Micro S_p-Open Sets in Micro Topological Spaces

M. Maheswari[1]
S. Dhanalakshmi[2]
N. Durgadevi[3]

Abstract

In this paper, a new class of open sets called Micro S_p- Open sets in Micro topological spaces are introduced and its fundamental properties are analyzed. Also, some operations on Micro S_p-open sets are investigated.

Keywords: Micro-open, Micro-Semi open, Micro-Pre-open, Micro-Pre closed and Micro S_p-open, Micro S_p-closed.

2010 Mathematics Classification:46S40, 34A07, 03E72[4]

---

[1] Research Scholar, Department of Mathematics, Sri Parasakthi College for Women, Courtallam. (Affiliated to Manonmaniam Sundaranar University, Tirunelveli, Tamilnadu).
[2] Research Scholar, Department of Mathematics, Sri Parasakthi College for Women, Courtallam. (Affiliated to Manonmaniam Sundaranar University, Tirunelveli, Tamilnadu).
[3] Assistant Professor, Department of Mathematics, Sri Parasakthi College for Women, Courtallam (Affiliated to Manonmaniam Sundaranar University, Tirunelveli, Tamilnadu).
Email: durgadevin@sriparasakthicollege.edu.in
[4]Received on July 20, 2022. Accepted on October 15, 2022. Published on January 30, 2023. doi: 10.23755/rm. v45i0.985. ISSN: 1592-7415. eISSN: 2282-8214.

1. Introduction

The concept of Nano topology was introduced by Lellis Thivagar [3] which is in terms of the lower and upper approximations and the boundary region of a subset of an universe. The notion of approximations and boundary region of a set was originally proposed by Pawlak [4] in order to introduce the concept of rough set theory. Chandrasekar [5] introduced the concept of micro topology which is a simple extension of Nano topology and he also studied the concepts of Micro pre-open and Micro semi-open sets. In 2010, Shareef introduced the class of semi open sets called S_p-open sets. Chandrasekar and Swathi [1] introduced Micro α-open in micro topological space. In this paper a new class of sets in Micro topological spaces called Micro S_p-Open set is introduced and some of its properties are derived.

2. Preliminaries

Definition 2.1. [4] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space, Let $X \subseteq U$, Then

(i) The Lower approximation of X with respect to R is the set of all objects which can be certain classified as X with respect to R and is denoted by $L_R(X)$. That is $L_R(X) = \bigcup_{X \in U}\{R(X) : R(X) \subseteq X\}$ where R(X) denotes the equivalence class determined by $X \in U$.

(ii) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is UR(X) = $\bigcup_{X \in U}\{R(X) : R(X) \cap X \neq \phi\}$.

(iii) The Boundary region of X with respect to R is the set of all objects, which can be classified as neither as X nor as not-X with respect to R and is denoted by $B_R(X)$. That is $B_R(X) = U_R(X) - L_R(X)$.

Definition2.2. [3] Let R be an equivalence relation on the universe U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ and $x \in U$. Then $\tau_R(X)$satisfies the following axioms.

(i) U and $\phi \in \tau_R(X)$.

(ii) The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

(iii) The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$. That is,$\tau_R(X)$ is a topology on U called the Nano topology on U with respect to X. Thus (U, $\tau_R(X)$) as called as Nano topological space. The elements of $\tau_R(X)$ are called as Nano open sets. A subset F of U is nano closed if its complement is nano open.

Definition 2.3. [5] Let (U,$\tau_R(X)$) be a Nano topological space. Then $\mu_R(X) = \{N \cup (N' \cap \mu):N, N' \in \tau_R(X)\}$ and $\mu \notin \tau_R(X)$) is called the Micro topology in U with respect to X. The triplet (U,$\tau_R(X)$, $\mu_R(X)$) is called Micro topological space and then elements of

$\mu_R(X)$ are called Micro open sets and the complement of a Micro open set is called a Micro closed set.

Definition 2.4. [1] Let $(U,\tau_R(X), \mu_R(X))$be a micro topological space and $A\subseteq U$. Then A is called

(i) Micro α-open if $A \subseteq$ Mic-Int (Mic-C*l* (Mic-Int (A)))

(ii) Micro pre-open if $A \subseteq$ Mic-Int (Mic-C*l* (A)).

(iii) Micro semi-open if $A \subseteq$ Mic-C*l* (Mic-Int (A)).

Definition 2.5. [5] Let $(U, \tau_R(X), \mu_R(X))$ be a Micro topological space. Let A and B be any two subsets of U. Then

(i) A is Micro open set if and only if Mic-Int(A)=A.

(ii) A is micro closed set if an only if Mic-C*l*(A) = A.

(iii) Mic-Int(U\A) = U\Mic-C*l*(A).

(iv) Mic-C*l*(U\A) =U\Mic-Int(A).

3. Micro S_p-Open Sets

Definition 3.1. Let $(U, \tau_R(X), \mu_R(X))$ be a Micro topological space and $A \subseteq U$. Then A is said to be Micro S_P-open (briefly Mic S_P-open) if for each $x\in A\in$Mic-SO (U, X), there exists a Micro pre-closed set F such that $X\in F \subseteq A$. The set of all Micro S_P-open sets is denoted by Mic S_P-O (U, X).

Definition 3.2. Let $(U, \tau_R(X), \mu_R(X))$ be a Micro topological space. A subset B of U is called Micro S_P-closed (briefly Mic S_P-closed) if and only if its complement is Micro S_P-open and Mic S_P-CL (U, X) denotes the collection of all Micro S_p-closed sets.

Example 3.3. Let U = {a, b, c, d} with U|R = {{a}, {c}, {b, d}}, X = {b, d} $\subseteq$ U then $\tau_R(X)$ = {U, ϕ, {b, d}}. If μ = {a}. Then $\mu_R(X)$ = {U,ϕ, {a}, {b, d}, {a, b, d}} and Micro S_p-open sets are {U, ϕ, {a, c}, {b, d}, {b, c, d}}

Remark 3.4. Every Micro S_p-open set is a Micro semi open set but the converse need not always be true as shown from the following example.

Example 3.5. In example 3.3, Mic-SO(U, X) = {U, ϕ, {a},{a, c}, {b, d}, {a, b, d}, {b, c, d}}.Mic S_P-O (U, X) = {U,ϕ, {a, c}, {b, d}, {b, c, d}} the set {a, b, d} is Micro semi-open but not Micro S_P-open.

Theorem 3.6. An arbitrary union of any family of Micro S_P-open sets is Micro S_P-open.
Proof: Let $\{A_i: i\in\Delta\}$ be a family of Mic S_P-open sets. If $x\in A$, then for each $x\in \bigcup_{i\in\Delta} A_i \subseteq$ Mic-SO (U, X) there exists a Micro pre-closed set F such that $x \in F \subseteq A_i \subseteq \bigcup_{i\in\Delta} A_i$ which

implies x $\in$ F $\subseteq \bigcup_{i\in\Delta} A_i$. Therefore $\bigcup_{i\in\Delta} A_i$ is Micro S_P-open.

Remark 3.7. From the above Theorem 3.6, arbitrary intersection of Micro S_P-closed sets of a Micro topological space is Micro S_P-closed as shown by the following example.

Example 3.8. In example 3.3, Mic S_P-O (U, X) = {U,ϕ, {a, c}, {b, d}, {b, c, d}} and Mic S_p-CL (U, X) = {U, ϕ, {b, d}, {a, c}, {a}}. Here {b, d} $\cap$ {b, c, d} = {b, d} which is a Micro S_P-closed set.

Remark 3.9. The intersection of any two Mic S_P-open sets need not be a Mic S_P-open set.
From example 3.3, {a, c}, {b, c, d} are Mic S_P-open sets but {a, c} $\cap$ {b, c, d} = {c} is not a Mic S_P-open set.

Proposition 3.10. If a subset A of a Micro topological space (U, τ_R(X), μ_R(X)) is Mic S_P-open. Then A is a Micro semi-open set and A is a union of Micro pre-closed sets.
Proof: Let A$\in$ Mic S_P-O (U, X) and A $\subseteq$ U. Then for each x $\in$ A $\in$ Mic-SO (U, X), there exists a Micro pre-closed set F containing x such that x $\in$ F $\subseteq$ A. Thus A is a Micro semi-open set and also F $\subseteq$ A which implies A is the union of Micro pre-closed sets.

Remarks 3.11. The converse of the above Proposition 3.10 need not be true as shown in the following example.

Example 3.12. In example 3.3, Mic-SO (U, X) = {U, ϕ, {a}, {a, c}, {b, d}, {a, b, d}, {b, c, d}} and Mic-PCL (U, X) = {U, ϕ, {b}, {c}, {d}, {a, c}, {b, c}, {c, d}, {a, b, c}, {a, c, d}, {b, c, d}}. Then {a, b, d} $\in$ Mic-SO (U, X) and {a, b, d} is not in the union of Micro Pre-closed sets, also {a, b, d} $\notin$ Mic S_P-O (U, X).

4. Operations on Micro S_P-open Sets

Definition 4.1. A point x$\in$U is said to be a Micro S_P-interior point of A if there exists a Micro S_p-open set V containing x such that V$\subseteq$A.
The set of all Mic S_p-interior points of A is said to be Micro S_p-interior of A and is denoted by Mic S_p-Int(A)**.**

Definition 4.2. Let A be any subset of a Micro topological space (U, τ_R(X), μ_R(X)). Then a point x$\in$U is in the Micro S_p-closure of A if and only if A $\cap$ H $\neq\phi$ for every
H$\in$ Mic S_p-O (U, X) containing x.
The intersection of all Micro S_p-closed sets containing H is called the Micro S_p-closure of F and is denoted by Mic S_p-C*l* (A).

Theorem 4.3. Let A be any subset of a Micro topological space (U, $\tau_R(X)$, $\mu_R(X)$). If a point $x \in$ Mic S_p-Int (A), then there exists $F \in$ Mic-PC*l* (U, X) containing x such that $F \subseteq$ A.

Proof: Suppose that $x \in$ Mic S_p-Int (A). Then $V \in$ Mic S_p-O (U, X) containing x such V $\subseteq$ A. since $V \in$ Mic S_p-O (U, X), then there exists $F \in$ Mic-PC*l* (U, X) containing x such that $F \subseteq U \subseteq A$. Hence $x \in F \subseteq A$.

Theorem 4.4. Let A be a subset of a Micro topological space (U,$\tau_R(X)$, $\mu_R(X)$). If $A \cap F \neq \phi$ for every $F \in$Mic-PC*l* (U, X) containing x, then $x \in$ Mic S_p-C*l* (A).

Proof: Let $V \in$ Mic S_p-O (U, X) containing x, then there exists a Micro pre-closed set F containing x such that $F \subseteq V$. Since $A \cap F \neq \phi$, $x \in$ Mic S_p-C*l* (A).

Theorem 4.5. For any two subsets A and B of a Micro topological space (U, $\tau_R(X)$, $\mu_R(X)$), the following properties are true.

(i) Mic S_p-Int (Mic S_p-Int (A)) = Mic S_p-Int (A).

(ii) Mic S_p-Int (A) = U – Mic S_p-C*l* (U – A).

(iii) If $A \subseteq B$, then Mic S_p-Int(A) $\subseteq$ Mic S_p-Int (B)

(iv) Mic S_p-Int (A)$\cup$ Mic S_p-Int (B) $\subseteq$ Mic S_p-Int ($A \cup B$).

(v) Mic S_p-Int ($A \cap B$) $\subseteq$ Mic S_p-Int (A) $\cap$ Mic S_p-Int (B).

Proof: Obvious.

The converse of (iii), (iv) and (v) of Theorem 4.5 need not be true as shown in the following example.

Example 4.6. Consider U = {p, q, r, s, t} with U|R = {{p, q, r}, {s}, {t}}, X = {p, q} $\subseteq$ U, then $\tau_R(X)$ = {U,ϕ, {p, q, r}}. If μ = {t}, then $\mu_R(X)$ = {U,ϕ, {t}, {p, q, r}, {p, q, r, t}} and Mic S_p-O(U, X) = {U, ϕ, {s, t}, {p, q, r}, {p, q, r, s}}.

(iii) If A = {s} and B = {q, r, t}, then Mic S_p-Int ({s}) = ϕ = Mic S_p-Int ({q, r, t}) but $A \not\subset B$.

(iv) Let A = {p, q} and B = {r, s}, then Mic S_p-Int ({p, q}) $\cup$ Mic S_p-Int ({r, s}) = $\phi \cup \phi = \phi$. But Mic S_p-Int ({p, q} $\cup$ {r, s}) = Mic S_p-Int ({p, q, r, s}) = {p, q, r, s} which implies that Mic S_p-Int ($A \cup B$) $\not\subset$ Mic S_p-Int (A) $\cup$ Mic S_p-Int (B).

(v) Consider A = {p, q, r, s} and B = {p, s, t}, then Mic S_p-Int ({p, q, r, s}) $\cap$ Mic S_p-Int ({p, s, t}) = {p, q, r, s} $\cap$ {s, t} = {s}. But Mic S_p-Int ({p, s, t} $\cap$ {p, q, r, s}) = Mic S_p-Int ({p, s}) =ϕ. Therefore, Mic S_p-Int (A) $\cap$ Mic S_p-Int(B) $\not\subset$ Mic S_p-Int($A \cap B$).

Theorem 4.7. For any two subsets A and B of a Micro topological space (U, $\tau_R(X)$, $\mu_R(X)$). the following properties are true.

(i) Mic S_p-C*l* (Mic S_p-C*l* (A)) = Mic S_p-C*l* (A).

(ii) Mic S_p-C*l* (A) = U – Mic S_p-Int (U – A).

(iii) If $A \subseteq B$, then Mic S_p-C*l* (A) $\subseteq$ Mic S_p-C*l* (B)

(iv) Mic S_p-C*l*(A) $\cup$ Mic S_p-C*l*(B) $\subseteq$ Mic S_p-C*l*(A$\cup$B).
(v) Mic S_p-C*l*(A$\cap$B) $\subseteq$ Mic S_p-C*l*(A) $\cap$ Mic S_p-C*l*(B).
Proof: Obvious, The converse of (iii), (iv) and (v) of Theorem 4.7 need not be true as shown in the following example.

Example 4.8. Consider U= {p, q, r, s, t} with U|R = {{p, q, r}, {s}, {t}}, X = {p, q} $\subseteq$ U, then τ_R(X) = {U,ϕ, {p, q, r}}. If μ = {t}, then τ_R(X) = {U, ϕ, {t}, {p, q, r}, {p, q, r, t}} and Mic S_p-O(U, X) = {U,ϕ, {s, t}, {p, q, r}, {p, q, r, s}}, Mic S_p-C*l* (U, X) = {U, ϕ, {p, q, r}, {s, t}, {t}}.
(iii) Let A = {p, t} and B = {q, s, t}, then Mic S_P-C*l* ({p, t}) = U and Mic S_p-C*l* ({q, s, t}) = U but A $\not\subset$ B.
(iv) Let A = {p, q, r} and B = {t}, then Mic S_p-C*l* ({p, q, r}) $\cup$ Mic S_p-C*l* ({t}) = {p, q, r} $\cup$ {t} = {p, q, r, t}. But Mic S_p-C*l* ({p, q, r} $\cup$ {t}) = Mic S_p-C*l* ({p, q, r, t}) = U. Therefore Mic S_p-C*l* (A$\cup$B) $\not\subset$ Mic S_p-C*l* (A) $\cup$ Mic S_p-C*l* (B).
(v)In general, for any closure operator C*l* (F) $\cup$ C*l* (E) = C*l* (F $\cup$ E) and for most of The closure operators C*l* (F) $\cap$ C*l* (E) $\neq$ C*l* (F$\cap$E). In the case of Mic S_P-closure operator, the equality sign need not hold for both the cases and it is justified by the following example. This obviously leads to the conclusion that Mic S_P-closure operator is not a Kuratowski's operator. For, let A = {r} and B = {p, t} then Mic S_P-C*l* ({r})$\cap$ Mic S_P-C*l* ({p, t}) = {p, q, r} $\cap$ U = {p, q, r}. But Mic S_p-C*l* ({r}$\cap$ {p, t}) = Mic S_p-C*l*(ϕ)= ϕ. Hence Mic S_P-C*l*(A) $\cap$ Mic S_P-C*l*(B) $\not\subset$ Mic S_P-C*l*(A$\cap$B).

5.Conclusion

In this paper Mic S_P -open sets and Mic S_P -closed sets are defined and some of their properties are discussed. This shall be extended in the future Research with some applications.

Acknowledgement

It is our pleasant duty to thank referees for their useful suggestions which helped us to improve our manuscript.

References

[1] Chandrasekar. S and Swathi. G: Micro α-open sets in Micro topological spaces, International Journal of Research in advent Technology, Vol.6, No. 10, October 2018.

[2] Hariwan. Z. Ibrahim: Micro β-open sets in Micro topology, Gen. Lett., 8(1) (2020),8-15.

[3] Lellis Thivagar. M and Richard. C: On Nano Forms of Weakly Open Sets, International Journal of Mathematics and Statistics Invention, 1/1 (2013), 31-37.

[4] Pawlak. Z: Rough Sets., International Journal of Information and Computer Sciences, 11(1982), 341-356.

[5] Sakkraiveeranan Chandrasekar: On Micro Topological Spaces, Journal of New Theory 26(2019), 23-31.

[6] Saravanakumar. D, Sathiyanandham. T and Shalini. V. C: NSp-open sets and NSp-closed sets in Nano Topological Spaces, International Journal of Pure and Applied Mathematics, Volume 113, No.12, 2017, 98-106.

[7] Shareef. A.H: S_p-open sets, S_p-continuity and S_p-continuity and S_p-compactness in topological spaces, M.Sc. Thesis, College of Science, Sulaimani Univ., 2007.

[8] Quays Hatem Imran: On Nano Semi Alpha Open Sets, Journal of Science and Arts, Year 17, No.2(39), pp 235-244, 2017.