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W_8 - Curvature tensor in generalized Sasakian-space-forms

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Abstract

The generalized Sasakian-space-forms and their properties have been examined by various researchers such as Alegre and Carriazo [2008], Prakasha [2012], Sarkar and Akbar [2014], Shanmukha et al. [2018], Sarkar and Sen [2012], Rajan and Singh [2020] and Sarkar and Sen [2012]. Motivated by the results of these works, we have proposed the idea of the W_8-curvature tensor in generalized Sasakian-space-forms. The main goal of this paper is to investigate the curvature properties of generalized Sasakian-space-forms that satisfy the conditions $\xi - W_8-$ flatness, $\phi - W_8-$semi-symmetric, $W_8 \cdot Q = 0$, $W_8 \cdot R = 0$ and to prove some interesting results.

Keywords: Sasakian-space-form, generalized Sasakian-space-form, $\phi-$recurrent, $\phi-$symmetric, $\phi-$semi-symmetric, W_8- curvature tensor, Einstein manifold, $\eta-$Einstein manifold.
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1 Introduction

The nature of a Riemannian manifold depends on the curvature tensor R of the manifold. It is well known that the sectional curvatures of a manifold determine its curvature tensor completely. Real space-forms are Riemannian manifolds with constant sectional curvature c, and their curvature tensor is given by

$$R(X,Y)Z = c\{g(Y,Z)X - g(X,Z)Y\}.$$

Representation for these spaces are hyperbolic spaces $(c < 0)$, spheres $(c > 0)$ and Euclidean spaces $(c = 0)$.
The Sasakian manifold determines the ϕ-sectional curvature of a Sasakian-space-form, and it has a specific form for its curvature tensor. The Kenmotsu and cosymplectic space-forms use the same notation. Alegre et al. [2004] developed and researched generalized Sasakian-space-forms in an effort to generalize such space -forms in a shared frame. A generalized Sasakian-space-form is an almost contact metric manifold $(M^{2n+1}, \phi, \xi, \eta, g)$, whose curvature tensor is given by

$$\begin{aligned} R(X,Y)Z = {} & f_1\{g(Y,Z)X - g(X,Z)Y\} + f_2\{g(X,\phi Z)\phi Y \\ & - g(Y,\phi Z)\phi X + 2g(X,\phi Y)\phi Z\} + f_3\{\eta(X)\eta(Z)Y \\ & - \eta(Y)\eta(Z)X + g(X,Z)\eta(Y)\xi - g(Y,Z)\eta(X)\xi\}. \end{aligned}$$

The Riemannian curvature tensor of a generalized Sasakian-space-form
M$^{2n+1}(f_1, f_2, f_3)$ is simply given

$$R = f_1R_1 + f_2R_2 + f_3R_3,$$

where f_1, f_2, f_3 are differential functions on $M^{2n+1}(f_1, f_2, f_3)$ and

$$R_1(X,Y)Z = g(Y,Z)X - g(X,Z)Y, \tag{1}$$

$$\begin{aligned} R_2(X,Y)Z = {} & g(X,\phi Z)\phi Y - g(Y,\phi Z)\phi X \\ & +2g(X,\phi Y)\phi Z, \ and \end{aligned} \tag{2}$$

$$\begin{aligned} R_3(X,Y)Z = {} & \eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X \\ & +g(X,Z)\eta(Y)\xi - g(Y,Z)\eta(X)\xi, \end{aligned} \tag{3}$$

where $f_1 = \frac{c+3}{4}, f_2 = f_3 = \frac{c-1}{4}$. Here c denotes the constant ϕ-sectional curvature. Numerous geometers, including Alegre and Carriazo [2008], De and Majhi [2015], De and Sarkar [2010], Kim [2006], Prakasha [2012], Sarkar and Akbar [2014], Sarkar and Sen [2012], Shanmukha et al. [2018], Singh [2016], have explored the characteristics of the generalized Sasakian-space-form. Numerous writers have addressed the idea of local symmetry of a Riemannian manifold in various ways and to different extents. Takahashi introduced the Sasakian manifold's locally ϕ-symmetry in Toshio [1977]. This is extended by De, Shaikh, and Sudipta to the notation of ϕ-symmetry in De et al. [2003], after which they introduce the notation of ϕ-recurrent Sasakian manifold. On the Kenmotsu manifold De et al. [2009], LP-Sasakian manifold Venkatesha [2008], W_8- Curvature Tensor in the Lorentzian Sasakian manifold Rajan and Singh [2020] and $(LCS)_n$-manifold Shaikh et al. [2008], the ϕ-recurrent condition was further studied.

In Tripathi and Gupta [2012] have define the W_8-curvature tensor, given by

$$W_8(X,Y)Z = R(X,Y)Z + \frac{1}{(n-1)}[S(X,Y)Z - S(Y,Z)X],$$

where R and S are curvature tensor and Ricci tensor of the manifold respectively.

A new class of almost contact Riemann manifold was presented by K. Kenmotsu [1972], sometimes referred to as a Kenmotsu manifold. Kenmotsu studied at the underlying characteristics of these manifolds local structure. Kenmotsu manifolds have a one-dimensional basis, a Kahler fibre, and are locally isometric to warped product spaces. According to Kenmotsu's research, a Kenmotsu manifold has a negative curvature of -1 if R(X,Y)Z = 0, where R is the Riemannian curvature tensor and R(X,Y)Z is the derivative of the tensor algebra at each point of the tangent space. Because odd dimensions hyperbolic spaces cannot admit Sasakian structures, unlike odd dimensional spheres, which are well known to do so, odd dimensional hyperbolic Kenmotsu structure is permitted in spaces. Normal Kenmotsu manifolds the almost contact Riemannian manifolds. Several properties of Kenmotsu manifold have been studied by many authors like Bagewadi et al. [2007], Blair [1976], Chaubey and Ojha [2010], DE [2008], Ingalahalli and Bagewadi [2012], Hui and Chakraborty [2017], Baishya and Chowdhury [2016], Nagaraja et al. [2018], Özgür [2006], Prakasha and Balachandra [2018], Ali Shaikh and Kumar Hui [2009], Sinha and Srivastava [1991].

These concepts served as our inspiration as we made an effort to research the characteristics of generalized Sasakian-space-form. The structure of the current paper is as follows. In section 2, we review some preliminary results. In section 3, we study $\xi - W_8 -$flat generalized Sasakian-space-forms. Section 4, deals with the $\phi - W_8 -$semi-symmetric condition in generalized Sasakian-space-form and found to be Einstein manifold. In section 5, we discuss generalized Sasakian-space-form satisfying $W_8 \cdot Q$ = 0 and also found to be Einstein manifold. Finally in the last section, we discuss the generalized Sasakian-space-form satisfying $W_8 \cdot R = 0$ and found to be $\eta-$Einstein Manifolds.

2 Generalized Sasakian-space-forms

The Riemannian manifold M^{2n+1} is called an almost contact metric manifold if the following result holds Blair [1976, 2002]:

$$\phi^2 X = -X + \eta(X)\xi, \tag{4}$$
$$\eta(\xi) = 1, \phi\xi = 0, \eta(\phi X) = 0, g(X, \xi) = \eta(X), \tag{5}$$
$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y), \tag{6}$$
$$g(\phi X, Y) = -g(X, \phi Y), g(\phi X, X) = 0, \tag{7}$$
$$(\nabla_X \eta)(Y) = g(\nabla_X \xi, Y), \forall X, Y \in (T_p M). \tag{8}$$

A almost contact metric manifold is said to be Sasakian if and only if Blair [1976], Sasaki [1965]

$$(\nabla_X \phi)Y = g(X, Y)\xi - \eta(Y)X, \tag{9}$$
$$\nabla_X \xi = -\phi X. \tag{10}$$

Again we know that Alegre et al. [2004] in (2n+1)-dimensional generalized Sasakian-space -form:

$$\begin{aligned} S(X, Y) = {} & (2nf_1 + 3f_2 - f_3)g(X, Y) \\ & - (3f_2 + (2n-1)f_3)\eta(X)\eta(Y), \end{aligned} \tag{11}$$
$$S(\phi X, \phi Y) = S(X, Y) + 2n(f_1 - f_3)\eta(X)\eta(Y), \tag{12}$$
$$QX = (2nf_1 + 3f_2 - f_3)X - (3f_2 + (2n-1)f_3)\eta(X)\xi, \tag{13}$$
$$r = 2n(2n+1)f_1 + 6nf_2 - 4nf_3, \tag{14}$$
$$R(X, Y)\xi = (f_1 - f_3)\{\eta(Y)X - \eta(X)Y\}, \tag{15}$$
$$R(\xi, X)Y = (f_1 - f_3)\{g(X, Y)\xi - \eta(Y)X\}, \tag{16}$$
$$\eta(R(X, Y)Z) = (f_1 - f_3)\{g(Y, Z)\eta(X) - g(X, Z)\eta(Y)\}, \tag{17}$$
$$S(X, \xi) = 2n(f_1 - f_3)\eta(X), \tag{18}$$
$$Q\xi = 2n(f_1 - f_3)\xi, \tag{19}$$

for any vector fields X,Y,Z where R,S,Q and r are the Riemannian curvature tensor, Ricci tensor, Ricci operator $g(QX, Y) = S(X, Y)$ and scalar curvature tensor of generalized Sasakian-space-forms in that order.

3 $\xi - W_8-$flat generalized Sasakian-space-form

In this section, we study $\xi - W_8-$flat in generalized Sasakian-space-form:

Definition 3.1. *A generalized Sasakian-space-form is said to be $\xi - W_8-$flat if*

$$W_8(X, Y)\xi = 0, \tag{20}$$

for any vector fields X, Y on M.

W_8-curvature tensor Tripathi and Gupta [2012] is defined as

$$W_8(X, Y)Z = R(X, Y)Z + \frac{1}{(n-1)}[S(X, Y)Z - S(Y, Z)X], \tag{21}$$

where R and S are curvature tensor and Ricci tensor of the manifold respectively. Replacing Z by ξ in (21), we get

$$W_8(X, Y)\xi = R(X, Y)\xi + \frac{1}{(n-1)}[S(X, Y)\xi - S(Y, \xi)X]. \tag{22}$$

By using (20) in (22), we get

$$R(X, Y)\xi + \frac{1}{(n-1)}[S(X, Y)\xi - S(Y, \xi)X] = 0. \tag{23}$$

By virtue of (15), (18) in (23) and on simplification, we obtained

$$(f_1-f_3)\{\eta(Y)X-\eta(X)Y\}+\frac{1}{(n-1)}[S(X, Y)\xi-2n(f_1-f_3)\eta(Y)X] = 0. \tag{24}$$

By taking inner product with ξ in (24) and on simplification, we have

$$S(X, Y) = 2n(f_1 - f_3)\eta(Y)\eta(X). \tag{25}$$

Hence above discussion, we state the following theorem:

Theorem 3.1. *If a generalized Sasakian-space-forms satisfying $\xi - W_8-$flat condition then the generalized Sasakian-space-form is a special type of $\eta-$Einstein manifolds.*

4 $\phi - W_8-$semi-symmetric condition in generalized Sasakian-space-form

In this section, we study $\phi - W_8-$semi-symmetric condition in generalized Sasakian-space-form:

Definition 4.1. *A generalized Sasakian-space-form is said to be $\phi - W_8-$semi-symmetric if*

$$W_8(X,Y) \cdot \phi = 0, \tag{26}$$

for any vector fields X,Y on M.

Now, (26) turns into

$$(W_8(X,Y) \cdot \phi)Z = W_8(X,Y)\phi Z - \phi W_8(X,Y)Z = 0. \tag{27}$$

From equation (21), we get

$$W_8(X,Y)Z = R(X,Y)Z + \frac{1}{(n-1)}[S(X,Y)Z - S(Y,Z)X]. \tag{28}$$

Replace Z by ϕZ in (28), we obtain

$$W_8(X,Y)\phi Z = R(X,Y)\phi Z + \frac{1}{(n-1)}[S(X,Y)\phi Z - S(Y,\phi Z)X]. \tag{29}$$

Making use of (28) and (29) in (27) and on simplification, we get

$$R(X,Y)\phi Z - \phi R(X,Y)Z + \frac{1}{(n-1)}[S(Y,Z)\phi X - S(Y,\phi Z)X] = 0. \tag{30}$$

Putting $X = \xi$ in (30) and by virtue of (16) and on simplification, we obtain

$$(f_1 - f_3)g(Y,\phi Z)\xi - \frac{1}{(n-1)}S(Y,\phi Z)\xi = 0. \tag{31}$$

Replace ϕZ by Z in (31) and on simplification, we get

$$S(Y,Z)\xi = (n-1)(f_1 - f_3)g(Y,Z)\xi. \tag{32}$$

By taking inner product with ξ in (32), we get

$$S(Y,Z) = (n-1)(f_1 - f_3)g(Y,Z). \tag{33}$$

Hence, we state the following theorem:

Theorem 4.1. *If a generalized Sasakian-space-form satisfying $\phi - W_8-$semi-symmetric condition then the generalized Sasakian-space-form is an Einstein manifolds.*

5 Generalized Sasakian-space-form satisfying $W_8 \cdot Q$ = 0

In this section, we study generalized Sasakian-space-form satisfying $W_8 \cdot Q$ = 0. Then we have

$$W_8(X,Y)QZ - Q(W_8(X,Y)Z) = 0. \tag{34}$$

Putting Y = ξ in (34), we obtain

$$W_8(X,\xi)QZ - Q(W_8(X,\xi)Z) = 0. \tag{35}$$

By virtue of (21) in (35), we get

$$\begin{aligned} R(X,\xi)QZ + \frac{1}{(n-1)}[S(X,\xi)QZ - S(\xi,QZ)X] - Q\{R(X,\xi)Z \\ + \frac{1}{(n-1)}[S(X,\xi)Z - S(\xi,Z)X]\} = 0. \end{aligned} \tag{36}$$

By using (16), (18) in (36), we obtain

$$\begin{aligned} -(f_1 - f_3)[g(X,QZ)\xi - \eta(QZ)X] \\ + \frac{1}{(n-1)}[2n(f_1-f_3)\eta(X)QZ - 2n(f_1-f_3)\eta(QZ)X] \\ -Q[-(f_1-f_3)g(X,Z)\xi - \eta(Z)X] \\ + \frac{1}{(n-1)}[2n(f_1-f_3)\eta(X)Z - 2n(f_1-f_3)\eta(Z)X] = 0, \end{aligned} \tag{37}$$

$$\begin{aligned} -(f_1-f_3)S(X,Z)\xi + (f_1-f_3)Q\eta(Z)X \\ + \frac{2n}{(n-1)}(f_1-f_3)\eta(X)QZ - \frac{2n}{(n-1)}(f_1-f_3)Q\eta(Z)X \\ +(f_1-f_3)g(X,Z)Q\xi - (f_1-f_3)\eta(Z)QX \\ - \frac{2n}{(n-1)}(f_1-f_3)\eta(X)QZ + \frac{2n}{(n-1)}(f_1-f_3)Q\eta(Z)X = 0. \end{aligned} \tag{38}$$

Using (19) and simplifying (38), we have

$$S(X,Z)\xi = 2n(f_1 - f_3)g(X,Z)\xi. \tag{39}$$

Taking inner product with ξ in (39) and on simplifaction, we have

$$S(X,Z) = 2n(f_1 - f_3)g(X,Z). \tag{40}$$

Hence, we state the following theorem:

Theorem 5.1. *A generalized Sasakian-space-form satisfying $W_8 \cdot Q$ = 0 is an Einstein manifolds.*

6 Generalized Sasakian-space-form satisfying $W_8 \cdot R$ = 0

In this section, we study generalized Sasakian-space-form satisfying $W_8 \cdot R$ = 0. Then we have

$$\begin{aligned} W_8(\xi,U)R(X,Y)Z - R(W_8(\xi,U)X,Y)Z \\ -R(X,W_8(\xi,U)Y)Z - R(X,Y)W_8(\xi,U)Z = 0. \end{aligned} \tag{41}$$

Putting Z = ξ in (41), we have

$$\begin{aligned} W_8(\xi,U)R(X,Y)\xi - R(W_8(\xi,U)X,Y)\xi \\ -R(X,W_8(\xi,U)Y)\xi - R(X,Y)W_8(\xi,U)\xi = 0. \end{aligned} \tag{42}$$

By using (15) in (42) and on simplification, we get

$$\begin{aligned} (f_1 - f_3)\eta(W_8(\xi,U)X)Y - (f_1 - f_3)\eta(W_8(\xi,U)Y)X \\ -R(X,Y)W_8(\xi,U)\xi = 0. \end{aligned} \tag{43}$$

By using (21) in (43), we get

$$\begin{aligned} (f_1 - f_3)\eta[R(\xi,U)X + \frac{1}{(n-1)}\{S(\xi,U)X - S(U,X)\xi\}]Y \\ -(f_1 - f_3)\eta[R(\xi,U)Y + \frac{1}{(n-1)}\{S(\xi,U)Y - S(U,Y)\xi\}]X \\ -R(X,Y)[R(\xi,U)\xi + \frac{1}{(n-1)}\{S(\xi,U)\xi - S(U,\xi)\xi\}] = 0. \end{aligned} \tag{44}$$

By using (16), (17), (18) in (44) and on simplification, we get

$$(f_1 - f_3)\{g(U,X)Y - g(U,Y)X\} + \frac{2n}{(n-1)}(f_1 - f_3)\eta(U)\eta(X)Y$$
$$-\frac{2n}{(n-1)}(f_1 - f_3)\eta(U)\eta(Y)X + \frac{1}{(n-1)}\{S(U,Y)X - S(U,X)Y\}$$
$$+R(X,Y)U = 0. \quad (45)$$

Putting Y = ξ in (45), we get

$$(f_1 - f_3)\{g(U,X)\xi - g(U,\xi)X\} + \frac{2n}{(n-1)}(f_1 - f_3)\eta(U)\eta(X)\xi$$
$$-\frac{2n}{(n-1)}(f_1 - f_3)\eta(U)\eta(\xi)X + \frac{1}{(n-1)}\{S(U,\xi)X - S(U,X)\xi\}$$
$$+R(X,\xi)U = 0. \quad (46)$$

By using (5), (16), (18) in (46) and on simplification, we get

$$S(X,U)\xi = 2n(f_1 - f_3)\eta(U)\eta(X)\xi. \quad (47)$$

By taking inner product with ξ in (47), we have

$$S(X,U) = 2n(f_1 - f_3)\eta(U)\eta(X). \quad (48)$$

Hence, we state the following theorem:

Theorem 6.1. *If a generalized Sasakian-space-form satisfies $W_8 \cdot R$ = 0, then the manifold is a special type of $\eta-$ Einstein manifolds.*

7 Conclusions

In this paper, we proposed the notion of the W_8-curvature tensor in generalized Sasakian-space-forms drawing inspiration from the generalized Sasakian-space-form and the W_8-curvature tensor. Several definitions are provided to support this new mechanism. The concept of generalized Sasakian-space-forms has been extensively studied by various authors,including Alegre et al. [2004], Prakasha [2012], Sarkar and Akbar [2014] and Shanmukha et al. [2018]. Their research has shed light on the properties and characteristics of these space-forms.

The main objective of this study is to investigate the curvature properties of the generalized Sasakian-space-form. The analysis begins by examining the $\xi - W_8-$ flat generalized Sasakian-space-forms which are revealed to be a specific type of Einstein manifolds. Furthermore, the paper explores the $\phi - W_8-$ semi-symmetric condition in the generalized Sasakian-space-form establishing it as an Einstein manifold. Additionally, the generalized Sasakian-space-form satisfying $W_8 \cdot Q = 0$ is discussed demonstrating its status as an Einstein manifold. Lastly, the paper presents the discovery of generalized Sasakian-space- forms that satisfy $W_8 \cdot R = 0$ and are identified as $\eta-$ Einstein manifolds.

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References

P. Alegre and A. Carriazo. Structures on generalized sasakian-space-forms. *Differential Geometry and its Applications*, 26(6):656–666, 2008.

P. Alegre, D. E. Blair, and A. Carriazo. Generalized sasakian-space-forms. *Israel journal of mathematics*, 141(1):157–183, 2004.

A. Ali Shaikh and S. Kumar Hui. On locally ϕ-symmetric β-kenmotsu manifolds. *Extracta Mathematicae*, 24(3):301–316, 2009.

C. Bagewadi et al. Some curvature tensors on a kenmotsu manifold. *Tensor. New series*, 68(2):140–147, 2007.

K. K. Baishya and P. Chowdhury. Kenmotsu manifold with some curvature conditions. *Annales Univ. Sci. Budapest*, 59:55–65, 2016.

D. Blair. *Riemannian Geometry of Contact and Symplectic Manifolds*. Birkhäuser Boston, 2002.

D. E. Blair. *Contact Manifolds in Riemannian Geometry*. Springer Berlin Heidelberg, Berlin, Heidelberg, 1976.

S. Chaubey and R. Ojha. On the m-projective curvature tensor of a kenmotsu manifold. *Differential Geometry-Dynamical Systems*, 12:52–60, 2010.

U. De and P. Majhi. ϕ-semisymmetric generalized sasakian space-forms. *Arab Journal of Mathematical Sciences*, 21(2):170–178, 2015.

U. De and A. Sarkar. On the projective curvature tensor of generalized sasakian-space-forms. *Quaestiones Mathematicae*, 33(2):245–252, 2010.

U. De, A. Shaikh, and S. Biswas. On ϕ-recurrent sasakian manifolds. *Novi Sad J. Math*, 33(2):43–48, 2003.

U. C. DE. On ϕ- symmetric kenmotsu manifolds. *International Electronic Journal of Geometry*, 1(1):33–38, 2008.

U. C. De, A. Yildiz, and F. Yaliniz. On ϕ-recurrent kenmotsu manifolds. *Turkish journal of Mathematics*, 33(1):17–25, 2009.

S. K. Hui and D. Chakraborty. Infinitesimal cl-transformations on kenmotsu manifolds. *Bangmod Int. J. Math. and Comp. Sci*, 3:1–9, 2017.

G. Ingalahalli and C. Bagewadi. A study on conservative c-bochner curvature tensor in k-contact and kenmotsu manifolds admitting semisymmetric metric connection. *International Scholarly Research Notices*, 2012:1–14, 2012.

K. Kenmotsu. A class of almost contact riemannian manifolds. *Tohoku Mathematical Journal, Second Series*, 24(1):93–103, 1972.

U. K. Kim. Conformally flat generalized sasakian-space-forms and locally symmetric generalized sasakian-space-forms. *Note di Matematica*, 26(1):55–67, 2006.

H. G. Nagaraja, D. L. Kiran Kumar, and V. S. Prasad. Ricci solitons on kenmotsu manifolds under d-homothetic deformation. *Khayyam Journal of Mathematics*, 4(1):102–109, 2018.

C. Özgür. On weakly symmetric kenmotsu manifolds. *Differ. Geom. Dyn. Syst*, 8: 204–209, 2006.

D. Prakasha. On generalized sasakian-space-forms with weyl-conformal curvature tensor. *Lobachevskii Journal of Mathematics*, 33:223–228, 2012.

D. Prakasha and S. Balachandra. On the conharmonic curvature tensor of kenmotsu manifolds with generalized tanaka-webster connection. *Miskolc Mathematical Notes*, 19(1):491–503, 2018.

Rajan and G. P. Singh. w_8-curvature tensor in lorentzian α-sasakian manifolds. *Turkish Journal of Computer and Mathematics Education*, 11(3):1061–1072, 2020.

A. Sarkar and A. Akbar. Generalized sasakian-space-forms with projective curvature tensor. *Demonstratio Mathematica*, 47(3):725–737, 2014.

A. Sarkar and M. Sen. On ϕ-recurrent generalized sasakian-space-forms. *Lobachevskii Journal of Mathematics*, 33(3):244–248, 2012.

S. Sasaki. *Almost Contact Manifolds*. Tohoku University, 1965.

A. Shaikh, T. Basu, and S. Eyasmin. On the existence of ϕ-recurrent $(lcs)_n$-manifolds. *Extracta Mathematicae*, 23(1):71–83, 2008.

B. Shanmukha et al. Some results on generalized sasakian-sapace-forms with quarter symmetric metric connection. *Asian Journal of Mathematics and Computer Research*, 25(3):183–191, 2018.

J. P. Singh. Generalized sasakian space forms with m-projective curvature tensor. *Acta Mathematica Universitatis Comenianae*, 85(1):135–146, 2016.

B. Sinha and A. Srivastava. Curvatures on kenmotsu manifold. *Indian J. pure appl. Math*, 22(1):23–28, 1991.

T. Toshio. Sasakian ϕ-symmetric spaces. *Tohoku Mathematical Journal, Second Series*, 29(1):91–113, 1977.

M. M. Tripathi and P. Gupta. On $(n(k), \xi)$-semi-riemannian manifolds: semisymmetries. *International Electronic Journal of Geometry*, 5(1):42–77, 2012.

B. C. Venkatesha. On concircular ϕ-recurrent lp-sasakian manifolds. *Differ. Geom. Dyn. Syst*, 10:312–319, 2008.