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Totally magic d-lucky number of graphs

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Abstract

In this paper we introduce a new labeling named as, totally magic d-lucky labeling, find the totally magic d-lucky number of some standard graphs like wheel, cycle, bigraph etc. and find the totally magic d-lucky number of some zero divisor graphs. A totally magic d-lucky labeling $t: V \to \{1,2,\ldots,p\}$ of a graph G = (V, E) is a labeling of vertices and label the graph's edges using the total label of its incident vertices in such a way that for any two different incident vertices u and v, their colors $d_t(u) = \sum_{v\in N(u)} t(v) +$ dgu, dtv= u∈Nvtu+ dg v are distinct and for any different edges in a graph, their weights $t(u) + t(v) + t(uv) \equiv 0 \pmod 2$ are same Where $d_g(u)$ represents the degree of u in a graph and $N(u)$ represents the open neighbourhood of u in a graph.

Keywords: Totally magic d-lucky labeling, totally magic d-lucky number, zero divisor graphs.

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1. Introduction

In [2], The idea of lucky labeling was first proposed by Czerwinski, Grytczuk, and Zelezny. In [1], The idea of "d-lucky labeling" was developed by Indira Rajasingh, D. Ahima Emilet, and D. Azhubha Jemilet. [1] Let l: V(G) → N is a vertex labeling. If for each pair of incident vertices of u and v, c(u) ≠ c(v) holds where c(u) = $d_g(u) + \sum_{v\in N(u)} l(v)$, c(v) = $d_g(u) + \sum_{u\in N(v)} l(u)$, $d_g(u)$ represents the degree of u and N(u) represents the open neighbourhood of the vertex u in a graph, then the labeling l is a d-lucky labeling. A graph's d-lucky number is the smallest value of labeling required to label the graph. Motivated by this labeling, we introduce Totally magic d-lucky labeling. A graph's total labeling is a mapping from the union of the vertex set and the edge set to positive integers. If the sum of the edge label and the label of the edge's end points has the same constant, the total labeling is said to be totally magic labeling. In [5,] we learned about the totally magic labeling. A totally magic d-lucky labeling $t: V \rightarrow \{1,2,\dots,p\}$ of a graph G = (V, E) is a labeling of vertices and label the edges of the graph by the sum of the labels of its incident vertices in such a way that for any two different incident vertices u and v, their colors $d_t(u) = \sum_{v\in N(u)} t(v) + d_g(u)$, $d_t(v) = \sum_{u\in N(v)} t(u) + d_g(v)$ are distinct and for any different edges in a graph, their weights $t(u) + t(v) + t(uv) \equiv 0 \pmod 2$ are same Where $d_g(u)$ represents the degree of u in a graph and $N(u)$ represents the open neighborhood of u in a graph.

2. Totally magic d-lucky labeling

In this section we introduce a new labeling named as the totally magic d-lucky labeling and apply it on the cycle, path, complete graph, bigraph, and wheel.

Definition 2.1 Define t: V(G) → {1, 2, …, p} and label the edges of E(G) as the label of the edge's incident vertices added together. The labeling is said to be Totally magic d-lucky labeling if $d_t(u) \neq d_t(v)$ and t(u) + t(v) + t(uv) ≡ 0 (mod 2) where u, v ∈ V(G) $d_t(u) = \sum_{v\in N(u)} t(v) + d_G(u)$ and $d_t(v) = \sum_{u\in N(v)} t(u) + d_G(v)$. The totally magic d-lucky number of G, tdln(G) is defined as the lowest value of p for which the graph G has totally magic d-lucky labeling.

Theorem 2.2. For a cycle graph C_n, tdln (C_n) = $\begin{cases} n & \text{if } n \text{ is odd} \\ 2 & \text{otherwise} \end{cases}$

Proof. Let G be the cycle graph.
Let V(G) = {v_i: 1≤ i ≤ n} and E(G) = {$v_i v_{i+1}$: 1≤ i ≤ n-1} ∪ {$v_n v_1$}
Case. (i).
When n ≡ 1 (mod 2).
Let t: V(C_n)→{1,2,...,p} defined by for 1≤i≤n-1
t(v_i)=i for 1≤ i ≤ n
Then the induced edge labelling is,
t($v_i v_{i+1}$) =2i+1 for 1≤ i ≤ n-1

$t(v_n v_1) = n+1$
We observe that,
$d_t(v_1) = n+4$
$d_t(v_i) = 2i+2$, for $2 \le i \le n-1$
$d_t(v_n) = n+2$
$d_t(v_i) \neq d_t(v_{i+1})$ and $d_t(v_1) \neq d_t(v_n)$
$t(v_i) + t(v_{i+1}) + t(v_i v_{i+1}) = 4i+2 \equiv 0 \pmod 2$ for $2 \le i \le n-1$ and
$t(v_1) + t(v_n) + t(v_1 vn) = 2n+2 \equiv 0 (mod2)$

case (ii).
When $n \equiv 0 \pmod 2$
Define $t: V(C_n) \to \{1,2, \ldots, p\}$ as follows,
for $1 \le i \le n$

$$t(v_i) = \begin{cases} 1, i \equiv 1 \pmod 2 \\ 2, i \equiv 0 \pmod 2 \end{cases}$$

Then the induced edge labelling is,
$t(v_i v_{i+1}) = 3$ for $2 \le i \le n-1$
$t(v_n v_1) = 3$
we observe that,
$d_t(v_i) = 6$ if i is odd
$d_t(v_i) = 4$ if i is even
$d_t(v_i) \neq d_t(v_{i+1})$
and $t(v_i) + t(v_{i+1}) + t(v_i v_{i+1}) = 6 \equiv 0 \pmod 2$
It can be easily verified that weights of the incident vertices are pair wise distinct and have the common totally magic d-lucky constant for its edges. Thus, the totally magic d-lucky number of cycle graph is 2. ∎

Theorem 2.3 Every path P_n has $tdln(P_n)=2$
Proof Let P_n be the path graph, $V(P_n)=\{v_i: 1 \le i \le n\}$ and
$E(P_n) = \{v_i v_{i+1}: \text{for } 1 \le i \le n\}$
Define $t: V(P_n) \to \{1,2,\ldots,p\}$ as follows:

$$t(v_i) = \begin{cases} 1, i \equiv 1 \pmod 2 \\ 2, i \equiv 0 \pmod 2 \end{cases}$$

Then the induced edge labelling is,
$t(v_i v_{i+1}) = 3$ for all edges in P_n
we observe that,
when n is even,
$d_t(v_1) = 3$
$d_t(v_i) = 4$ if $i \equiv 0 (mod 2)$
$d_t(v_i) = 6$ if $i \equiv 1 (mod 2)$
$d_t(v_n) = 2$
$d_t(v_i) \neq d_t(v_{i+1})$ for all i
and $t(v_i) + t(v_{i+1}) + t(v_i v_{i+1}) = 6 \equiv 0 (mod 2)$.
When n is odd
$d_t(v_1) = 3 = d_t(v_n)$

$d_t(v_i) = 4$ if $i \equiv 0 \pmod 2$
$d_t(v_i) = 6$ if $i \equiv 1 \pmod 2$
$d_t(v_i) \neq d_t(v_{i+1})$ for all i
and $t(v_i) + t(v_{i+1}) + t(v_iv_{i+1}) = 6 \equiv 0 \pmod 2$.
Hence tdln $(P_n) = 2$. ■

Theorem 2.4 For a complete graph k_n, tdln $(k_n) = n$
Proof In complete graph K_n, Each and every pair of vertices are close together.
Define $t:V(K_n) \rightarrow \{1,2,\ldots,p\}$ as follows:
$t(v_i) = i : 1 \leq i \leq n$
Then the induced edge labelling is,
$t(v_iv_j) = i+j$ for all edges in P_n
we observe that,
for $1 \leq i \leq n$
$d_t(v_i) = \frac{(n^2+3n-2i-2)}{2}$
$d_t(v_i) \neq d_t(v_j)$
and $t(v_i) + t(v_j) + t(v_iv_j) = 2(i+j) \equiv 0 \pmod 2$.
tdln $(K_n) = n$. It is simple to confirm that the colors of the pair wise incident vertices are distinct and that the sum of the labels for each edge and the incident vertices of its edges is even. ■

Theorem 2.5 For a bigraph $K_{m,n}$, tdln $(K_{m,n}) = 1$.
Proof A bigraph's vertices can be divided into two separate subsets, V_1 and V_2, and each edge of the bigraph connects a point on each subset. $K_{m,n}$ indicates a bigraph.
Let $V(K_{m,n}) = V_1 \cup V_2$ where $V_1 = \{u_1, u_2, \ldots, u_m\}$ and $V_2 = \{v_1, v_2, \ldots, v_n\}$ and $E(K_{m,n}) = \{u_iv_j : u_i \in V_1\ v_j \in V_2, 1 \leq i \leq m, 1 \leq j \leq n\}$.
Define $t: V(K_{m,n}) \rightarrow \{1, 2, \ldots, p\}$ as follows: $t(u_i) = 1$, $t(v_j) = 1$
Then the induced edge labeling is
$t(u_iv_j) = 2$ for all edges in $K_{m,n}$
We observe that,
$d_t(u_i) = 2n$,
$d_t(v_j) = 2m$,
$t(u_i) + t(v_j) + t(u_iv_j) = 2(i+j) \equiv 0 \pmod 2$.
It is obvious that all incident vertices have pair wise different colors and that all of the edges in the $K_{m,n}$ graph have the same totally magic d-lucky constant.
Hence tdln $(K_{m,n}) = 1$. ■

Theorem 2.6 For a wheel graph W_n, tdln $(W_n) = \begin{cases} n-1 & \text{if n is odd} \\ n & \text{otherwise} \end{cases}$.
Proof A wheel graph is obtained by joining a vertex to all the vertices of a cycle graph. It is denoted by W_n for n>3, where n is the number of vertices in the graph.
Let $V(W_n) = \{u_i : 1 \leq i \leq n\}$ and $E(W_n) = \{u_1u_i : 2 \leq i \leq n\} \cup \{u_iu_{i+1} : 2 \leq i \leq n\}$
Case(i) When $n \equiv 1 \pmod 2$

Define a labeling $t:V(W_n) \rightarrow \{1,2,\ldots,p\}$ as follows:
$t(u_1) = 1$, $t(u_i) = i-1$ for $2 \leq i \leq n$
Then the induced edge labelling is
$t(u_1u_i) = i$ for $2 \leq i \leq n$
We observe that,
$d_t(u_1) = \frac{n^2+n-2}{2}$
$d_t(u_2) = n+5$
$d_t(u_i) = 2i+2$ for $3 \leq i \leq n-1$
$d_t(u_n) = n+3$
$d_t(u_1) \neq d_t(u_i)$ for $2 \leq i \leq n$
$d_t(u_i) \neq d_t(u_{i+1})$ for $2 \leq i \leq n-1$
$d_t(u_n) \neq d_t(u_1)$
$t(u_1)+t(u_i)+t(u_1u_i) = 2i \equiv 0 \pmod 2$;
$t(u_i)+t(u_{i+1})+t(u_iu_{i+1}) = 4i-2 \equiv 0 \pmod 2$, for $2 \leq i \leq n-1$;
$t(u_1)+t(u_n)+t(u_n) = 2n \equiv 0 \pmod 2$
Hence tdln $(W_n) = n-1$

Case (ii) When $n \equiv 0 \pmod 2$
Define $t:V(W_n) \rightarrow \{1,2,\ldots,n\}$ as follows:
$t(u_i) = i$ for $1 \leq i \leq n$
Then the induced edge labelling is
$t(u_1u_i) = 1 + i$ for $2 \leq i \leq n$;
$t(u_iu_{i+1}) = 2i+1$ for $2 \leq i \leq n-1$
$t(u_nu_2)=n+2$
we observe that,
$d_t(u_1) = \frac{n^2+3n-4}{2}$
$d_t(u_2) = n+7$
$d_t(u_i) = 2i+4$ for $3 \leq i \leq n-1$
$d_t(u_n) = n+5$
$d_t(u_1) \neq d_t(u_i)$ for $2 \leq i \leq n$
$d_t(u_i) \neq d_t(u_{i+1})$ for $2 \leq i \leq n$
$d_t(u_n) \neq d_t(u_1)$
for $2 \leq i \leq n$
$t(u_1) + t(u_{i+1}) + t(u_1u_{i+1}) = 2i+2 \equiv 0 \pmod 2$,
$t(u_i) + t(u_{i+1}) + t(u_iu_{i+1}) = 4i+2 \equiv 0 \pmod 2$,
$t(u_2) + t(u_n) + t(u_2u_n) = 2n+4 \equiv 0 \pmod 2$.
Hence tdln $(W_n)= n$
It is simple to confirm that all incident vertices' colors are pairwise different and preserve the totally magic d-lucky constant for all of the graph's edges W_n. ■

3. Totally magic d-lucky number of some zero divisor graphs

In this part, the totally magic d-lucky number of some zero divisor graphs is examined.

Theorem. 3.1 For $R = Z_k$, $k = mn$, $m=2,3$ and $n>3$ be a prime number, $tdln(\Gamma(R)) = 1$
Proof Consider $G_0=\Gamma(R)$ where $R=Z_k$, $k=mn$
Case(i)
when $m=2, n>3$ be a prime. By the definition of zero divisor graph,
Assume $V(G_0) = \{2, 4... 2(n-1), n\} = \{v_i: 1 \leq i \leq n\}$, $E(G_0) = \{v_i v_n: v_i \in V(\Gamma(R)) - \{v_n\}\}$.
We have $d_g(v_i) = m-1$, $d_g(v_n) = n-1$, $1 \leq i \leq 2(n-1)$
Define $t:V(G_0) \rightarrow \{1,2,\dots,p\}$ as follows:
$t(v_i) = 1$ for $1 \leq i \leq n$
Then the induced edge labeling is,
$t(e) = 2$ for all edges e in G_0
we observe that,
$d_t(v_i) = 2m-2$,
$d_t(v_n) = 2n-2$
$d_t(v_i) \neq d_t(v_n)$ for $1 \leq i \leq n-1$ and
$t(v_i) + t(v_n) + t(v_i v_n) = 4 \equiv 0 \pmod 2$
Hence $tdln(G_0) = 1$
Case(ii)
when $m=3$, $n>3$ be a prime.
In this graph, we have $V(G_0) = V_1(G_0) \cup V_2(G_0)$ where $V_1(G_0) = \{n, 2n\}$,
$V_2(G_0) = \{3i: 1 \leq i \leq n-1\}$ and $E(G_0) = \{uv: u \in V_1(G_0), v \in V_2(G_0)\}$.
Hence $d_g(u) = n-1$ for all $u \in V_1(G_0)$, $d_g(v) = m-1$, for all $v \in V_2(G_0)$ and $|E(G_0)| = 2n-2$
Define a labeling $t: V(G_0) \rightarrow \{1,2,\dots,p\}$ as follows:
$t(u) = 1$ for all $u \in V_1(G_0)$
$t(v) = 1$ for all $v \in V_2(G_0)$
Then the induced edge labeling is,
$t(uv) = 2$ for all $uv \in E(G_0)$
We observe that,
$d_t(u) = 2n-2$,
$d_t(v) = 2m-2$
$d_t(u) \neq d_t(v)$ for all $u \in V_1$, for all $v \in V_2$ and
$t(u) + t(v) + t(uv_i) = 4 \equiv 0 \pmod 2$ for all $uv \in E(G_0)$
Hence $tdln(G_0) = 1$. ■

Theorem 3.2 For $R = Z_k$, $k = m^2 n$, $n > 3$ be a prime number,

$$tdln(\Gamma(R)) = \begin{cases} 1 \text{ when } m = 2 \\ 2 \text{ when } m = 3 \end{cases}$$

Proof Assume $G_0 = \Gamma(R)$

Case(i) When m=2, In this case(G_0) has partitioned into two sets $V_1(G_0)$,$V_2(G_0)$. $V_1(G_0)$ contains the multiples of n in Z_k, $V_2(G_0)$ contains the multiples of m excluding 2n in Z_k.
Let $V_1(G_0) = \{r_1, r_2, r_3\}$ and $V_2(G_0) = \{s_1, s_2, \ldots, s_{n-1}, s_{n+1}, \ldots, s_{2n-1}\}$
$|V(G_0)| = 2n+1$
$E(G_0) = \left\{r_i s_j : i \in \{1,3\}, s_j = \{4,8,\ldots,(4m-4)\}\right\} \cup \left\{r_2 s_j : \text{for all } s_j \in V_2(G_0)\right\}$.
$|E(G_0)| = 4n-4$
Hence $d_g(r_i) = n-1$, for $i \in \{1,3\}$;
$d_g(r_2) = 2n-2$;
$d_g(s_j) = m+1$, $s_j \in \{4,8,\ldots,4n-4\}$;
$d_g(s_j) = m-1$, $s_j \in V_2(G_0) - \{4, 8, \ldots, 4n-4\}$
Define a labeling $t: V(G_0) \rightarrow \{1, 2, \ldots, p\}$ as follows:
$t(r_i) = 1$ for $1 \le i \le 3$;
$t(s_j) = 1$ for $1 \le j \le 2n-2$;
Then the induced edge labelling is,
$t(r_i s_j) = 2$ for all $r_i s_j \in E(G_0)$
We observe that,
$d_t(r_i) = 2n-2$, $i \in \{1,3\}$,
$d_t(r_2) = 4n-4$,
$d_t(s_j) = 2m-2$, $s_j \in V(\Gamma(R))-\{4,8,\ldots,4n-4\}$,
$d_t(s_j) = 2m+2$, $s_j \in \{4,8,\ldots,4n-4\}$
$d_t(r_i) \neq d_t(s_j)$ for all$r_i \in V_1(G_0)$, $s_j \in V_2(G_0)$ and
$t(r_i) + t(s_j) + t(r_i s_j) = 4 \equiv 0 \pmod 2$ for all edges in G_0
Hence $tdln(G_0)=1$.
Case(ii) when m = 3,
In this case, the vertex set of G_0 partitioned into two sets V_1 and V_2.
Where$V_1 = \{n, 2n, 3n, 4n, 5n, 6n, 7n, 8n\} = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8\}$and
$V_2 = \{3, 6, 9, \ldots, 9n-3\} - \{n, 2n\} = \{v_1, v_2, v_3, \ldots, v_{3n-1}\}$.
$E(G_0) = \{u_i v_i$: for all $u_i \in V_1$, $v_i \in \{9,18,27,\ldots,9(n-1)\} \cup \{u_i v_i : u_i \in \{3n,6n\}\ v_i \in V_2\} \cup \{u_3,u_6\}$.
Hence $d_g(u_i) = n-1$ for all $u_i \in V_1-\{3n,6n\}$;
$d_g(u_i) = 3n-2$, $i=\{3,6\}$;
$d_g(v_i) = 8$ for all $v_i \in \{9,18,\ldots,9(n-1)\}$;
$d_g(v_i) = 2$, $v_i \in V_2-\{9,18,\ldots,9(n-1)\}$.
Define the labelling $t: V(G_0) \rightarrow \{1,2,\ldots,p\}$ as follows:
$t(u_i) = 1$, for $1 \le i \le 8, u_i \in V_1$;
$t(u_6) = 2$, $u_6 \in V_1$;
$t(v_i) = 1$, for all $v_i \in V_2$.
Then the induced edge labellings are,
$t(u_i v_i) = 2$ for all $u_i v_i \in E(G_0)$
$t(u_3 u_6) = 3$
$t(u_6 v_i) = 3$ for all $v_i \in V_2$
We observe that,
$d_t(u_i) = 2n-2$,

$d_t(u_3) = 6n-3$,
$d_t(u_6) = 6n-4$,
$d_t(v_i) = 5$ for all $v_i \in V_2(G_0)-\{9,8,\ldots,9(n-1)\}$
$d_t(v_i) = 17$, $v_i \in \{9,8,\ldots,9(n-1)\}$
$d_t(u_i) \neq d_t(v_i)$,
$d_t(u_3) \neq d_t(u_6)$ and
$t(u_i)+t(v_i)+t(u_iv_i) = 4 \equiv 0 \pmod 2$ for all edges in G_0
$t(u_3) + t(u_6) + t(u_3u_6) = 6 \equiv 0 \pmod 2$
$t(u_6) + t(v_i) + t(u_6v_i) = 3$ for all $v_i \in V_2$
It can be easily verified that weights of all the incident vertices are distinct and all the edges of the graph have common totally magic d-lucky constant.
Hence tdln $(G_0) = 2$. ■

Theorem 3.3 Let R = $\prod_{i=1}^{k} Z_{{m_i}^{n_i}}$ be a commutative ring with unity. For the zero-divisor graph Γ(R), tdln(Γ(R)) = M-1 where M = $(m_1, m_2, m_3, \ldots, m_k)$, m_i's are distinct prime numbers, n_i's are positive integers.
Proof Consider G_0 = Γ(R) be a zero-divisor graph of commutative ring R = $\prod_{i=1}^{k} Z_{{m_i}^{n_i}}$ where m_i's are prime numbers and n_i's are positive integers.
The vertex set of G_0 consists of different blocks**,**
$V(G_0) = \bigcup B_{x_1,x_2,\ldots,x_k}$
where $(x_1,x_2,\ldots,x_k) \neq (0, 0\ldots, 0)$ and $(x_1,x_2,\ldots,x_k) \neq (n_1,n_2,\ldots,n_k)$.
$B_{x_1,x_2,\ldots,x_k}$ = **{**$(u_1,u_2,\ldots,u_k)$: u_i=0 if $x_i = n_i$ and ${m_i}^{x_i} \mid u_i$ and ${m_i}^{x_i+1} \nmid u_i$ if $x_i \in \{0,1,2,\ldots,n_i-1\}\}$
All the vertices in $B_{x_1,x_2,\ldots,x_k}$ are adjacent to all the vertices in $B_{y_1,y_2,\ldots,y_k}$ if $x_i + y_i \geq n_i$ for all i= 1,2,…,k.
The vertices in $B_{x_1,x_2,\ldots,x_k}$ form a clique in G_0 if $2x_i \geq n_i$ for all i= 1,2,..,k
Hence we have, for each $u \in B_{x_1,x_2,\ldots,x_k}$,
$d_g(u) = -2 + \prod_{i=1}^{k} {m_i}^{x_i}$ if $B_{x_1,x_2,\ldots,x_k}$ is clique ;
$d_g(u) = -1 + \prod_{i=1}^{k} {m_i}^{x_i}$ if $B_{x_1,x_2,\ldots,x_k}$ is not a clique.
Define a labeling t: $V(G_0) \to \{1, 2, \ldots, p\}$as follows,
Label the vertices of the block as 1 if the block is not form a clique. If the block is form a clique, label the vertices of clique u_j as $1 \leq j \leq (|B_{x_1,x_2,\ldots,x_k}| = q\ )$,
where $|B_{x_1,x_2,\ldots,x_k}| = \prod_{i=1}^{k} \varphi({m_i}^{n_i-x_i})$,
Then the induced edge labellings are,
if the block is not form a clique,
t(e) = 2 for all edge e in this block
if the block is form a clique,
$t(u_ju_{j+1}) = 2j+1$ for all $1 \leq j \leq q-1$,
$t(u_qu_1) = q+1$

Let T = Max ($|B_{x_1,x_2,\ldots,x_k}|$) if $B_{x_1,x_2,\ldots,x_k}$form a clique
We observe that, for each u∈ $B_{x_1,x_2,\ldots,x_k}$,
we have, ,

$$d_t(u) = \begin{cases} -1 + \prod_{i=1}^{k} m_i^{x_i} + \sum_{v\in N(u)} t(v) & \text{if } B_{x_1,x_2,\ldots,x_k} \text{ is not a clique} \\ -2 + \prod_{i=1}^{k} m_i^{x_i} + \sum_{v\in N(u)} t(v) & \text{if } B_{x_1,x_2,\ldots,x_k} \text{ forms a clique} \end{cases}$$

t(u)+t(v)+t(uv) ≡ 0 (mod 2) for all uv ∈ $E(G_0)$
Hence tdln(G_0) = T= M-1 where M=Max($m_1, m_2, \ldots, m_k$).
It can be easily verified that colors of all the incident vertices are pairwise distinct and have the common constant for all the edges of the our given graph. ■

4. Conclusions

In this paper, we introduced a new labeling, totally magic d-lucky labeling, found the totally magic d-lucky number of some standard graphs and some zero divisor graphs. In future, we use this labeling in some other graphs**.**

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