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Generalized Model for Spectral Efficiency of Cellular Systems

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نموذج عام لكفاءة الطيف في الانظمة الخلوية

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خلاصة: يستخدم البحث نموذج رياضي عام لدراسة كفاءة الطيف في الانظمة الخلوية. ياخذ هذا النموذج بنظر الاعتبار تاثيرات التداخل للصف الاول والثاني، التوزيع العشوائي للمستخدمين على المنطقة المغطاة بالكامل من قبل النظام الخلوي، وكذلك تاثيرات التداخل بسبب تعدد الطرق للموجات. كما يدرس النموذج تاثير تجزئة منطقة للنظام، الخلوي على قيمة كفاءة الطيف. كما يعالج البخحث تأثير نصف قطر الخلية وحجم المجموعة على كفاءة الطيف، وتاثير الحالات المختلفة لظروف انتشار الموجات على هذه الكفاءة. ان نتائج المحكاة المقدمة في هذا البحث توضح انه بالامكان الحصول على تحسن في كفاءة الطيف بمقدار يصل ٧٠٪ عند استخدام نظام بستة مقاطعمقارنة مع نظام الاتجاه الواحدز كما يوضح انه ازدياد حجم المجموعة يؤدي الى تناقص في كفاءة الطيف بمعدل يتناسب مع الجذر التربيعي لحجم المجموعة.

المفردات المفتاحية: كفاءة الطيف، التقطيع، الانظمة الخلوية، حجم المجموعة، انظمة فقدان طريق الانتشار.

Abstract: A general mathematical model was used to study the spectral efficiency of cellular systems. The model took into consideration effects of first and second tier interference. The random distribution of users across the whole area covered by the cellular system as well as the shadowing and multipath effects were taken into account. The influence of sectorization on value of the spectral efficiency was also studied. The model investigated the effects of the cell radius and cluster size on the spectral efficiency. The influence of different propagation conditions on the spectral efficiency was also considered. Results of simulation showed that an improvement of up to 70% can be achieved in the spectral efficiency, when using a six-sector system in comparison with the omni-directional system. Also, the spectral efficiency was shown to decay by a rate proportional to the square of the radius. It was also shown that spectral efficiency improved in severe propagation conditions. Using a higher value for the cluster size decreased the spectral efficiency by a rate proportional to the square root of the cluster size.

Keywords: Spectral efficiency, Sectorization, Cellular systems, Cluster size, Path loss models

1. Introduction

Spectral efficiency is one of the most important issues in wireless systems. Because of limited available frequency spectrum, current cellular radio systems adopt the concept of frequency reuse to utilize the same frequency repeatedly at different locations. However, while frequency reuse provides a more efficient use of the limited available spectrum, it also introduces unavoidable co-channel interference. A large frequency reuse distance can enhance the channel quality by reducing co-channel interference, but will decrease the system spectral efficiency. One challenge for cell engineering is to optimize the tradeoff among channel quality, system efficiency, and the costs of infrastructure and user terminals.

The main aim of this paper is to build a generalized mathematical model that can be used to study effect of different parameters on the spectral efficiency of microcellular and macrocellular systems. The used model investigates also, the effect of using cell sectorization on the spectral efficiency. Cell sectorization is an economic way to increase spectral efficiency. Every cell is divided into multiple sectors, and each sector is served by a dedicated antenna (Fig. 1). The co-channel interference in such a system will be decreased as a single omni-directional

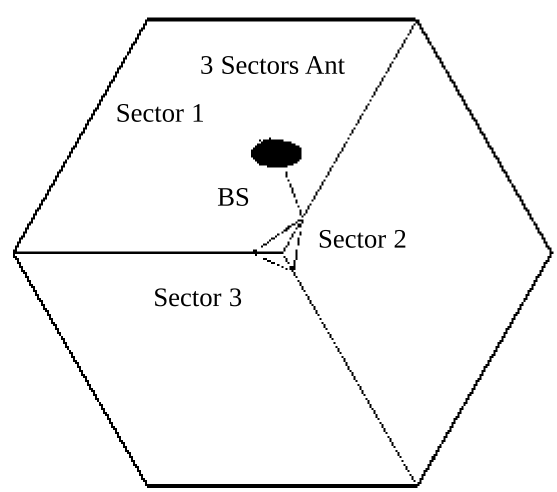

Figure 1. Diagram showing principles of cell sectorization

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antenna at the base station is replaced by several directional antennas, each radiating within a specified sector. A given cell will receive interference and transmit with only a fraction of the available co-channel cells.

The proposed model in this paper is an extension to the model proposed by (Hammerschmidt *et al*. 1997) and modified by (Walke 2002; Alouni and Goldsmith, 1999). It is based on: (a) the two-slope path loss model that considers the effect of distance on value of received signals and interferers, (b) Suzuki model (Shankar, 2002) that considers the effect of shadowing and multipath, (c) a uniform distribution of users in the whole area under consideration, (d) interference from first and second tiers so that effect of all considerable interferences are included, (e) adaptive Simpson quadrature (Gander and Gautschi, 2000) to solve the model equation.

2. Channel and System Models

The mechanisms which govern radio propagation in cellular systems are complex and diverse, but they can be attributed to three basic propagation mechanisms; reflection, diffraction and scattering. The total signal received at the receiver is, therefore, the phasor superposition of a number of reflected, diffracted and scattered signal components. As a result of the above three propagation mechanisms, the received signal can be characterized by three nearly independent phenomena including path loss variation with distance, slow lognormal shadowing, and fast multipath fading (Andersen *et al*. 1995). These phenomena are illustrated in Fig. 2.

2.1 Path loss model

The difference (in dB) between the transmitted signal level and the signal in the general area of the receiver (area mean power) is referred to as the pathloss (see Fig. 2c). Path loss is due to the decay of the intensity of a propagating radio-wave. In the following analyses and simulations, the two-slope path-loss model derived from Erceg *et al*. (1992) will be used to obtain the average received power as a function of distance. According to this model, the average path loss is given by:

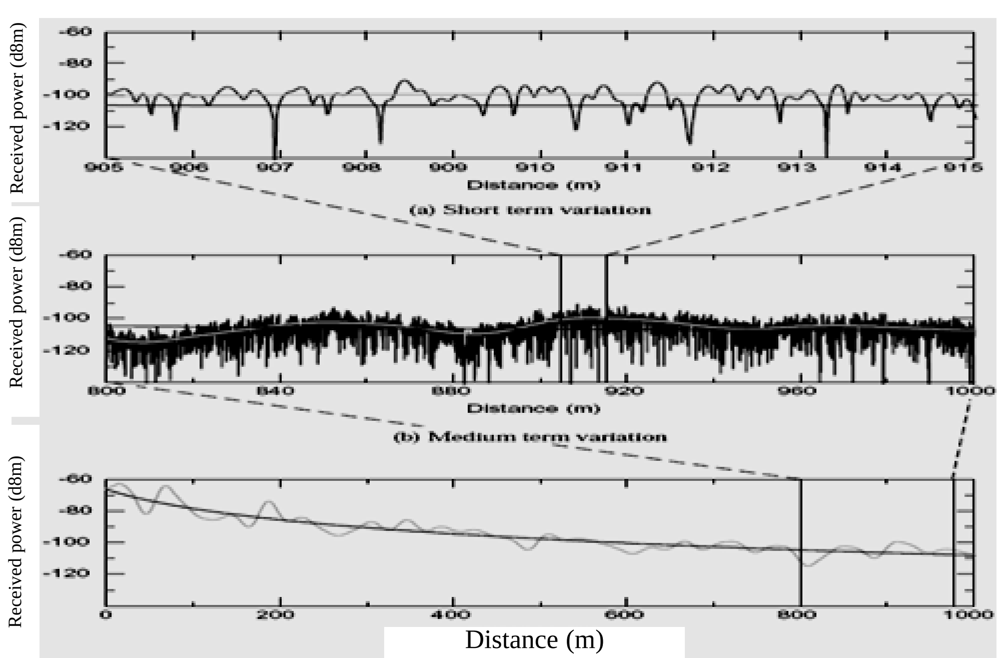
$$P_l = \begin{cases} 10\,\alpha \log\left(\dfrac{4\pi r}{\lambda}\right) & r \le r_b \\ 10\,\alpha \log\left(\dfrac{4\pi r}{\lambda}\right) + 10\,\beta \log\left(\dfrac{r}{r_b}\right) & r > r_b \end{cases} \tag{1}$$

$$P_l = 10 \log_{10}(P_t / P_r) \tag{2}$$

$$r_b = \frac{4 h_t h_r}{\lambda} \tag{3}$$

where; r is the transmitter-receiver distance; r_b is the break point distance; h_t and h_r are transmitter and receiver antenna heights, respectively; λ is the wavelength; α is the path loss exponent when the propagation distance $r \le r_b$ while $(\beta + \alpha)$ is the path loss exponent when $r > r_b$; P_t is the transmitted power, and P_r is the received power.

The simulation introduced in this paper uses the following values for the above parameters. The path loss exponent α ranges from 2 to 3, while β ranges from 1 to 2. Transmitter antenna height h_t is taken as 10 m in microcellular system and as 30 m in macrocellular system, while h_r is always assumed 2 m. These values meet the practical limits of cellular systems.



(c) Long term variation

Figure 2. Variation of received power with distance in a cellular system. Part a is a zoom from b which is a zone from c

2.2 Lognormal shadowing

The variability associated with large scale environmental obstacles leads to the local mean power fluctuating about a constant area mean power over medium distances (~100 m) (see Fig. 1). This phenomenon is known as shadowing and arises due to obstacles in the propagation path such as buildings, hills and foliage. Experiments and analysis indicated that this variability can be approximated by a lognormal distribution (Parsons, 2001). By lognormal it is meant that the local-mean signal, $\bar{P}$, expressed in logarithmic values has a normal probability distribution function of the form (Parsons, 2001);

$$f(\bar{P}) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[\frac{-(\bar{P}-m)^2}{2\sigma^2}\right] \quad (4)$$

where; m and σ are the mean and standard deviation of the normal distribution respectively, and are measured in dB. The m value is normally approximated by the area mean power, P, although strictly speaking, they are only identical when σ = 0 dB (Linnartz, 1993). If the local mean signal is expressed in watts, then the corresponding distribution is a lognormal distribution of the form (Parsons, 2001):

$$f(\bar{P}) = \frac{10}{\log_e(10)\bar{P}\sqrt{2\pi}\sigma} \exp[-\alpha] \quad (5)$$

where $\alpha = \dfrac{(10\log_{10}(\bar{P}) - m)^2}{2\sigma^2}$

Note that the terms m and σ in Eq. (5) are the mean and standard deviation of the normal distribution presented in Eq. (4) and they retain their logarithmic units. The shadowing standard deviation, σ, of the normal distribution tends to be dependent on frequency and the nature of the environment in the vicinity of the terminal. In most cases, σ is relatively independent of transmission path length and antenna height (Rowe and Williamson, 1986). Generally, the variability of the shadowing is greater in more urbanized areas. Mogensen, *et al*. (1991) have reported σ values to be 6.5 to 8.2 dB in urban areas. In this paper, σ was assumed to be 7 dB.

2.3 Rayleigh fading

In addition to distance-dependent path loss and environmental shadowing, the signal received at a terminal receiver will undergo significant envelope fading over distances of a few tens of wavelengths (see Fig. 2a). This fluctuation is attributable to multipath propagation, whereby the signal propagates not by one, but by many different paths to the receiver. Several multipath components of the transmitted signal arriving at the receiver at the same time will each have different time delays, and therefore, different phase shifts. These add either constructively or destructively to produce a random phase shift. This creates rapid changes in the signal strength, an effect known as Rayleigh fading (Parsons, 2001). It is important to realize that the Rayleigh distribution has been found to model the variability of the received signal envelope when the envelope is expressed in volts. If the signal envelope is expressed in watts, the appropriate statistical distribution is no longer the Rayleigh distribution, but rather the exponential distribution. Hence, if P is the total momentary power received, it will have an exponential probability density function of the form (Parsons, 2001):

$$f(P) = \frac{1}{\bar{P}} \exp\left(\frac{-P}{\bar{P}}\right) \quad (6)$$

If the momentary received power, P, is expressed in dB, then the probability distribution function is given by (Shepherd & Milnarich, 1973):

$$f(P[\mathrm{dB}]) = \frac{\log_e(10)}{10\bar{P}} \exp\left(\frac{\log_e(10)P}{10} - \frac{10^{P/10}}{\bar{P}}\right) \quad (7)$$

2.4 Lognormal shadowing and Rayleigh fading

In the previous subsections, it was shown that over relatively small distances, the signal is well described by Rayleigh statistics, with the local mean over a somewhat larger area being lognormally distributed. It is of interest, therefore, to examine the overall distribution of the received signal in these larger areas. It might be reasonable to expect the distribution to be a mixture of Rayleigh and lognormal, which has been shown by investigators to be the case by Parsons and Ibrahim (1983) examined the Nakagami-*m* and Weibull distributions, which both contain the Rayleigh distribution as a special case, but came to the conclusion that the statistics of the radio signal can be represented by a mixture of Rayleigh and lognormal statistics in the form of a Rayleigh distribution with a lognormally varying mean. This combined distribution is normally referred to as the Suzuki distribution. The probability density function of the momentary received signal power, P, (in watts) in this case is given by (Shanker, 2002);

$$f(P) = \int_0^\infty \frac{10\exp\left(\frac{-P}{\bar{P}}\right)}{\log_e(10)\bar{P}^2\sqrt{2\pi}\sigma} \exp(-\alpha)d\bar{P} \quad (8)$$

3. Carrier-to-Interference Ratio

To simplify the analyses, the following assumptions have been made in the co-channel interference model. First, the system is considered to be interference-limited in which the thermal noise power is negligible relative to the co-channel interference power. Thus, the ratio of carrier power-to-noise plus interference power reduces to the carrier-to-interference power ratio (CIR). Therefore, the desired user CIR, γ_d , can be written as:

$$\gamma_d = \frac{P_d}{P_I} = \frac{P_d(r_d)}{\sum_{i=1}^{n} P_i(r_i) + \sum_{j=1}^{2n} P_j(r_j)} \tag{9}$$

where P_d is the received power level (in *W*) from the desired mobile at a distance r_d from its BS; P_I is the total interfering power (in *W*). P_i (r_i) and P_j (r_j) are the received power levels (in *W*) from the *i*-th and *j*-th interfering mobile located within the first and second tiers, respectively and at distances of r_i and r_j from the desired mobiles' BS. Also, *n* and *2n* are numbers of active co-channel cells in the first and second tiers respectively. It is clear from Eq. (9) that effects of interferers from the first tier as well as from the second tier are included in the calculations. This is one of the main differences between the model presented in this paper and other models which neglected effect of the second tier interferes.

Refering to Fig. 3b, it is clear that n is equal to 6 but in case of sectorization with *s* sectors, it is modified to:

$$n = 6/s \tag{10}$$

Assume that r and θ define the polar position of a mobile unit relative to its base station, then values of r_i and r_j can be calculated from:

$$r_i = \sqrt{D_1^{\,2} + r^2 + 2D_1 r \sin(\varphi)} \tag{11}$$

$$r_j = \sqrt{D_2^{\,2} + r^2 + 2D_2 r \sin(\varphi)} \tag{12}$$

where: D_1 is the reuse distance (see Fig. 3a) and it is equal to (Walke, 2002):

$$D_1 = \sqrt{3N}R \tag{13}$$

where: R is radius of the cell, and N is the cluster size. D_2 is the distance to the second tier (see Fig. 3b) and it can be proven to be equal to;

$$D_2 = 3 * \sqrt{N}R \tag{14}$$

Throughout this paper, it will be assumed that the co-channel interfering signals add up incoherently since this leads to a more realistic assessment of the co-channel interference in cellular systems (Prasad & Kegel, 1991). Also, since the signal powers of both the desired and interfering mobiles experience fluctuations due to multipath fading, shadowing, and the random location of users in their respective cells, γ_d is also a random variable which depends on the distribution of P_d and P_1.

4. Users Location Model

For analytical convenience, all the mobiles (desired and interfering users) are assumed to be mutually independent and uniformly distributed in their respective cells. Thus, the probability density function (PDF) of the mobile's polar coordinates (r, θ) relative to their BS's are (Alouni & Goldsmith, 1999):

$$f_\tau(r) = \frac{2(r - R_o)}{(R - R_o)^2} \tag{15}$$

$$f_\theta(\theta) = \frac{1}{2\pi} \tag{16}$$

where:

$R_o \le r \le R$ and $0 \le \theta \le 2\pi$

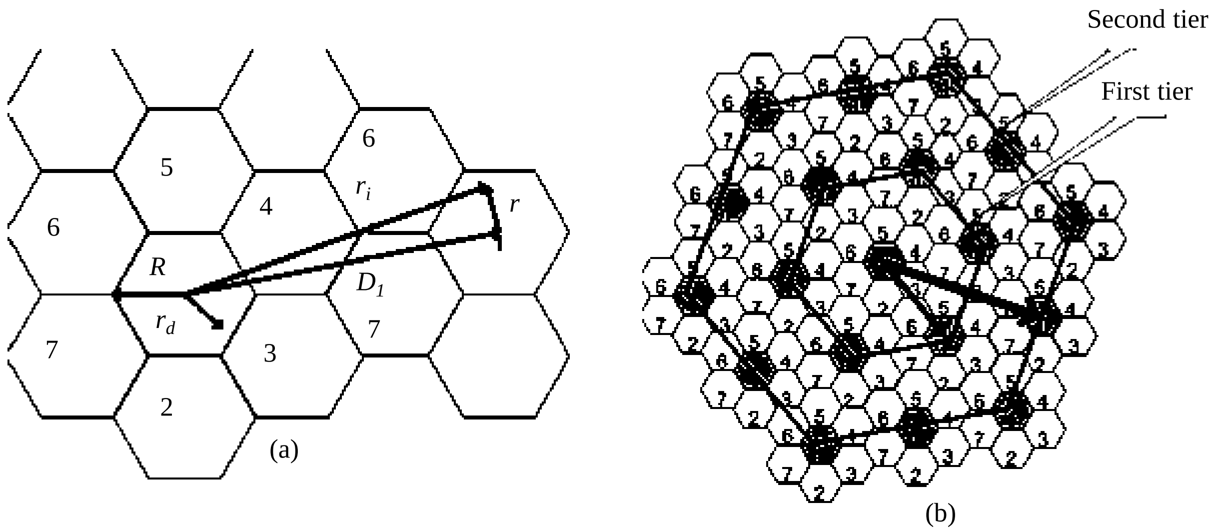


Figure 3. A 7-cell reuse system which shows: (a) the cell radius R and the distance r_i from a random MS to the desired BS; and (b) the first and second tier cochannels at distances D_1 and D_2 (thick arrow in the diagram), respectively from the desired BS. Notice the 6 and 12 cochannels that exist in the first and second tiers, respectively.

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R_o corresponds to the closest distance the mobile can be from the BS antenna, and is taken to be equal to 20 m for microcellular systems and 100 m for macrocellular systems.

5. Spectral Efficiency

In this paper, the concept of spectral efficiency (SE) for fully loaded systems will be considered. In the full - loaded system, the cell's resource (serviced channels) is fully and the number of interferers is constant and equal to *n*, as seen in Eq. (9). SE of a cell is defined as the sum of the maximum bit rates/Hz/unit area supported by a cell's BS (Alouni & Goldsmith, 1999). Since frequencies are reused at a distance D1, which covers the area of a single cluster of size N. The area covered by one cluster is ($3\sqrt{3}NR^2/2$). SE [b/s/Hz/m^2] is therefore given by;

$$SE = \frac{\sum_{k=1}^{N_S} C_k}{W(3\sqrt{3}NR^2/2)} \tag{17}$$

where: N_s is the total number of active serviced channels per cell; C_k [b/s] is the maximum data rate of the *k*-th user; and *W* [Hz] is the total allocated bandwidth per cell. C_k is taken to be the Shannon capacity of the *k*-th user in the cell which depends on γ_k, the received CIR of that user, and W_k, the bandwidth allocated to that user. *SE* quantifies the tradeoff between the increased system efficiency induced by a small frequency reuse and the decreased capacity of each user resulting from the corresponding increase in co-channel interference

For a constant γ_k , C_k is given by Shannon's formula:

$$C_k = W_k \log_2(1+\gamma_k) \tag{18}$$

However, γ_k is not constant in our system since both the interference and signal power of the *k*-th user will vary with mobile's locations and propagation conditions. When γ_k varies with time, C_k equals to the average channel capacity of the k-th user (Alouni & Goldsmith, 1999), and it is given by:

$$\overline{C}_k = W_k \int_0^{+\infty} \log_2(1+\gamma_k) f(\gamma_k)\, d_{\gamma k} \tag{19}$$

where $f(\gamma_k)$ is the PDF of the k^{th} user's CIR. It is possible to define the average spectral efficiency $\overline{SE}$ [b/s/Hz/m^2] as the sum of the maximum average data rates/Hz/unit area for the system, given by eqn.17, with C_k replaced by $\overline{C_k}$. Assuming that all users are assigned the same bandwidth then $\overline{C_k}$ becomes the same for all users and will be replaced by $\overline{C}$. $\overline{SE}$ can therefore be written as;

$$\overline{SE} = \frac{2 N_s \overline{C}}{3\sqrt{3} W N R^2} \tag{20}$$

Substituting for $\overline{C}$ from eqn.19 in to eqn.20 yields;

$$\overline{SE} = \frac{2}{3\sqrt{3} N R^2} \int_0^{+\infty} \log_2(1+\gamma) f_\gamma(\gamma)\, d_\gamma \tag{21}$$

where it has been assumed for TDMA or FDMA systems.

$$W = N_s W_k \tag{22}$$

6. Results and Discussion

The models discussed in previous sections were used to study effect of different parameters on the spectral efficiency of microcellular and macrocellular systems. The simulations assumed different types of sectorization. The investigations were made at the two frequencies of 900 MHz and 2 GHz bands that are typically used in modern cellular systems such as the global system for mobile communication (GSM) and the universal mobile telecommunication system (UMTS). Also, the effect of different propagation conditions on spectral efficiency wass considered.

Figure 4 shows variations of $\overline{SE}$ with cell radius in microcellular and macro-cellular systems at a frequency of 900 MHz. In Fig. 4, the results of using different number of sectors were compared to each other and to the case of using an omni-directional antenna, which is shown in this figure as the case of *s*=1. It was clear that increasing number of sectors per cell causes an increase in $\overline{SE}$ by up to 70% in comparison with the system that uses an omni-directional antenna (see Figs. 4a and 4c). $\overline{SE}$ decreased rapidly with increase in the cell radius. When using a curve fitting, it was possible to show that the decay in $\overline{SE}$ was proportional to the square value of the cell radius. When the cell radius increased, the co-channel interference decreased. This effect enhanced the $\overline{SE}$ value. On the other hand, increasing the cell radius increased the area covered by a constant number of channels, which reduced $\overline{SE}$. It seems that the second parameter had the higher influence on $\overline{SE}$ and the overall effect was towards decreasing $\overline{SE}$.

In the model presented in this paper, the effect of the second tier interferes is included alongwith the effect of the first tier interferes. To show the error that may occur when neglecting effect of the second tier interferers, the simulation was repeated for one case that is of a microcellular system with six sectors. The results are shown in Fig. 4a. It is obvious that the error could be as high as 30%. This justifies the general model presented in this paper.

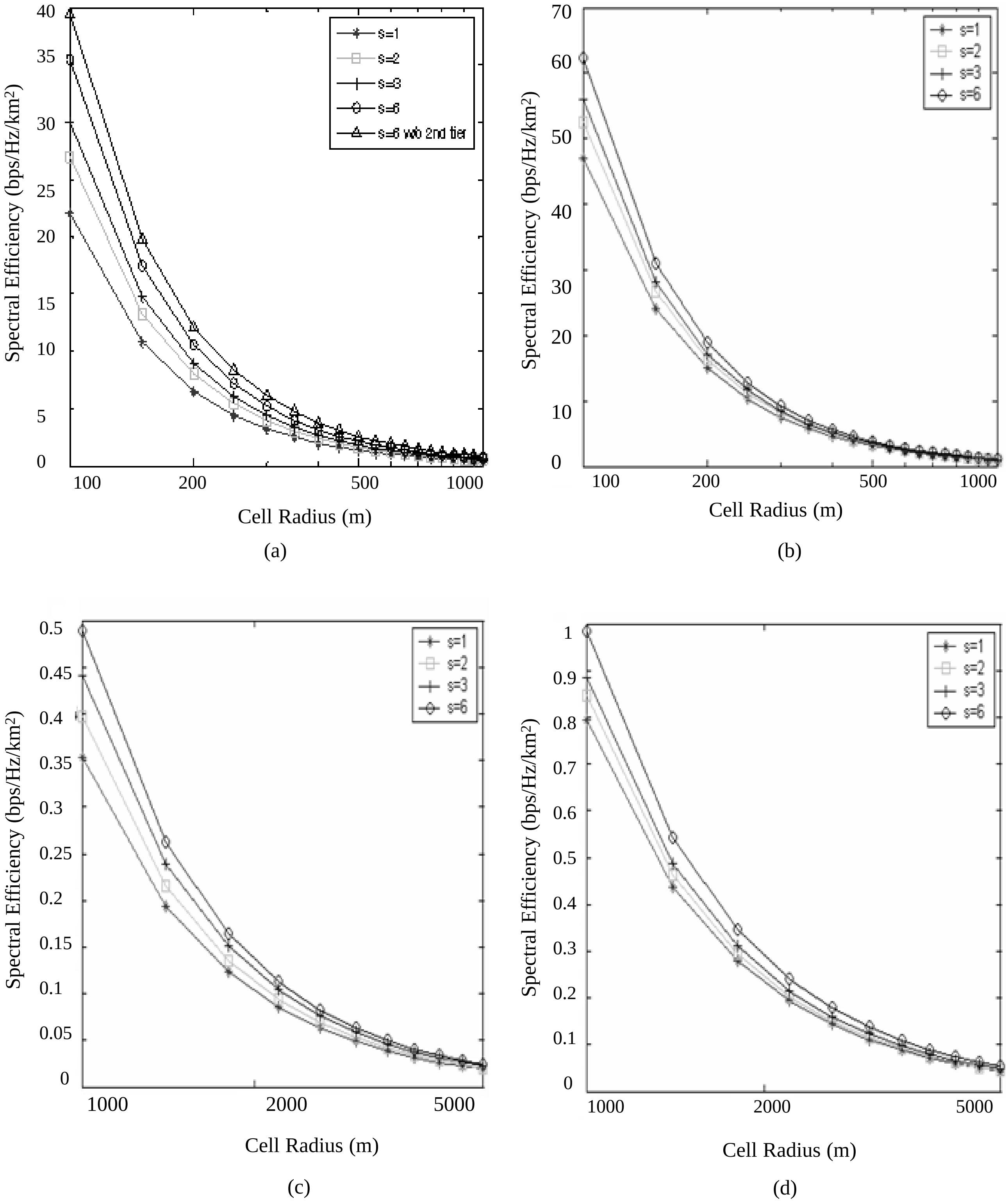

Figure 4: Variation of spectral efficiency with cel l radius for different sectorization values at f=900 MHz: (a) Microcellular system with α =2, β =1; (b) Microcellular system with α =3, β =2; (c) Macrocellular system with α =2, β =1; and (d) Macrocellular system with α =3, β =2. Results of simulation by a model that neglects effect of the second tier interference is shown in (a) for the case *s*=6.

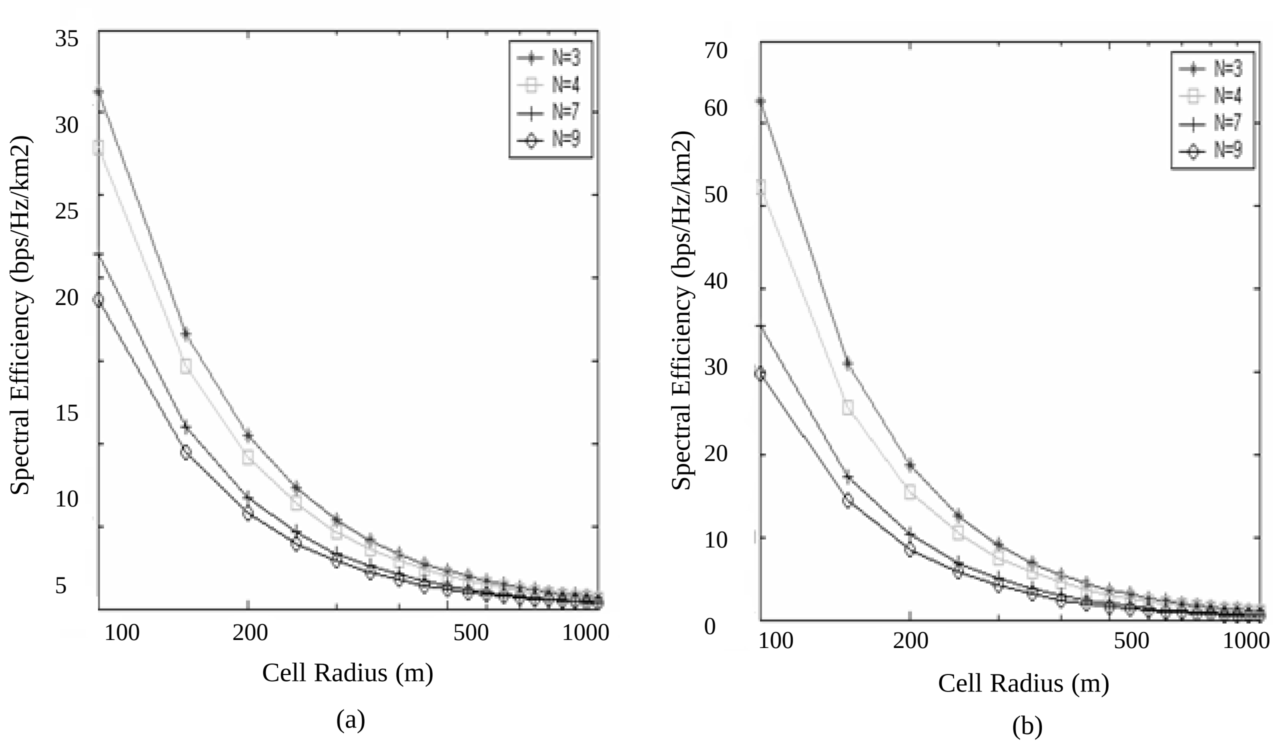

Figure 5. Variation of spectral efficiency with cell radius for different cluster size values; (a) f=900 MHz, α =2, β =1, *s*=1; and (b) f=900 MHz, α =2, β =1, *s*=6.

Figure 5 shows variations of $\overline{SE}$ with the cell radius for different values of cluster size *N*. It is shown that when *N* increases $\overline{SE}$ decreases by a rate which can be proven using curve fitting to be proportional to the square root of *N*. The lower number of cells per cluster, the greater the number of channels that can be used per cell. However, frequencies reused within a short distance increase the co-channel interference. As shown in Fig. 5, the overall effect is that reducing the cluster size enhances the spectral efficiency. Hence, from a design viewpoint, the smallest possible value of *N* is desirable in order to maximize $\overline{SE}$.

7. Conclusions

In this paper, a general mathematical model was presented to study the effect of cell sectorization on the spectral efficiency of cellular systems. The presented model took into consideration the effect of all valuable interference, *i.e.* in the first as well as in the second tier regions. The model considered also the random distribution of users over the whole area. The combined effects of the lognormal fading due to shadowing and Rayleigh fading due to the multipath propagation were included through the use of Suzuki Model. The effect of using different values of cluster size on the spectral efficiency was also investigated.

Results of simulation indicated that an improvement of up to 70% in the spectral efficiency value can be obtained through the use of a six-sector system in comparison with the omni-directional system. It was shown that the spectral efficiency variation was inversely proportional to the square value of the cell radius. It improved as the path loss exponent increased. An increasing cluster size would cause a decay in the spectral efficiency by a rate proportional to the square root of the cluster size.

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