,\emptyset,\{d,e\},\{a,d,e\},\{b,c,d,e\}\}$. $N_o\,S^*\alpha\,C(U,\tau_R(X))$ =$\{U,\emptyset,\{d\},\{e\},\{d,e\},$ $\{a,d,e\},\{b,c,d\},\{b,c,d,e\}\}$. $N_o\,P\,C(U,\tau_R(X))=\{U,\emptyset,\{b\},\{c\},\{d\},\{e\},\{b,d\},$ $\{b,e\}$,{c,d},$\{c,e\},\{d,e\},\{a,d,e\},\{b,c,e\},\{b,d,e\},\{c,d,e\},\{a,b,d,e\},\{a,c,d,e\}$ $\{b,c,d,e\}\}$. Clearly the subset $\{b,c,d\}$ is nano semi* α-closed but not nano pre closed and the subsets $\{b\},\{c\}$, $\{b,d\},\{b,e\}$,{c,d},$\{c,e\}$, $\{b,c,e\}$,$\{b,d,e\}$, $\{c,d,e\}$, $\{a,b,d,e\}$, $\{a,c,d,e\}$ are nano pre- closed but not nano semi* α-closed.

Remark 4.18: The concept of nano semi* α-closed sets and nano θ- closed sets are independent as shown in the following example.

Example 4.19: Let U=$\{a,b,c,d\}$,U/R = $\{\{a\},\{d\},\{b,c\}\}$. Let X= $\{a,c\}$. $\tau_R{}^c\,(X)$ = {U,∅, {d},$\{a,d\},\{b,c,d\}\}$. Then $N_o\,S^*\,\alpha\,C(U,\tau_R(X))$=$\{U,\emptyset,\{d\},\{a,d\},\{b,c,d\}\}$. $N_o\,\theta C(U,\tau_R(X))=\{U,\emptyset,\{a\},\{b,c\},\{a,b,c\}\}$.Clearly the subsets $\{d\},\{a,d\},\{b,c,d\}$ are nano semi* α-closed but not nano θ- closed and the subsets $\{a\},\{b,c\},\{a,b,c\}$ are nano θ-closed but not semi* α-closed.

5.Nano semi* α- interior and nano semi* α-closure

Definition 5.1: The **nano semi*α- interior** of A is defined as the union of all nano semi*****α - open sets contained in A. It is denoted by s*αN_oInt(A).

Definition 5.2: Let A be a subset of U. A point u in U is called a **nano semi*α-interior point** of A if A contains a nano semi* α-open set containing u.

Theorem 5.3: If A is any subset of a nano topological space $(U, \tau_R(X))$,then
(i) s*αN_oInt(A) is the largest nano semi*α-open set contained in A.
(ii)A is nano semi*α-open if and only if s*αN_oInt(A)=A.
Proof: (i) Being the union of all nano semi*α-open subsets of A, by theorem 3.4, s*αN_oInt(A) is nano semi*α-open and contains every nano semi*α-open subsets of A. (ii) A is nano semi*α-open implies s*αN_oInt(A)=A is obvious from definition 5.1. On the other hand, suppose s*αN_oInt(A)=A. Hence by (i) s*αN_oInt(A) is nano semi*α-open and hence A is nano semi*α-open.
Theorem 5.4: In any nano topological space (U,τ_R(X), if A and B are subsets of U, then the following results hold:
i) s*αN_oInt (∅)=∅
ii) s*αN_oInt(U)=U
iii) s*αN_oInt(A)⊆A
iv) A⊆B⇒ s*αN_oInt(A)⊆s*αN_oInt(B)
v) s*αN_oInt(s*αN_oInt(A)) = s*αN_oInt(A).
vi) N_oInt(A)⊆s*αN_oInt(A)⊆sαN_oInt(A)⊆A
vii) s*αN_oInt(A∪ B)⊇s*αN_oInt(A)∪s*αN_oInt(B)
viii) s*αN_oInt(A∩B) ⊆s*αN_oInt(A)∩s* αN_oInt(B)
Proof: (i), (ii), (iii) and (iv) follows from definition 5.1. By theorem 5.3(i), s*αN_oInt(A) is nano semi*α-open and by theorem 5.3(ii), s*αN_oInt(s*αN_oInt(A)) = s*αN_oInt(A). Thus (v) proved. (vi) follows from theorem 3.11 and 3.17. (vii)Since A⊆A∪B, from statement (iv) we have s*αN_oInt(A) ⊆ s*αN_oInt(A∪B). Similarly, s*αN_oInt(B) ⊆ s*αN_oInt(A∪B). Then s*αN_oInt(A∪ B) ⊇ s*αN_oInt(A) ∪ s*αN_oInt(B).
(viii) Since $A \cap B \subseteq A$,from statement (iv) we have s*αN_oInt($A \cap B$) ⊆ s*αN_oInt(A).Similarly s*αN_oInt($A \cap B$) ⊆ s*αN_oInt(B). Therefore s*αN_oInt($A \cap B$) ⊆ s*αN_oInt(A)∩ αN_oInt(B).

Remark 5.5: In Theorem 5.4(vi), each of the inclusions may be strict and equality may also hold. This can be seen from the following examples:

Example 5.6: Let $U = \{a,b,c,d,e\}$,$U\backslash R = \{\{a\},\{d\},\{b,c\}\}$,$X = \{a,c\}$.Then $\tau_R(X) = \{U,\emptyset,\{a\},\{b,c\},\{a,b,c\}\}andN_o\, S^*\alpha\, O(U,\tau_R(X)) = \{U,\emptyset,\ \{a\},\ \{a,\ e\},\ \{b,\ c\},$ $\{a,\ b,\ c\},\{a,b,c,d\},\{a,b,c,e\}\}$ Let $A = \{b,c\}$.Then $N_o\, int\, (A) =$ $\mathrm{s}^*\alpha\, N_o\, \mathrm{int(A)} = \mathrm{s}\,\alpha\, N_o\, \mathrm{int(A)} = \{\mathrm{b,c}\} = \mathrm{A}$. Let $B = \{a,e\}$.Then N_o int (B)=$\{a\}$, $\mathrm{s}^*\alpha\, N_o\, \mathrm{int(B)} = \{a,e\}$, $\mathrm{s}\,\alpha\, N_o\, \mathrm{int(B)} = \{a,e\}$. Here $N_o\ int(B) \subsetneq \mathrm{s}^*\alpha\, N_o\, \mathrm{int(B)} = \mathrm{s}\,\alpha\, N_o\, \mathrm{int(B)} = \mathrm{B}$. Let $C = \{b,d\}$. Then $N_o\, int\, (C) = \emptyset$,$\mathrm{s}^*\alpha\, N_o\, \mathrm{int(C)} = \emptyset$, $\mathrm{s}\,\alpha\, N_o\, \mathrm{int(C)} = \emptyset$. Let $D = \{a,c,d,e\}$,Then $N_o\, int\, (D) = \{a\}$,$\mathrm{s}^*\alpha\, N_o\, \mathrm{int(D)} = \{a,e\}$,$\mathrm{s}\,\alpha\, N_o\, \mathrm{int(D)} = \{a,d,e\}$. Here $N_o\, int\, (D) \subsetneq \mathrm{s}^*\alpha\, N_o\, \mathrm{int(D)} \subsetneq \mathrm{s}\,\alpha\, N_o\, \mathrm{int(D)} \subsetneq \mathrm{D}$. Let $E = \{b,c,d,e\}$.Then $N_o\, int\, (E) = \{b,c\}$,$\mathrm{s}^*\alpha\, N_o\, \mathrm{int(E)} = \{\mathrm{b,c}\}$,$\mathrm{s}\,\alpha\, N_o\, \mathrm{int(E)} = \{\mathrm{b,c,d,e}\}$. Here $N_o\, int\, (E) = \mathrm{s}^*\alpha\, N_o\, \mathrm{int(E)} \subsetneq \mathrm{s}\,\alpha\, N_o\, \mathrm{int(E)} = \mathrm{E}$.

Remark 5.7: In Theorem 5.5(vii) and (viii), each of the inclusions may be strict and equality may also hold. This can be seen from the following examples:

Example 5.8: Let $U = \{a,b,c,d\}$,$U \setminus R = \{\{a\},\{c\},\{b,d\}\}$,X=$\{a,b\}$. Then $\tau_R(X) = \{U,\emptyset,\{a\},\{b,d\},\{a,b,d\}\}$ and $N_o\,S^*\,\alpha O\,(U,\tau_R(X))$ = $\{U,\emptyset,\{a\},\{a,c\},\{b,d\},$ $\{a,b,d\},\{b,c,d\}\}$.
Let$A = \{a,b\}$, B = $\{b,d\}$.Then $A \cup B = \{a,b,d\}$ and $s^*\alpha N_o\text{int}(A) = \{a\}$, $s^*\alpha\,\text{Nint}(B) = \{b,d\}$, $s^*\alpha N_o\text{int}(A \cup B) = \{a,b,d\}$. Therefore $s^*\alpha N_o\text{int}(A) \cup s^*\alpha N_o\text{int}(B) = \{a,b,d\} = s^*\alpha N_o\text{int}(A \cup B)$.
LetC= $\{a,b\}$,$D = \{a,d\}$. Then $C \cup D = \{a,b,d\}$ and $s^*\alpha N_o\text{int}(C) = \{a\}$, $s^*\alpha N_o\text{int}(D) = \{a\}$, $s^*\alpha N_o\text{int}(C \cup D) = \{a,b,d\}$
Therefore $s^*\alpha N_o\text{int}(C) \cup s^*\alpha N_o\text{int}(D) \subsetneq s^*\alpha N_o\text{int}(C \cup D)$.
Let $E = \{a,b,d\}$,$F = \{a,c,d\}$.Then $E \cap F = \{a,d\}$ and $s^*\alpha N_o\text{int}(E) = \{a,b,d\}$, $s^*\alpha N_o\text{int}(F) = \{a,c\}$,$s^*\alpha N_o\text{int}(E \cap F) = \{a\}$.
Therefore $s^*\alpha N_o\text{int}(E \cap F) = s^*\alpha N_o\text{int}(E) \cap s^*\alpha N_o\text{int}(F)$.
Let G= $\{a,b,c\}$, $H = \{b,c,d\}$.Then $G \cap H = \{b,c\}$ and $s^*\alpha N_o\text{int}(G) = \{a,c\}$, $s^*\alpha N_o\text{int}(H) = \{b,c,d\}$, $s^*\alpha N_o\text{int}(G \cap H) = \emptyset$.
Therefore $s^*\alpha N_o\text{int}(G \cap H) \subsetneq s^*\alpha N_o\text{int}(G) \cap s^*\alpha N_o\text{int}(H)$.

Definition 5.9: If A is a subset of a nano topological space U, the Nano semi$^*\alpha$-closure of A is defined as the intersection of all nano semi$^*\alpha$-closed sets in U containing A. It is denoted by $s^*\alpha\,\text{Ncl}(A)$.

Theorem 5.10: If A is any subset of a nano topological space (U, $\tau_R(X)$) ,then
(i)$s^*\alpha\,\text{Ncl}(A)$is the smallest nano semi*α-closed set in U containing A.
(ii)A is nano semi*α-closed if and only if $s^*\alpha N_o\text{cl}(A) = A$.
Proof: (i) Since $s^*\alpha\,\text{Ncl}(A)$ is the intersection of all nano semi* α-closed subsets of U containing A, by theorem 4.3, it is nano semi* α-closed and it is contained in every nano semi*α-closed set containing A and hence it is the smallest nano semi*α-closed set in U containing A. (ii)If A is nano semi*α-closed, then $s^*\alpha N_o\text{cl}(A) = A$ is obvious. Conversely, let $s^*\alpha N_o\text{cl}(A) = A$, By (i) $s^*\alpha N_o\text{cl}(A)$ is nano semi*α-closed and hence A is nano semi*α-closed.

Theorem 5.11: In any nano topological space (U,τ_R(X), if A and B are subsets of U, then the following results hold:
(i)s*αN_ocl(∅)=∅
(ii)s*αN_ocl(U)=U
(iii)A⊆s*αN_ocl(A)
(iv)A⊆B⇒s*αN_ocl(A)⊆s*αN_ocl(B)
(v)s*$\alpha N_o cl$(s*$\alpha N_o cl$(A))= s*$\alpha N_o cl$(A).
(vi)A⊆sαN_ocl(A)⊆s*αN_ocl(A)⊆ αN_ocl(A)
(vii)s*αN_ocl(A∪ B)⊇s*αN_ocl(A) ∪ s*αN_ocl(B)
(viii)s*αN_ocl(A∩B)⊆s*αN_ocl(A)∩s*αN_ocl(B)

Proof: (i), (ii), (iii) and (iv) follows from definition 5.7.
From theorem 5.10(i) s*αN_ocl(A) is the nano semi*α-closed and from theorem5.10(ii) s*$\alpha N_o cl$(s*$\alpha N_o cl$(A))= s*$\alpha N_o cl$(A).This proves (v).
(vi) follows from theorem 4.6 and 4.9. (vii) Since $A \subseteq A \cup B$, from statement (iv) we have s*Ncl(A) ⊆ s*αNcl(A∪B). Similarly, s*αN_ocl(B) ⊆ s*αN_ocl(A∪ B). Then s*αN_ocl(A∪B) ⊇s*αN_ocl(A) ∪ s*αN_ocl(B)
(viii) Since $A \cap B \subseteq A$,from statement (iv) we have s* αN_ocl($A \cap B$) ⊆ s* αN_ocl(A).Similarly s* αN_ocl($A \cap B$) ⊆ s* αN_ocl(B). Therefore s* αN_ocl($A \cap B$) ⊆ s* αN_oInt(A)∩ s*αN_ocl(B).

Remark 5.12: In Theorem 5.11(vi), each of the inclusions may be strict and equality may also hold. This can be seen from the following examples:

Example 5.13: Let $U = \{a, b, c, d, e\}$, $U \backslash R = \{\{a\}, \{d\}, \{b, c\}\}$, $X = \{a, c\}$,
Then $\tau_R(X) = \{U \quad \emptyset, \{a\}, \{b, c\}, \{a, b, c\}\}$ and $N_o\, S^*\, \alpha C\, \left(U, \tau_R(X)\right) = \{U, \emptyset, \{d\}, \{e\}, \{d, e\}, \{a, d, e\}$, $\{b, c, d\}, \{b, c, d, e\}\}$. $N_o\, S\, \alpha C\, \left(U, \tau_R(X)\right) = \{U, \emptyset, \{a\}, \{a, e\}, \{b, c\}\, , \{a, b, c\}, \{a, b, c, d\}$, $\{a, b, c, e\}\}$.
Let $A = \{a, b, c\}$.Then s*αN_o cl(A)= sαN_o cl(A)= αN_o cl(A)=U.
Let $B = \{a, e\}$.Then s*αN_o cl(B)= $\{a, d, e\}$, sαN_o cl(B)= $\{a, e\}$,αN_o cl(A) $= \{a, \mathrm{d}, \mathrm{e}\}$
Here $B = \mathrm{s}\alpha N_o\, \mathrm{cl(B)} \subsetneq \mathrm{s} * \alpha N_o\, \mathrm{cl(B)} = \alpha N_o\, \mathrm{cl(A)}$.
Let$C = \{b, c, d\}$.Thens*αN_o cl(C)= $\{b, c, d\}$,sαN_o cl(C) $= \{\mathrm{b, c, d}\}$,αN_o cl(C) $=$ $\{\mathrm{b, c, d, e}\}$.
Here $C = \mathrm{s}\alpha N_o\, \mathrm{cl(C)} = \mathrm{s} * \alpha N_o\, \mathrm{cl(C)} \subsetneq \alpha N_o\, \mathrm{cl(C)}$.
Let $D = \{c, d, e\}$. Then s*αN_o cl(D)= $\{b, c, d, e\}$, sαN_o cl(D) $= b, c, d, e\}$, αN_o cl(A)= $\{\mathrm{b, c, d, e}\}$.
Here $D \subsetneq \mathrm{s}\alpha N_o\, \mathrm{cl(D)} = \mathrm{s} * \alpha N_o\, \mathrm{cl(D)} = \alpha N_o\, \mathrm{cl(D)}$.
Let E= $\{a\}$.Then s*αN_o cl(E)= $\{a, d, e\}$, sαN_o cl(E) $= \{a\}$, αN_o cl(E)= $\{a, \mathrm{d, e}\}$.
Here $E = \mathrm{s}\alpha N_o\, \mathrm{cl(E)} \subsetneq \mathrm{s} * \alpha N_o\, \mathrm{cl(E)} = \alpha N_o\, \mathrm{cl(E)}$.

6. Conclusions

In this article, we have introduced nano semi*α-open sets and nano semi *α-closed sets in nano topological spaces and studied their characterizations with other nano open sets. A diagramatic explanation gives a clear explanation of this article.

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