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Research Paper

Subclasses of Analytic Functions with Negative Coefficients Involving q-Derivative Operator

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Abstract
Let A denote the class of functions f which are analytic in the open unit disk U. The subclass of A consisting of univalent functions is denoted by M. In this paper, we also consider a subclass of M which is denoted by V, consisting of functions with negative coefficients. In addition, this paper also studies the q-derivative operator. By combining the ideas, this paper introduced three subclasses of A with negative coefficients involving q-derivative. Furthermore, the coefficient estimates, growth results and extreme points were obtained for all of these classes.

Keywords
Analytic, Univalent, q-Derivative Operator

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1. INTRODUCTION

We denote A as the class of functions which has a Maclaurin series expansion of the form

$$f(\delta) = \delta + \sum_{\tau=2}^{\infty} a_\tau \delta^\tau. \tag{1}$$

The function f is analytic in the open unit disk $U = \{\delta \in \mathbb{C}: |\delta|<1\}$.

While we use M to represent the subclass of A and it is consisting of univalent functions. In recent times, there are quite a number of researchers have studied different subclasses of A which associated with q-derivative (see Breaz and Cotîrlă, 2021; Ibrahim, 2020; Jabeen et al., 2022; Janteng et al., 2020; Khan et al., 2022; Karahuseyin et al., 2017; Murugusundaramoorthy et al., 2015; Najafzadeh, 2021; Oshah and Darus, 2015; Rashid and Juma, 2022; Shilpa, 2022).

From (Jackson, 1909; Aral et al., 2013), we have the q-derivative of a function $f \in A$ which given by (1) with $0 < q < 1$ as

$$D_q(f(\delta)) = \frac{f(q\delta) - f(\delta)}{(q-1)\delta}, q \neq 1, \delta \neq 0, \tag{2}$$

$D_q(f(0)) = f'(0)$. From (2), we can get

$$D_q(f(\delta)) = 1 + \sum_{\tau=2}^{\infty} [\tau]_q a_\tau \delta^{\tau-1},$$

where $[\tau]_q = \frac{1-q^\tau}{1-q}$. As $q \to 1$, $[\tau]_q \to \tau$. For a function $j(\delta) = 2\delta^\tau$,

$$D_q(j(\delta)) = D_q(2\delta^\tau) = 2\left(\frac{1-q^\tau}{1-q}\right)(\delta^{\tau-1}) = 2[\tau]_q \delta^{\tau-1}$$

$$\lim_{q\to 1}\left(D_q(j(\delta))\right) = \lim_{q\to 1}\left(2[\tau]_q \delta^{\tau-1}\right) = 2\tau\delta^{\tau-1} = j'(\delta)$$

where j' is the ordinary derivative.

Furthermore, we denote V as a class with negative coefficients and a subclass of M, consisting of the following functions

$$f(\delta) = \delta - \sum_{\tau=2}^{\infty} a_\tau \delta^\tau \tag{3}$$

where $a_\tau \geq 0$.

For $f \in V$, there are some significant researchers for example in (Halim et al., 2005), the authors studied the class $M_S^*V(\eta, \vartheta)$ consisting of starlike functions with respect to (w.r.t) symmetric points. Besides, there are various studies for example in (Al-Abbadi and Darus, 2010; Al Shaqsi and Darus, 2007; Atshan and Ghawi, 2012; Bucur and Breaz, 2020; Choo and Janteng, 2013; Halim et al., 2006; Janteng and Halim, 2009; Najafzadeh and Salleh, 2022; Oluwayemi et al., 2022; Porwal et al., 2022).

In this paper, by considering functions $f \in V$ and q-derivative operator, we introduce the classes $M^*_{S,q}V(\eta,\vartheta)$, $M^*_{C,q}V(\eta,\vartheta)$ and $M^*_{SC,q}V(\eta,\vartheta)$. The coefficient estimates, growth results, and extreme points are obtained for these classes.

First, we give the definitions for the 3 classes. We note that as $q \to 1$, we obtain the classes which were introduced by (Halim et al., 2005).

Definition 1. A function $f \in M^*_{S,q}V(\eta,\vartheta)$ if and only if it satisfies

$$\left|\frac{\delta D_q f(\delta)}{f(\delta)-f(-\delta)}-1\right| < \vartheta\left|\frac{\eta\delta D_q f(\delta)}{f(\delta)-f(-\delta)}+1\right|$$

for $0 \le \eta < 1$, $0 < \vartheta < 1$, $0 \le \frac{2(1-\vartheta)}{1+\eta\vartheta} < 1$ and $\delta \in U$.

Definition 2. A function $f \in M^*_{C,q}V(\eta,\vartheta)$ if and only if it satisfies

$$\left|\frac{\delta D_q f(\delta)}{f(\delta)+\overline{f}(\overline{\delta})}-1\right| < \vartheta\left|\frac{\eta\delta D_q f(\delta)}{f(\delta)+\overline{f}(\overline{\delta})}+1\right|$$

for $0 \le \eta < 1$, $0 < \vartheta < 1$, $0 \le \frac{2(1-\vartheta)}{1+\eta\vartheta} < 1$ and $\delta \in U$.

Definition 3. A function $f \in M^*_{SC,q}V(\eta,\vartheta)$ if and only if it satisfies

$$\left|\frac{\delta D_q f(\delta)}{f(\delta)-\overline{f}(-\overline{\delta})}-1\right| < \vartheta\left|\frac{\eta\delta D_q f(\delta)}{f(\delta)-\overline{f}(-\overline{\delta})}+1\right|$$

for $0 \le \eta < 1$, $0 < \vartheta < 1$, $0 \le \frac{2(1-\vartheta)}{1+\eta\vartheta} < 1$ and $\delta \in U$.

2. RESULTS

Now, we give the properties for the 3 classes. First, we proceed with the coefficient estimates for $f \in M^*_{S,q}V(\eta,\vartheta)$.

Theorem 1. Let $f \in V$. A function $f \in M^*_{S,q}V(\eta,\vartheta)$ if and only if

$$\sum_{\tau=2}^{\infty}\left(\frac{(1+\vartheta\eta)[\tau]_q}{\vartheta(2+\eta)-1}+\frac{\vartheta\left(1-(-1)^\tau\right)-\left(1-(-1)^\tau\right)}{\vartheta(2+\eta)-1}\right)a_\tau \le 1 \quad (4)$$

for $0 \le \eta < 1$, $0 < \vartheta < 1$ *and* $0 \le \frac{2(1-\vartheta)}{1+\eta\vartheta} < 1$.

proof. Initially, we may prove the 'if' part first. We apply the method in (Clunie and Keogh, 1960). So, we write

$$\begin{aligned}
&\left|\delta D_q f(\delta)-(f(\delta)-f(-\delta))\right|-\vartheta\left|\eta\delta D_q f(\delta)+(f(\delta)-f(-\delta))\right| \\
&=\left|-\delta-\sum_{\tau=2}^{\infty}\left([\tau]_q-\left(1-(-1)^\tau\right)\right)a_\tau\delta^\tau\right|-\vartheta\Bigg|(2+\eta)\delta- \\
&\quad\sum_{\tau=2}^{\infty}\left([\tau]_q\eta+1-(-1)^\tau\right)a_\tau z^\tau\Bigg| \le \sum_{\tau=2}^{\infty}\left([\tau]_q-\left(1-(-1)^\tau\right)\right)a_\tau r^\tau \\
&\quad+r-\vartheta(2+\eta)r+\sum_{\tau=2}^{\infty}\vartheta\left([\tau]_q\eta+1-(-1)^\tau\right)a_\tau r^\tau \\
&\quad<\Bigg[\sum_{\tau=2}^{\infty}\left([\tau]_q-\left(1-(-1)^\tau\right)\right)a_\tau+1-\vartheta(2+\eta) \\
&\quad+\sum_{\tau=2}^{\infty}\vartheta\left([\tau]_q\eta+1-(-1)^\tau\right)a_\tau\Bigg]r \\
&=\Bigg[\sum_{\tau=2}^{\infty}((1+\eta\vartheta)[\tau]_q+\vartheta\left(1-(-1)^\tau\right) \\
&\quad-\left(1-(-1)^\tau\right))a_\tau-(\vartheta(2+\eta)-1)\Bigg]r
\end{aligned}$$

By considering inequality (4), we get

$\sum_{\tau=2}^{\infty}\left((1+\vartheta\eta)[\tau]_q+\vartheta(1-(-1)^\tau)-(1-(-1)^\tau)\right)a_\tau-(\vartheta(2+\eta)-1) \le 0,$

and by applying this inequality, we obtain

$$\begin{aligned}
&\left|\delta D_q f(\delta)-(f(\delta)-f(-\delta))\right|-\vartheta\left|\eta\delta D_q f(\delta)+(f(\delta)-f(-\delta))\right| \\
&=\Bigg[\sum_{\tau=2}^{\infty}\left((1+\eta\vartheta)[\tau]_q+\vartheta(1-(-1)^\tau)-(1-(-1)^\tau)\right)a_\tau \\
&\quad-(\vartheta(2+\eta)-1)\Bigg]r \le 0
\end{aligned}$$

Thus,

$$\left|\frac{\frac{\delta D_q f(\delta)}{f(\delta)-f(-\delta)}-1}{\frac{\eta\delta D_q f(\delta)}{f(\delta)-f(-\delta)}+1}\right| < \vartheta$$

and hence $f \in M^*_{S,q}V(\eta,\vartheta)$. Conversely, let

$$\left|\frac{\frac{\delta D_q f(\delta)}{f(\delta)-f(-\delta)}-1}{\frac{\eta\delta D_q f(\delta)}{f(\delta)-f(-\delta)}+1}\right| = \left|\frac{-1-\sum_{\tau=2}^{\infty}\left([\tau]_q-\left(1-(-1)^\tau\right)\right)a_\tau\delta^{\tau-1}}{(2+\eta)-\sum_{\tau=2}^{\infty}\left([\tau]_q\eta+1-(-1)^\tau\right)a_\tau z^{\tau-1}}\right| < \vartheta.$$

Since we know that the function f is analytic, continuous and non constant in U, then we apply the maximum modulus principle, so we can get

$$\left|\frac{-1-\sum_{\tau=2}^{\infty}\left([\tau]_q-(1-(-1)^\tau)\right)a_\tau\delta^{\tau-1}}{(2+\eta)-\sum_{\tau=2}^{\infty}\left([\tau]_q\eta+1-(-1)^\tau\right)a_\tau\delta^{\tau-1}}\right|$$
$$=\frac{\left|1+\sum_{\tau=2}^{\infty}\left([\tau]_q-(1-(-1)^\tau)\right)a_\tau\delta^{\tau-1}\right|}{\left|(2+\alpha)-\sum_{\tau=2}^{\infty}\left([\tau]_q\eta+1-(-1)^\tau\right)a_\tau\delta^{\tau-1}\right|}$$
$$\leq\frac{1+\sum_{\tau=2}^{\infty}\left([\tau]_q-(1-(-1)^\tau)\right)|a_\tau||\delta|^{\tau-1}}{(2+\eta)-\sum_{\tau=2}^{\infty}\left([\tau]_q\eta+1-(-1)^\tau\right)|a_\tau||\delta|^{\tau-1}}$$
$$\leq\frac{1+\sum_{\tau=2}^{\infty}\left([\tau]_q-(1-(-1)^\tau)\right)a_\tau r^{\tau-1}}{(2+\eta)-\sum_{\tau=2}^{\infty}\left([\tau]_q\eta+1-(-1)^\tau\right)a_\tau r^{\tau-1}}=f(r).$$

Since $f\in M^*_{S,q}V(\eta,\vartheta)$ and $0<r<1$, we obtain

$$\frac{1+\sum_{\tau=2}^{\infty}\left([\tau]_q-(1-(-1)^\tau)\right)a_\tau r^{\tau-1}}{(2+\eta)-\sum_{\tau=2}^{\infty}\left([\tau]_q\eta+1-(-1)^\tau\right)a_\tau r^{\tau-1}}<\vartheta. \qquad (5)$$

Then, we let $r\to 1$ in (5), we gain

$$1+\sum_{\tau=2}^{\infty}\left([\tau]_q-(1-(-1)^\tau)\right)a_\tau\leq\vartheta\Bigg((2+\eta)-\sum_{\tau=2}^{\infty}\left([\tau]_q\eta+1-(-1)^\tau\right)a_\tau\Bigg)$$

and hence $\sum_{\tau=2}^{\infty}\left(\frac{(1+\vartheta\eta)[\tau]_q}{\vartheta(2+\eta)-1}+\frac{\vartheta(1-(-1)^\tau)-(1-(-1)^\tau)}{\vartheta(2+\eta)-1}\right)a_\tau\leq 1$ as required. This completes the proof of the theorem.

Corollary 1. If $f\in M^*_{S,q}V(\eta,\vartheta)$ then

$$a_\tau\leq\frac{\vartheta(2+\eta)-1}{(1+\vartheta\eta)[\tau]_q+\vartheta(1-(-1)^\tau)-(1-(-1)^\tau)},\ \tau\geq 2.$$

Proof. From Theorem 1, if $f\in M^*_{S,q}V(\eta,\vartheta)$ then

$$\sum_{\tau=2}^{\infty}\left(\frac{(1+\vartheta\eta)[\tau]_q+\vartheta(1-(-1)^\tau)-(1-(-1)^\tau)}{\vartheta(2+\eta)-1}\right)a_\tau\leq 1$$

for $0\leq\eta<1$, $0<\vartheta<1$ and $0\leq\frac{2(1-\vartheta)}{1+\eta\vartheta}<1$.
Since

$$\left(\frac{(1+\vartheta\eta)[\tau]_q+\vartheta(1-(-1)^\tau)-(1-(-1)^\tau)}{\vartheta(2+\eta)-1}\right)a_\tau$$
$$\leq\sum_{\tau=2}^{\infty}\left(\frac{(1+\vartheta\eta)[\tau]_q+\vartheta(1-(-1)^\tau)-(1-(-1)^\tau)}{\vartheta(2+\eta)-1}\right)a_\tau$$
$$\leq 1,$$

we obtain that $a_\tau\leq\frac{\vartheta(2+\eta)-1}{(1+\vartheta\eta)[\tau]_q+\vartheta(1-(-1)^\tau)-(1-(-1)^\tau)}$. The proof is completed.

Next, by applying similar way of methods, we may get the coefficient properties for the functions which belongs to $M^*_{C,q}V(\eta,\vartheta)$ and $M^*_{SC,q}V(\eta,\vartheta)$. The results are shown in Theorem 2 and Theorem 3.

Theorem 2. Let $f\in V$. A function $f\in M^*_{C,q}V(\eta,\vartheta)$ if and only if

$$\sum_{\tau=2}^{\infty}\left(\frac{(1+\vartheta\eta)[\tau]_q}{\vartheta(2+\eta)-1}+\frac{2(\vartheta-1)}{\vartheta(2+\eta)-1}\right)a_\tau\leq 1$$

for $0\leq\eta<1$, $0<\vartheta<1$ *and* $0\leq\frac{2(1-\vartheta)}{1+\eta\vartheta}<1$.

Corollary 2. If $f\in M^*_{C,q}V(\eta,\vartheta)$ then

$$a_\tau\leq\frac{\vartheta(2+\eta)-1}{(1+\vartheta\eta)[\tau]_q+2(\vartheta-1)},\quad\tau\geq 2.$$

Theorem 3. Let $f\in V$. A function $f\in M^*_{SC,q}V(\eta,\vartheta)$ if and only if

$$\sum_{\tau=2}^{\infty}\left(\frac{(1+\vartheta\eta)[\tau]_q}{\vartheta(2+\eta)-1}+\frac{\vartheta(1-(-1)^\tau)-(1-(-1)^\tau)}{\vartheta(2+\eta)-1}\right)a_\tau\leq 1$$

for $0\leq\eta<1$, $0<\vartheta<1$ *and* $0\leq\frac{2(1-\vartheta)}{1+\eta\vartheta}<1$.

Corollary 3. If $f\in M^*_{S,q}V(\eta,\vartheta)$ then

$$a_\tau\leq\frac{\vartheta(2+\eta)-1}{(1+\vartheta\eta)[\tau]_q+\vartheta(1-(-1)^\tau)-(1-(-1)^\tau)},\ \tau\geq 2.$$

After that, we may get the growth property for functions in the class $M^*_{S,q}V(\eta,\vartheta)$ in the next part.

Theorem 4. Given that a function f be defined by (4) and belongs to the class $M^*_{S,q}V(\eta,\vartheta)$. Then for $\{\delta:0<|\delta|=r<1\}$,

$$r-\frac{\vartheta(2+\eta)-1}{[2]_q(1+\vartheta\eta)}r^2\leq|f(\delta)|\leq r+\frac{\vartheta(2+\eta)-1}{[2]_q(1+\vartheta\eta)}r^2.$$

proof. First, it is obvious that

$$\frac{[2]_q(1+\vartheta\eta)}{\vartheta(2+\eta)-1}\sum_{\tau=2}^{\infty}a_\tau\leq\sum_{\tau=2}^{\infty}\Bigg(\frac{(1+\vartheta\eta)[\tau]_q}{\vartheta(2+\eta)-1}+\frac{\vartheta(1-(-1)^\tau)-(1-(-1)^\tau)}{\vartheta(2+\eta)-1}\Bigg)a_\tau$$

and as $f\in M^*_{S,q}V(\eta,\vartheta)$, we use the inequality in Theorem 1 and it gives

$$\sum_{\tau=2}^{\infty} a_\tau \leq \frac{\vartheta(2+\eta)-1}{[2]_q(1+\vartheta\eta)}. \tag{6}$$

From (4) with $|\delta| = r$ $(r < 1)$, we can gain

$$|f(\delta)| \leq r + \sum_{\tau=2}^{\infty} a_\tau r^\tau \leq r + \sum_{\tau=2}^{\infty} a_\tau r^2$$

and

$$|f(\delta)| \geq r - \sum_{\tau=2}^{\infty} a_\tau r^\tau \geq r - \sum_{\tau=2}^{\infty} a_\tau r^2.$$

Lastly, by considering the inequalities (6), we may gain the result of Theorem 4.

In the next part, we shall gain the growth results for functions that belongs to $M^*_{C,q}V(\eta,\vartheta)$ and $M^*_{SC,q}V(\eta,\vartheta)$ by using a similar method. The results are shown in Theorem 5 and Theorem 6.

Theorem 5. Given that a function f be defined by (4) and belongs to the class $M^*_{C,q}V(\eta,\vartheta)$. Then for $\{z : 0 < |\delta| = r < 1\}$,

$$r - \frac{\vartheta(2+\eta)-1}{([2]_q-1)+\vartheta([2]_q\eta+2)} r^2 \leq |f(\delta)| \leq r + \frac{\vartheta(2+\eta)-1}{([2]_q-1)+\vartheta([2]_q\eta+2)} r^2.$$

Theorem 6. Given that a function f be defined by (4) and belongs to the class $M^*_{SC,q}V(\eta,\vartheta)$. Then for $\{z : 0 < |\delta| = r < 1\}$,

$$r - \frac{\vartheta(2+\eta)-1}{[2]_q(1+\vartheta\eta)} r^2 \leq |f(\delta)| \leq r + \frac{\vartheta(2+\eta)-1}{[2]_q(1+\vartheta\eta)} r^2.$$

Finally, we consider extreme points for these 3 classes.

Theorem 7. Let $f_1(\delta) = \delta$ and $f_\tau(\delta)$

$$=\delta - \frac{\vartheta(2+\eta)-1}{(1+\vartheta\eta)[\tau]_q + \vartheta(1-(-1)^\tau) - (1-(-1)^\tau)}\delta^\tau, \tau \geq 2.$$ Then $f \in M^*_{S,q}V(\eta,\vartheta)$ if and only if $f(\delta) = \sum_{\tau=1}^{\infty}\lambda_\tau f_\tau(\delta)$ where $\lambda_\tau \geq 0$ and $\sum_{\tau=1}^{\infty}\lambda_\tau = 1$.

proof. We adopt the technique by (Silverman, 1975), we assume that

$$\begin{aligned} f(\delta) &= \sum_{\tau=1}^{\infty}\lambda_\tau f_\tau(\delta) \\ &= \delta - \sum_{\tau=2}^{\infty}\lambda_\tau\left(\frac{\vartheta(2+\eta)-1}{(1+\vartheta\eta)[\tau]_q + \vartheta(1-(-1)^\tau) - (1-(-1)^\tau)}\right)\delta^\tau. \end{aligned}$$

Next since

$$\begin{aligned} &\sum_{\tau=2}^{\infty}\lambda_\tau\left(\frac{\vartheta(2+\eta)-1}{(1+\vartheta\eta)[\tau]_q + \vartheta(1-(-1)^\tau) - (1-(-1)^\tau)}\right) \\ &\left(\frac{(1+\vartheta\eta)[\tau]_q + \vartheta(1-(-1)^\tau) - (1-(-1)^\tau)}{\vartheta(2+\eta)-1}\right) \\ &= \sum_{\tau=2}^{\infty}\lambda_\tau = 1 - \lambda_1 \leq 1. \end{aligned}$$

Therefore by Theorem 1, $f \in M^*_{S,q}V(\eta,\vartheta)$. Conversely, suppose $f \in M^*_{S,q}V(\eta,\vartheta)$. Since

$$a_\tau \leq \frac{\vartheta(2+\eta)-1}{(1+\vartheta\eta)[\tau]_q + \vartheta(1-(-1)^\tau) - (1-(-1)^\tau)}, \tau \geq 2,$$

we may set

$$\lambda_\tau = \left\{\frac{(1+\vartheta\eta)[\tau]_q + \vartheta(1-(-1)^\tau) - (1-(-1)^\tau)}{\vartheta(2+\eta)-1}\right\} a_\tau, \tau \geq 2$$

and

$$\lambda_1 = 1 - \sum_{\tau=2}^{\infty}\lambda_\tau.$$

Then

$$\begin{aligned} &\sum_{\tau=1}^{\infty}\lambda_\tau f_\tau(\delta) \\ &= \lambda_1 f_1(\delta) + \sum_{\tau=2}^{\infty}\lambda_\tau f_\tau(\delta) \\ &= \delta - \sum_{\tau=2}^{\infty}\lambda_\tau\delta + \sum_{\tau=2}^{\infty}\lambda_\tau\delta - \sum_{\tau=2}^{\infty}a_\tau\delta^\tau \\ &= f(\delta). \end{aligned}$$

Hence, we complete the proof.

By using a similar method, we obtain the extreme points for the other 2 classes.

Theorem 8. Let $f_1(\delta) = \delta$ and

$$f_\tau(\delta) = \delta - \frac{\vartheta(2+\eta)-1}{(1+\vartheta\eta)[\tau]_q + 2(\vartheta-1)}\delta^\tau, \tau \geq 2. \quad \text{Then}$$
$$f \in M^*_{C,q}V(\eta,\vartheta) \quad \text{if and only if}$$
$$f(\delta) = \sum_{\tau=1}^{\infty} \lambda_\tau f_\tau(\delta) \quad \text{where} \quad \lambda_\tau \geq 0 and \sum_{\tau=1}^{\infty} \lambda_\tau = 1.$$

Theorem 9. Let $f_1(\delta) = \delta$ and

$$f_\tau(\delta) = \delta - \frac{\vartheta(2+\eta)-1}{(1+\vartheta\eta)[\tau]_q + \vartheta(1-(-1)^\tau) - (1-(-1)^\tau)}\delta^\tau, \tau \geq 2. \quad \text{Then} \quad f \in M^*_{SC,q}V(\eta,\vartheta) \quad \text{if and only if}$$
$$f(\delta) = \sum_{\tau=1}^{\infty} \lambda_\tau f_\tau(\delta) \quad \text{where} \quad \lambda_\tau \geq 0 \quad \text{and} \quad \sum_{\tau=1}^{\infty} \lambda_\tau = 1.$$

3. CONCLUSIONS

In this paper, we introduced 3 new subclasses of A with negative coefficients involving q-derivative and obtained their results for the coefficient estimates, growth results and extreme points.

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