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Mathematics Education of Pre-Service Teachers: As Reflected in Methods Course Syllabus

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Introduction

The Merriam- Webster's dictionary defines *syllabus* as a summary, a course of study, or an outline. Matejka and Kurke (1994) proposed that there are four key functions of a syllabus. The syllabus represents a legal agreement between the instructor and the student, the student and the university, and the instructor and the university; it is a communication device regarding the learning outcomes and goals of a program of study; it is a plan or a description of events to occur within the course; and it is a cognitive map outlining a way in which knowledge will be shaped by the content of the course.

Burkhardt, Fraser, and Ridgway (1990) provide comprehensive definitions for various kinds of *curricula* where the term *ideal curriculum* may add to the understanding of the role of a syllabus. According to these authors,

The *ideal curriculum* is what experts propound; because it is not firmly grounded in relevant experience. The ideal curriculum is fundamentally speculative but important in defining directions for change that should be pursued. The *implemented curriculum* is what teachers actually teach in the classroom; because teachers vary enormously in their capabilities, hence, there is a wide distribution of implemented curricula. The *achieved curriculum* is what the students actually learn; its distribution is even wider across many variables. The *tested curriculum* is determined by the spectrum of tests which vary public credibility, and through that, influence what happens in classrooms (Burkhardt et al., 1990, p. 5).

A syllabus can be viewed as a representation of what Burkhardt, Fraser, and Ridgway call "ideal curriculum" (Burkhardt et al., 1990, p. 5) and what Deng (2011) refers to as one of the major constituents of programmatic curriculum. The syllabus is therefore an important first glimpse for students to know what matters most to a discipline and to their learning by providing information on the topics to be studied, and the planned activities of the course.

The ideal curriculum is particularly important in mathematics teacher education. It signals important theory to practice connections from research that are relevant for advancing learning and understanding for students. In this paper, we concern ourselves

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with the course content in mathematics method courses, also referred to as mathematics pedagogy and school mathematics. As we describe shortly, we specifically adopt a framework of analysis that is derived from research literature in mathematics education. The objective of this study is to examine the extent to which key opportunities to learn mathematics method courses by pre-service mathematics teachers, as recommended by researchers, is evidenced in course syllabi. The question guiding the research was: *How are recommendations derived from research for the mathematics education of pre-service teachers reflected in mathematics methods course syllabi?*

Some studies have attempted to identify ideal curriculum. For example, the recent Teacher Education and Development Study (TEDS-M) (Tatto, 2013) provided cross-national data and shared terminology on what, and how, teachers learn in teacher education programs around the world (Tatto, Lerman, & Novotna, 2010; Blömeke, 2014). Others have investigated specific components of teacher education such as mathematics concepts and knowledge pre-services teachers need for teaching and the links between these components (Hill, Rowan & Ball, 2005). This understanding contributes to conversations on teacher knowledge and practices as well as to conversations on improving the future teachers' effectiveness in the classroom.

Certain studies have focused on areas such as course goal, course content, course structure, instructional approaches, and assessment (see for examples, Little, 2009; Monoranjan, 2015; Sinay, & Nahornick, 2016). TEDS-M also administered a content test questionnaire to teacher candidates (Tatto, 2013). This questionnaire was administered to determine the "effectiveness of a course/content arrangement" [in teacher education programs] (Hsieh, Law, Shy, Wang, Hsieh, & Tang, 2011, p. 180) by focusing on "organization of sequences, links of the course/content, and whether the courses/content met the needs of future teachers" (Hsieh et al., p. 180).

TEDS-M not only studied the achieved curriculum by future teachers at the end of their teacher-education programs (TEP), it also collected data on "syllabi and sample assignments from teacher education mathematics curricular" (CMEC, 2010, p. 7). Canãdas, Gómez, and Rico (2013) maintain that analyzing the content dimensions of the syllabi of courses offered by teacher education institutions was a useful data source, which was more important than using student and instructor self-reported data on the learned curriculum. In certain studies, instructors investigated syllabi (e.g., Burton, 2003) for the course which they taught, or syllabi at their institutions (e.g., Corlu, 2013), or syllabi in their own countries (e.g., Canãdas et al., 2013). Also, Canãdas et al. (2013) studied the content for Spanish primary teachers' training programs. We found few studies on syllabi of mathematics pedagogy or mathematics knowledge at the university level that offered a comparative analysis on proposed ideal curriculum. Our research hopes to contribute in this way.

This study serves three goals: (a) introducing an instrument that may be useful in supporting the conceptual thinking of instructors when developing mathematics methods course content, (b) providing an examination of the extent to which key assertions within the field related to the mathematics education for future teachers materialize in course content, and (c) serving as a study of recommended elements for teacher education. It also has the potential to suggest ways to strengthen teacher preparation courses.

Our study examines syllabi for mathematics teacher-education courses in six countries. Cross-national studies are particularly important in an international climate of educational reform that emphasizes the need to promote learning for all students (Tatto et al., 2010). In short, we examine trends identified in course syllabi that signal what is deemed as important across the discipline. Our analysis provides these findings as well as commentary on areas that appear to be underrepresented.

Theoretical Framework

Cross-national comparative studies on mathematics teacher education have only emerged in the past decade. It is, therefore, commonplace for a study on this topic to develop its own framework for analysis (Blömeke, 2014). In line with the framework development, Blömeke, Suhl, Kaiser, and Döhrmann (2012) raised its importance and a need to review research internationally to discuss such issues.

Variations seem to exist in the literature among the components of pedagogy courses. For instance, Tatto et al. (2013) developed a framework on teacher education using data from 20 countries to analyze approaches to content in teacher education programs. Their framework which classified the overall curricular structure of teacher education, specifically inquiring whether mathematics content knowledge (CK), pedagogical content knowledge (PCK), or pedagogical knowledge (PK), were successfully addressed in the programs. On the question of content and pedagogy, Canãdas et al. (2013) following TEDS-M, considered four major categories of CK that should be taught to teachers in teacher preparation programs–school mathematics content knowledge, tertiary mathematics content knowledge, mathematics pedagogy and general pedagogy. These four categories of CK were considered the content categories of the seven broad opportunities to learn (OTL) for teachers (Tatto, 2013; Tatto & Senk, 2011). The seven OTLs identified altogether were: Mathematics content knowledge (MCK)–school and tertiary, mathematics education, general education pedagogy, teaching for diversity, reflection on practice, school experiences and the field experience, as well as overall coherence of the teacher education program. Table 1 summarizes categories identified in TEDS-M, Blömeke (2014), NCTM (2012), and Wang and Tang (2013) alongside the categories identified in our study of course syllabi.

Table 1: *Classification of mathematics pedagogical (content) knowledge, MPCK*

Mathematics education (Blömeke 2014; Monroe, 1984; NCTM 2012; Tatto, 2013; Tatto & Senk, 2011; Wang & Tang, 2013).	**This study & MTEd**
Teaching issues (e.g., planning, reflection, and foundations of mathematics)	Pedagogical content knowledge
Curriculum (e.g., school content and assessment)	Content knowledge Assessment
Learning issues (e.g., development of mathematics thinking, planning),	Technology Mathematical tasks
Contexts (e.g., equity and diversity)	Policy and politics of mathematics education Equity and diversity
Affective issues (e.g., motivational issues)	Affective issues in pedagogy and content
Professional competencies (e.g., teacher inquiry)	Reflection Lesson as unit of study
Field Experiences and Clinical Practices	Theory and practice connections

Blömeke et al. (2012) and Hsieh (2013) argue that examination of both teaching and assessment of future teachers would generate more insight into the relationship between teacher-education programs and teacher learning. Furthermore, it is clear from studies using TEDS-M data that opportunities to learn (together with teacher backgrounds), were closely positively correlated to teacher performance on the tests of mathematics knowledge and mathematics pedagogical knowledge (Canãdas et al., 2013). Still other distinctions such as teacher learning, and teacher quality are possible (Blömeke, 2014).

This current research is an extension of a study that proposed an evaluative tool, the *Mathematics Teacher Educator (MTEd) Instrument* (see Appendix A; Kotsopoulos, Morselli, & Purdy, 2011). The MTEd Instrument includes 11 categories of research related to mathematics pre-service teacher education and the rationale for these derives from the literature as described above and outlined in Table 1. These 11 are: reflection, mathematical tasks, lesson study, assessment, theory and practice connections, policy and politics of mathematics education, equity and diversity, affect, content knowledge, pedagogical content knowledge, and technology. We state up front, that other categories or classifications likely exist. Several of these categories also appeared independently in the TEDS-M framework. Given the contributions we hope to make in this paper, these categories are viewed by us as broad and yet encompassing enough to capture this field area of research. Our literature review consists of brief overviews of each of the 11 categories. More robust reviews are contained within the citations in each category.

Literature Review

Reflection

Artzt and Armour-Thomas (2002) contended that for teachers to develop their teaching practice, they must engage in reflection before, during, and after implementing a lesson. In the same way, prospective teachers must participate in those same forms of reflection throughout their teacher preparation program (Chung-Shu, 2006; García, Sánchez, & Escudero, 2007). Researchers argue that reflection helps teacher candidates to: Become active learners; think about mathematics; improve their professional skills; contribute to their understanding, especially of teachers' professional knowledge; and serve as teaching for diversity. Although reflection on practice in the TEDS-M framework was conceptualized to be outside of the opportunity in mathematics pedagogy, we argue that reflection is an integral component [tool] of pre-service mathematical methods courses.

Mathematical Tasks

NCTM (2012) breaks content into content knowledge and mathematical practices. It includes a distinct category of professional knowledge and skills about teachers continuing to learn. Pre-service teachers need to engage in mathematical tasks that allow them to develop a deeper understanding of mathematical content and student learning processes (Ching-Shu, 2006; Watson & Sullivan, 2008; Zaslavsky, 2007). Watson and Sullivan distinguish between *classroom tasks for students*, that is, "questions, situations and instructions teachers might use when teaching students" (p. 109) and *tasks for teachers*, that is, "the mathematical prompt" (p. 109) that teachers employ to increase their knowledge levels. Mathematical tasks for teachers serve as a basis for providing meaningful help to pre-service teachers to develop a blend of mathematical and pedagogical knowledge (Chapman, 2007).

Lesson Study

Teachers increasingly engage in inquiry or research to obtain reliable and updated information for their lessons. Teacher engagement in inquiry involves avenues such as collaborative inquiry, communities of inquiry, design research, lesson study and learning study (Lerman, 2014). In this study, we focus on lesson study, which is a process that is aimed at improving teaching practices among pre-service teachers by allowing them to collaboratively plan lessons, [to] observe one or more teachers while teaching the jointly planned lessons, and [to] collectively reflect upon the lessons (Dumm & Mojeed , 2015; Post & Varoz, 2008; Stigler & Hiebert, 1999).

Assessment

Generally accepted in mathematics education is the need for pre-service teachers to engage in assessment practices through the act of analyzing students' level of mathematization. Additionally, it is important for teachers to engage in a variety of assessment strategies (Cunningham & Bennett, 2009; Ketterlin-Geller & Yovanoff, 2009) since authentic assessment practices can encourage and support further student learning and teachers' own understanding of students' mathematical thinking.

Theory and Practice

Through theory-to-practice activities pre-service teachers make connections between research and practice when learning to teach (McDonnough & Matkins, 2010; Tsafos, 2010). They are given opportunities to engage in research that allows them to make practical connections between educational theory and practice.

Policy and Politics

Policy documents and educational reforms (e.g., the New Math reform of the Post-Sputnik era) pervade mathematics education. Curriculum and teaching standards and guidelines for teacher preparation programs (NCATE, 2008; NCTM, 2000), the institutional curriculum (Deng, 2011) that according to Doyle (1992) guides programmatic curriculum for teacher education. Apple (1992) and Johnston (2007) have verified the importance of pre-service teachers exploring and engaging with policy documents as a means of becoming familiar with regional and educational standards. Further, mathematics by itself according to Gutiérrez (2013), "operates with a kind of formatting power on our lives" and critical mathematics education researchers underscore the importance of engaging preservice teachers in exploring the politics of teaching mathematics.

Equity and Diversity

Numerous factors have been identified as contributing to the marginalization of certain populations of students, such as: pedagogical factors (Esmonde, 2009), Eurocentric mathematics (D'Ambrosio, 1985; Skovsmose, 1990), and teacher preparation (Sleeter, 2001). This research points to some serious deficiencies in the learning environments of several students, specifically when it comes to how teachers are prepared during their pre-service teacher education (Bartolo, Smyth, Swennen, & Klink, 2008). As a result, prospective teachers need to explore concepts of equity and diversity, so that they learn strategies for meeting the needs of diverse learners. The TEDS-M framework adequately addresses this concern by making ways of teaching diverse students an integral part of mathematics method courses for pre-service teachers.

Affect

Goldin (2002) stresses that the affective system is not merely an auxiliary to cognition; it is central to what the cognitive represents. Traditionally, four key components of affect are studied: emotions, beliefs, conceptions, and attitudes. Beliefs about teaching and learning mathematics deeply influence teachers' instructional practice (Philipp, 2007; Thompson, 1992). Affect influences teaching practice as much as the social context and the teachers' level of thought and reflection (Ernest, 1989) to the extent that, sometimes, it is difficult to distinguish beliefs from knowledge because "teachers treat their beliefs as knowledge" (Thompson, 1992, p. 127). Therefore, it is important for pre-service teachers to become aware of and begin to challenge and potentially change any unhelpful beliefs during their teacher preparation programs.

Content Knowledge

Over the last decade, numerous scholars have attempted to articulate the sorts of content knowledge required by future mathematics teachers, where the outgrowth of this research has become known as *mathematics for teaching* (M*f*T) (Adler & Davis, 2006; Ball, Hill, & Bass, 2005; Ball, Thames, & Phelps, 2008). Stylianides and Stylianides (2009) define M*f*T as the "mathematical content that is important for teachers to know and be able to use in order to manage successfully the mathematical issues that come up in their practices" (p. 161). MK*f*T allows pre-service teachers to know, implement mathematical content, and also solve mathematical problems (Ball & Bass 2000; Stylianides & Stylianides, 2009). The reason for acquiring the content knowledge is that most elementary teachers are noted to "have had little or no mathematics [education] since high school, and have found their high school mathematics difficult" (Jonker, 2008, p. 328). Therefore, to improve the effectiveness of teaching and students work, mathematics knowledge for teaching (Hurrell, 2013) would be an important aspect of pre-service teachers' education.

Pedagogical Content Knowledge

Much of TEDS-M study research focused on the knowledge of content learned in mathematics pedagogy courses. Shulman (1986) defined two different components of teachers' knowledge: content knowledge, also known as M*f*T as stated earlier, and pedagogical content knowledge (PCK). M*f*T and PCK are complementary pieces of the knowledge puzzle necessary for teaching mathematics. It appears that prospective pre-service teachers need opportunities to examine, develop, and analyze various pedagogical strategies to gain knowledge about the components of M*f*T and PCK including instructionally sound representations and how to approach students' learning difficulties (Shulman,1986).

Technology

According to Niess (2005), research regarding technology integration in mathematics teacher education has focused primarily on ways of using technology to enhance teaching and learning. Mistretta (2005) noted that this integration has brought a lot of enhancement into the teaching and learning environments. Freiman (2014) breaks down learning technologies into: Microworlds, Virtual Learning Communities, Applications and Task Designs, Mobile Learning, and Games. Educational reforms around technology have promoted the integration of various facets of these technologies into all classrooms (Greenhow, Robelia, & Hughes, 2009; Jonassen, Howland, Marra, & Crismond, 2008; Xiao & Carroll, 2007). Some of the teacher education practices championed in classroom technology include: a professor demonstrating or modeling the use of mathematics technology (Picha, 2018; Sturdivant, Dunham, & Jardine, 2009), opportunities for pre-service teachers to study mathematics technology (da Ponte, Oliveira, & Varandas, 2002), and to engage in authentic implementation of mathematics technology (Lin, 2008). Studies have shown that when pre-service teachers are provided with the opportunity to observe, investigate and implement technology, they maximize their current knowledge of technology integration in a mathematics classroom (Blubaugh, 2009; Niess, 2001). Even though this category did not appear in the TEDS-M, as evidenced in the review above, it is a major category in mathematics education research.

Methods

Participants

Mathematics methods course syllabi (English only) for pre-service teachers were solicited from professors/instructors of mathematics teacher education courses through two listserv mailing lists: The Psychology of Mathematics Education (PME) Listserv and the Canadian Mathematics Education Study Group (CMESG) Listserv. In total, 147 syllabi were submitted. Although our intent was to obtain syllabi from every continent, we were unable to do so despite numerous invitations.

The full syllabus set underwent a preliminary filtering to exclude (a) syllabi that were not consistent with the methods courses under investigation in this research (e.g., syllabi related to practicum/field experience, enrichment mathematics, mathematic content exclusive of pedagogy, graduate courses that had a narrow/conceptual focus), and (b) syllabi with less than 30 or more than 49 hours of instructional time. Hours of instruction were bracketed to ensure that appropriate comparisons were made across courses of similar length versus short courses or full year courses. This considerably reduced the data set. Multiple syllabi from one institution were not excluded, given that differences existed between the syllabi when examined; that is, different instructors prepared different versions of mathematics methods courses reflecting differing perspectives on essential components. Two researchers and one graduate student coded the syllabi and analyzed the data. The two researchers are specialists in mathematics education. Interrater reliability was determined by independently reviewing coding by two coders and any disagreements were resolved.

Data Sources

In total 31 syllabi were analyzed from six different countries, three of which–Canada, Malaysia, United States—also participated in TEDS-M. The mean length of the syllabi in the final sample was 9.3 pages and the mean number of course hours was 37.9 hours. Each of these courses spanned one academic term (approximately 12 to 13 weeks) and was deemed to be as close an approximation of similar hours as possible. To maintain anonymity for the course instructors, the syllabi were referred to by country and a sequential number (e.g., Canada 1, Canada 2, etc.).

The final syllabus data set was then broken down into two categories, elementary and secondary. Elementary refers to syllabi used in courses that prepare teachers to teach kindergarten to grade eight. Secondary refers to syllabi used in courses that prepare teachers to teach grades nine through to twelve. This grade breakdown between elementary and secondary reflects common groupings in Canada. The final dataset included 19 elementary syllabi and 12 secondary syllabi (see Table 2).

Table 2: Final Syllabus Dataset Sorted by Country (elementary vs. secondary)

		Level	
Country	Syllabi	Elementary	Secondary
Australia	5	5	0
Canada	6	6	0
Italy	2	1	1
Malaysia	1	0	1
New Zealand	3	2	1
United States of America	14	5	9
Total	31	19	12

Materials

The MTEd instrument used in this study is an analytical rubric that uses level one through to level four, to evaluate the extent to which the categories emphasized in the mathematics education literature and research in the literature review is evidenced in a syllabus. Low, moderate, and high tags were assigned to each syllabus based upon the cumulative level of the syllabus obtained using the MTEd Instrument. In a similar manner, Corlu (2013) developed and applied an analytical rubric to assess STEM courses at a university. The cumulative level was determined by adding up the levels from each of the individual categories, with one point for each level one, two points for each level two, three points for each level three, and four points for each level four. A syllabus was tagged as showing *low* evidence of research if it scored below 22 (total of one point for each of the 11 categories), *moderate* evidence of research if it scored between 22 and 32 (range of potential points if all categories scored below a level three), and *high* evidence of research if it scored higher than 32 (range of potential scores reflecting inclusion of one or more categories at a level four).

Data Analysis

Each of the syllabi contained in the final syllabus data set (n = 31) were coded using the MTEd Instrument. Items on the syllabus could be coded as representing two different categories or multiple instances of the same category. If multiple pieces of evidence were found for one category within one syllabus, then the highest level noted for that category was recorded. If no pieces of evidence were found for a category within one syllabus, then the category was given a level one. A ten percent reliability test of coding was conducted, and inter-rater reliability was 90%.

Descriptive statistics were computed to summarize overall levels across the eleven research areas analyzed. Qualitative examples of categories were identified. Correlation analysis was conducted to examine the relationship between category levels and overall levels assigned to each syllabus in elementary-only and secondary-only. Finally, a *Mann-Whitney U* test was conducted between two groups found in the final data set, elementary and secondary, to see if the distribution of levels varied in a statistically significant way.

Results

An overall level of *high* was achieved by eight syllabi since their overall score was 33 or higher. An overall level of *moderate* was achieved by nineteen syllabi since their overall score was between 22 and 32. An overall level of *low* was achieved by four syllabi since their overall score was 21 or lower. The overall score assigned to the syllabi ranged from 17.0 to 38.0. Of the eight syllabi that scored *high*, six were elementary, and two were secondary. Of the four syllabi that scored *low*, one was elementary, and three were secondary. Consequently, the evidence of research in the course syllabi was *moderate* overall.

Descriptive analysis (see Table 3) of the 31 course syllabi revealed variation across the research areas identified on the MTEd Instrument. Mathematical tasks (M = 1.77, SD = 0.80) and affect (M = 1.68, SD = 0.87) were the lowest represented on the syllabi. Additionally, equity (M = 2.94, SD = 1.41) and technology (M = 3.84, SD = 1.44) had the greatest amount of variance. Conversely, the three categories that showed low variance were: theory (M = 2.61, SD = 0.67), policy (M = 3.03, SD = 0.75), and content (M = 2.71, SD = 0.64).

Table 3: *Overall Descriptive Statistics of Syllabi (n = 31*)

				Range	
	N	*M*	*SD*	*Min.*	*Max.*
Course Hours	31	37.94	5.31	30.00	49.00
MTEd Categories					
Reflection	31	2.52	1.03	1.00	4.00
Tasks	31	1.77	0.80	1.00	4.00
Lesson Study	31	2.71	1.07	1.00	4.00
Assessment	31	2.16	0.90	1.00	4.00
Theory	31	2.61	0.67	2.00	4.00
Policy	31	3.03	0.75	1.00	4.00
Equity	31	2.94	1.41	1.00	4.00
Affect	31	1.68	0.87	1.00	4.00
Content	31	2.71	0.64	1.00	3.00
Pedagogy	31	3.03	1.02	1.00	4.00
Technology	31	3.84	1.44	1.00	4.00
Overall Score	31	28.00	5.28	17.00	38.00

Qualitative examples of the cells of the MTEd Instrument are provided in Table 4. The table does not distinguish between elementary and secondary examples. As we will show later, differences in means across cells between elementary and secondary syllabi were not significant except for content. Therefore, only one table of examples is provided.

Table 4: Qualitative Examples of MTEd Instrument Cells

Categories	Level 1	Level 2	Level 3	Level 4
Reflection	*(no reference to reflection)*	"[teaching assignment] Comments will focus on how successful the sequence of lessons was including in areas for improvement, and possible directions of future lessons." (Australia 4) *(only one type of reflection – posteri)*	"[teaching assignment] Following your peer teaching session you will view your lesson on tape and write a 2-3-page self-analysis/reflection paper using feedback from the instructor and students in the class." (USA 13) *(two types of reflection – initeri while watching tape and posteri after viewing)*	"[assignment] …weekly reflective journal" (Canada 1) *(a weekly journal is ongoing and thus requires all 3 types of reflection – priori, initeri, and posteri)*
Mathematical Tasks	*(no reference to mathematical tasks)*	"[lesson planning assignment] At least two examples of how to solve the problem you have chosen for the main part of the lesson. Solutions (showing various approaches) to the questions you are assigning for work-time and/or	"[assignment] Activity of problem solving to find an operation that is commutative and not associative. Activity of problem solving: the sum of the first 100 numbers. Activity of problem solving: the magic square." (Italy 1 - translated)	"[assignment] Three problem-solving assignments will be given to you to complete. The main goals of these assignments are for you to become a better problem solver yourself, to identify and develop strategies for solving problems…to reflect on your own approach and style in problem solving." (USA 4) *(extensive opportunity to engage in mathematical tasks)*

		homework." (Canada 6) *(opportunity to engage in only pupil level tasks)*	*(some opportunity to engage in mathematical tasks)*	
Lesson Study	*(no reference to lesson planning)*	"[assignment] …developing a unit of mathematics study (individually)… include…lesson plans (minimum of five)" (Canada 1) *(individual planning lessons but they are not enacted or reflected upon)*	"[course objectives] Design and implement a mathematics lesson in collaboration with practicum teacher." (USA 7) *(collaborative lesson planning and implementing those lessons but no reflection piece)*	"[assignment] Plan and teach a mathematics lesson…collaborate with your mentor teacher on a lesson that you will be responsible to teach. After conducting your lesson, you need to write a reflection on your assessment of the lesson" (USA 4) *(planned collaboratively, presented, and reflected upon)*
Assessment	*(no reference to engaging in assessment)*	"[student outcomes] …by the end of this course, students should be able to describe a variety of formative and summative assessment techniques" (Canada 1) *(limited opportunity to analyze student level work because the candidate is only required to describe assessment techniques)*	"[course content] Assessment of children's mathematical understanding, performance, and disposition." (USA 4) *(some opportunity to analyze the different aspects of student level work)*	"[student outcomes] Developing understanding of curriculum in context by assessing students' work, mathematical problems and/or texts." (USA 10) *(extensive opportunity to analyze student level work and other aspects of the mathematics program)*

Theory and Practice Connections	*(reference only to textbook and no other research)*	"[assignment] …article and reading summary paragraph." (Canada 1) *(limited opportunity to engage with research since highly structured introduction to research literature)*	"[assignment] …assume responsibility for reading, reporting on, and presenting three practitioners' articles…presentation should include an overview of the concepts…along with the group's critique or reflections." (USA 7) *(some opportunity to engage with research through course being grounded in research but no chance to engage in their own inquiry/research)*	"[section under each class schedule with research links] Linking Theory and Practice" (Canada 3) "[inquiry project assignment] … engage in teacher/action research…actively involved in asking questions aimed at understanding or improving teaching." (Canada 3) *(extensive and authentic engagement in research with links to current research and engagement in their own inquiry/research)*
Policy and Politics of Mathematics Teaching	*(no reference to curriculum documents or political aspect of education)*	"[course description] The course provides participants the opportunity to be familiar with the organization of mathematics through the BC' s math curriculum" (Canada 1) *(limited evidence of policy exploration*	"[lesson topic] …familiarization with the content standards of NCTM, the Ontario Curriculum, and additional Ministry documents (e.g., Expert Panel reports and support documents)." and a list of supplementary journal readings (Canada 6)	"[course objective] Critique national assessment practices and tasks for mathematics" and a list of supplementary journal readings (Australia 1) *(extensive evidence of policy exploration due to additional readings and critique of national standards)*

		since curriculum document stated but no extra journal readings required)	*(some evidence of policy exploration due to additional readings and one class discussion)*	
Equity and Diversity	*(no reference to the exploration of equity and diversity issues)*	"[learning objectives] …developed an understanding of…suitable teaching approaches for addressing anxiety and other mathematical phobias." (Australia 5) *(limited evidence of equity exploration due to narrow focus on mathematics specific phobias and not the diverse needs of contemporary students)*	"[lesson topic] Multicultural Mathematics" (Canada 1) *(a topic for a class but not an overriding concept for the entire course)*	"[course objectives] …apply their understanding of student differences and needs in the classroom to promote quality mathematics for all students." (USA 9) *(equity statement and an overriding concept for the entire course)*
Affect	*(no reference to addressing affect issues)*	"[course assignment] Mathematics Autobiography… write your ideas, attitudes and beliefs about mathematics…" (USA 13)	"[generic skill] Students will develop… confidence in addressing personal conceptual and skill-based knowledge of mathematics during class activities." (Australia 4)	"[course framework] Reflecting Professionally - How does my relationship to math, my math thinking, and my teaching change over time?" (Canada 2) *(addresses, challenges, and potentially changes affect)*

		(addresses affect but does not try to challenge or potentially change it	*(addresses affect and challenges students' confidence, but it does not try to potentially change affect)*	
Content Knowledge	*(no reference to content knowledge exploration)*	"[course schedule] Algebraic Thinking [and] Geometry" (USA 10) *(engaged in only two selective components of content knowledge)*	"[course schedule] Geometry and Measurement [and] Number Concepts and Operations [and] Patterns and Place Value, Fractions [and] Percent, and Decimals, Statistics and Probability Data Analysis." (Canada 1) *(engaged in content knowledge at student grade level but not taken beyond it)*	*(no syllabi received on this level since none evidenced engagement in broader ranges of content knowledge beyond the level of instruction of the students)*
Pedagogical Content Knowledge	*(no reference to pedagogical discussion)*	"[learning outcomes] …on successful completion of this course, students should be able to access strategies to implement…relevant pedagogy." (Australia 2) *(examine pedagogical*	"[course description] …pragmatic activities involving the development and implementation of effective teaching and learning strategies." (USA 12) *(examine and develop pedagogical strategies but no analysis of*	"[course objectives] Be immersed in, discuss when and how, and implement the use of different instructional strategies appropriate for teaching mathematics, including whole class, small group, cooperative learning, and individual instruction." (USA 7) *(examine, develop, and analyze pedagogical strategies)*

		strategies but no development or analysis of pedagogical strategies)	*pedagogical strategies)*	
Technology	*(no reference to the use of technology)*	"[course topic] Technology" (Canada 5) *(didactic method of technology investigation since technology is limited to a course topic to be covered by the professor)*	"[assignment] …lesson plans…one based on the use of technology" (Canada 1) *(some evidence of investigation into technology but limited to one lesson plan opposed to integrating technology into an entire unit)*	"[technology use statement] Utilize technology as a resource for your own learning and the learning of children." (USA 4) *(extensive evidence of investigation into technology since it is an overriding concept for the entire course)*

Correlations in the Elementary Syllabus Data Set

Correlation analysis of elementary syllabi revealed a statistically significant very strong positive relationship between technology and overall score (*r* = .841, *p* = .000). Additionally, numerous statistically significant strong positive relationships were found including: reflection and pedagogy (*r* = .492, *p* = .016), reflection and overall score (*r* = .484, *p* = .018), lesson study and assessment (*r* = .571, *p* = .005), lesson study and technology (*r* = .418, *p* = .038), lesson study and overall score (*r* = .666, *p* = .001), assessment and technology (*r* = .477, *p* = .020), assessment and overall score (*r* = .649, *p* = .001), theory and policy (*r* = .520, *p* = .011), policy and overall score (*r* = .521, *p* = .011), equity and technology (*r* = .586, *p* = .004), equity and overall score (*r* = .575, *p* = .005), and pedagogy and overall score (*r* = .579, *p* = .005). Finally, a statistically significant moderate positive relationship was found between policy and pedagogy (*r* = .392, *p* = .048)

Correlation analysis of elementary syllabi also revealed statistically significant strong negative relationships including: course hours and lesson study (*r* = -.446, *p* = .028), course hours and assessment (*r* = -.459, *p* = .024), mathematical tasks and equity (*r* = -.490, *p* = .017), and mathematical tasks and content (*r* = -.403, *p* = .044). Therefore, mathematical tasks were negatively related to a focus on equity and content, and more course hours did not suggest more lesson study or more evidence of assessment. Important to note, course hours and overall score were negatively related and not statistically significant (*r* = -2.99, *p* = n.s.).

Correlations in the Secondary Syllabus Data Set

Correlation analysis of secondary syllabi revealed a statistically significant very strong positive relationship between technology and overall score (*r* = .772, *p* = .002). Additionally, numerous statistically significant strong positive relationships were found including: course hours and theory (*r* = .507, *p* = .046), course hours and affect (*r* = .510, *p* = .045), reflection and lesson study (*r* = .641, *p* = .012), reflection and overall score (*r* = .600, *p* = .020), lesson study and assessment (*r* = .647, *p* = .012), lesson study and overall score (*r* = .562, *p* = .029), assessment and theory (*r* = .554, *p* = .031), assessment and policy (*r* = .514, *p* = .044), assessment and overall score (*r* = .664, *p* = .009), policy and overall score (*r* = .586, *p* = .023), equity and pedagogy (*r* = .553, *p* = .031), equity and technology (*r* = .555, *p* = .031), equity and overall score (*r* = .499, *p* = .049), and pedagogy and technology (*r* = .635, *p* = .013).

Correlation analysis of secondary syllabi also revealed statistically significant strong negative relationships between mathematical tasks and theory (*r* = -.696, *p* = .046) and mathematical tasks and affect (*r* = -.507, *p* = .046). More evidence of mathematical tasks was negatively related to evidence of theory or affect components to the syllabi. Important to note, course hours and overall score were not related and not statistically significant for secondary syllabi as well (*r* = .084, *p* = n.s.).

Mann-Whitney U

A Mann-Whitney U test was conducted between the elementary data-set and the secondary data-set and the results indicated that evidence of content was greater in the elementary syllabi (Mean Rank = 18.11) than in the secondary syllabi (Mean Rank = 12.67), *U* = 74.00, *p* = 0.18, *r* = .42 and this was statistically significant. There were no other statistically significant differences found across any of the other MTEd Instrument categories. Therefore, other than around content, evidence of research representing the MTEd Instrument categories across both the elementary and secondary syllabi was consistent.

Discussion

Results suggest that only *moderate* levels of the dominant areas of research in mathematics teacher education were found in the syllabi that were analyzed in this research. Technology and assessment were the only categories correlated across both datasets to overall score. The elementary and secondary course syllabi only differed in content, where elementary syllabi were shown to have a higher overall level. Finally, course hours were not related to overall score. This implies that secondary courses likely include more content, and this perhaps makes intuitive sense. These findings also imply that an overall score, which would suggest high levels of evidence of the categories explored in this research, are not influenced by simply more classroom time.

It is important to note from this study's result that while the analysis of course syllabi showed *moderate* or no evidence of correlations between certain categories, such as course hours and tasks, opportunities to learn these may still emerge when instructors connect to them in classroom practices in the implemented curriculum. However, the absence of this evidence has led to questions about the importance of transparency of course content for students. Numerous scholars have argued that this sort of transparency is essential and that it may indeed be why some institutions have policy statements regarding course syllabi (Matejka & Kurte, 1994). Conversely, it is important to also note that certain items may be stated in a syllabus but may not necessarily be implemented in the classroom, while other items may be implemented that may not be stated at the onset of the syllabus.

The categories of mathematical tasks and affect have the lowest mean levels and are thus the two categories least represented on mathematics teacher educators' syllabi. It has been proposed that pre-service teachers engage in mathematical tasks to develop a deeper understanding of content and learning processes (Chapman, 2007; Watson & Sullivan, 2008). Yet, this does not appear to be widely evident on the syllabi examined. This finding is worth sharing because mathematical tasks is one of the important attributes to assist teachers in engaging students in a meaningful learning. One possible reason for the lack of representation of mathematical tasks on mathematics teacher education syllabi might be that research on pre-service education in this area is still at an early stage. This is not to say that there is no historical research on the importance of mathematical tasks, but rather that current research is focusing more on pre-service mathematical tasks, and giving consideration to the importance of both student-and teacher-level mathematical tasks (Watson & Sullivan, 2008).

In contrast, research on affect is robust and stretches across many years, which makes it unusual that so few syllabi reference affect. Interestingly, affect is proposed to influence teaching practice as much as the social context and the teachers' level of thought and reflection (Ernest, 1989; Watson & Sullivan, 2008). Perhaps the only justification for the lack of representation of affect in mathematics teacher education syllabi is that sometimes it becomes difficult to distinguish beliefs from knowledge, because most "teachers treat their beliefs as knowledge" (Thompson, 1992, p. 127). As a result, it could be inferred that mathematics teacher educators overlook affect when planning their pre-service teacher preparation programs because the syllabi already represents, at least implicitly, their orientation towards affect.

Equity and technology have the greatest amount of variance in terms of the levels received from syllabus to syllabus, and this is not surprising. This may be because most syllabi either mentioned equity and/or technology once in their overriding course goals section or not at all. Conversely, the three categories that showed the least amount of variance about the levels received from syllabus to syllabus were theory, policy, and content. The low variance of levels received suggests that mathematics education is a political endeavor that is closely prescribed by policy and needs to be followed by teachers and taught to pre-service teachers.

Mathematics teacher education research around content knowledge has been extensive and vigorous, particularly over the past decade (Adler & Davis, 2006; Ball, 2000; Ball et al., 2005; Ball & Grevholm, 2008; Stylianides & Stylianides, 2009). As stated in the literature review, numerous scholars have attempted to articulate the sorts of content knowledge required by future mathematics teachers. Many outgrowths of this have occurred (Ball & Grevholm, 2008; Blömeke 2014; NCTM, 2012; Tatto, 2013; Wang & Tang, 2013). Overall, the extensive research available on theory-to-practice connections and content knowledge may explain why pre-service teacher educators include these areas of research in their program and thus, why these two categories showed the least amount of variance about the levels received from syllabus to syllabus.

Content knowledge was the only category on the MTEd Instrument that displayed a statistically significant difference between elementary and secondary syllabi (i.e., content knowledge was observed more on elementary syllabi than on secondary syllabi). This may be due in part because secondary teachers likely have more background in mathematics education discipline and thus, mathematics teacher educators may assume that content knowledge is not necessarily a crucial aspect of their teacher education program. This finding is in line with research which notes that elementary teachers, on the other hand, tend to have a smaller number of mathematics courses, and rather explore more content knowledge and in ways that make it less difficult (Jonker, 2008).

The literature on assessment points out that opportunities for pre-service teachers to engage in the analysis of student level diagnostic, formative, and summative assessment tasks allow them to gain the necessary knowledge and understanding needed to teach mathematics (Cunningham & Bennett, 2009; Ketterlin-Geller & Yovanoff, 2009; Xu & Liu, 2009). It could be argued then that assessment weaves through many stages of the teaching and learning processes and thus, an explanation for the relationship between assessment and overall score can begin with the realization that assessment is embedded into some of the eleven categories on the MTEd Instrument (e.g., lesson study, pedagogy, equity).

Educational reforms around technology have endorsed the advantages of integrating information and communication technologies (ICT) into all classrooms (Chai, Koh, & Tsai, 2010; Greenhow et al., 2009; Jonassen et al., 2008; Tan et al., 2006; Xiao & Carroll, 2007). This type of mass adoption of technology into all facets of teaching and learning is a relatively new and an evolving concept as it is still in its formative stages as stated by Tsai and Tsai (2019). One can therefore conclude that a mathematics teacher education course that integrates technology into its program demonstrates an approach to pre-service teacher education that is grounded in current research. Moreover, pre-service mathematics teacher education courses that incorporate technology into the classroom may also incorporate other educational reforms into their program and thus, a high degree of current research in their pre-service teacher education course syllabi may also be evident.

Another surprising result was that no statistically significant positive correlations (at p = .01 or below) existed between course hours and overall score. An increase in course hours did not potentially yield high score or more lesson study according to this research. Whereas this result may be interpreted that more time spent in a course may not necessarily lead to greater opportunities to learn for a pre-service teacher, it could mean that certain courses focus on in-depth opportunities for selected categories, leaving other categories for other methods courses.

When considering statistically significant correlations (at p = .01 or below) between course hours and individual MTEd Instrument categories, we see that one statistically significant strong negative relationship appears between mathematical tasks and course hours within the full data set. Again, this result could be interpreted that when course hours increase, the level for mathematical tasks activities decreases, and vice versa. A plausible interpretation could be that this category could have been the focus on another method course or experience.

Limitations of the study

There are some limitations in the study that will be addressed. First, the sample may not have been fully representative of all mathematics methods courses in a program, at a university or in the country, and may not have been vigorous enough to generalize the result. Hence, a larger sample size may have been needed, given the methodology adopted to reach data saturation. Second, the instrument may need further development both in category content and design to obtain deeper and richer data for generalization of results. It may be argued that the categories are not fully representative of the field. While this may be true, the preliminary contributions of this work and this instrument are viewed by us as still noteworthy.

The MTEd Instrument used a rating scale that was limited to level one through level four, which may have caused a compression of trends due to its small range. Alternatively, it could be argued that a different or a larger ranged grading scale may be appropriate. We recognize the limitations that the MTEd Instrument weighted all the categories equally when it could be argued that some of the eleven categories are more deferentially important to mathematics teacher education in different contexts. In future research, it would be important to use MTEd categories together with categories arising from more recent studies. Furthermore, the study does not consider how the syllabus is implemented in the classroom or the implications for pre-service teachers' learning and their subsequent practices in their own classrooms. We agree with Hora and Ferrare (2013) that firsthand observations of classroom practice and activities, the achieved and tested curriculum (Burkhardt et al., 1990) or classroom curriculum (Deng, 2011) would capture multiple dimensions of what is learned in mathematics methods courses and how this learning comes to life in practice.

These limitations should not diminish the importance of this preliminary work, given the important first message that a syllabus provides to a student about a discipline and their learning.

Suggestions for further research

Further research that validates the instrument would be important extensions of this work. For instance, the strong evidence of technology applied across the dataset, may demonstrate that technology is an excellent indicator of overall score on the MTEd Instrument. Hence, this measure could be used in place of an elaborate rubric to quickly evaluate the extent to which a pre-service teacher mathematics education program reflects current research. The MTEd instrument could also be very useful for instructors to evaluate intentionally if what is proposed to be optimal for learning in the research is reflected in their curricular plans and goals. The MTEd has established a way for teacher educators to self-evaluate what is included in their syllabi. Moreover, the more correlated categories in both data sets–technology and assessment–could be further studied to consider what they entail.

On the other hand, three categories–mathematical tasks, affect, and content knowledge–on the MTEd Instrument did not have any correlation with the overall score of the syllabi. The most surprising result is related to content knowledge. There is extensive research available on content knowledge and its importance in teacher development (e.g., Ball, 2000; Ball et al., 2005; Ball & Grevholm, 2008; Blömeke, 2014; NCTM, 2012; Tatto, 2013; Wang & Tang, 2013). So, it is surprising that no correlation exists between content knowledge and overall score. This raises a lot of research questions that would need further research, which would in turn replicate the present study.

It would also be of great interest to explore variation in teacher practices considering pre-service teachers who participated in courses that exhibited *low*, *moderate*, and *high* evidence of research, such as theory and practice connections, policy and politics of mathematics teaching within their course syllabi.

Finally, certain categories reflected on the syllabus may not result in implemented curriculum (Burkhardt et al., 1990) in the classroom; and, conversely, those categories not listed in the syllabi may nevertheless have been implemented. Further research would be useful to explore implemented curriculum in mathematics teacher education programs.

Conclusions

This study examined paper syllabi from six countries to analyze the intended learning experiences and the course effectiveness in mathematics education courses of pre-service teachers. Although pre-service teachers take a variety of courses, the focus of this research was limited to mathematics education courses.

This research found that recommendations in research related to the mathematics education of pre-service teachers were moderately represented in the course syllabi analyzed (according to the MTEd Instrument). Technology and assessment were the only two categories that proved to be correlated in both datasets, elementary and secondary syllabi differed on content, where elementary syllabi were shown to have a higher overall level. Moreover, in terms of representation on the syllabus, equity and technology had the greatest amount of variance, followed by three categories—theory, policy, and content— that showed low variance with mathematical tasks; whereas affect categories had the lowest representation on the syllabi.

Lastly course hours are not related to overall score, which suggests that more course hours may not necessarily result in pre-service teachers gaining qualitative differences in knowledge and understanding about research-informed practice. This study recommends future research to further examine the relationship between the number of course hours a mathematics teacher education course offers and the level of knowledge and understanding that pre-service teachers receive from that course.

Appendix A: MTEd. Instrument

Low evidence of research (overall score less than 22)	**Moderate** evidence of research (overall score from 22 to 32)	**High** evidence of research (overall score more than 32)

Categories	**Level 1**	**Level 2**	**Level 3**	**Level 4**
1. Reflection	**No** opportunities to engage in reflection.	Opportunities to engage in only **one** type of reflection.	Opportunities to engage in only **two** types of reflection.	Opportunities to engage in all **three** types of reflection (priori, initeri, and posteri).
2. Mathematical Tasks	**No** direct engagement with mathematical tasks.	Opportunities to engage only in **either** pupil or pre-service level tasks.	Some opportunities to engage in **both** types of tasks.	**Extensive** opportunities to engage in **both** types of tasks.
3. Lesson Study	**No** lesson planning.	Developing lesson plans **individually or collaboratively** that are **not** enacted. [No reflection piece]	Developing lesson plans **individually or collaboratively** that are **presented** to the class. [No reflection piece]	Developing lesson plans **collaboratively** that are **presented** to the class and **reflected** upon.
4. Assessment	**No** opportunities to engage in assessment.	**Limited** opportunities to engage in assessment and analyze pupil level mathematization.	**Some** opportunities to engage in assessment and analyze pupil level mathematization.	**Extensive** opportunities to engage in assessment and to analyze pupil level mathematization.
5. Theory and Practice Connections	**No** opportunities to engage with research. [e.g., only the textbook – no references to other research]	**Limited** opportunities to engage with research through course readings and discussions. [e.g., attempt made to introduce students to research literature – highly structured or select]	**Some** opportunities to engage with research through course readings and discussions (course is **somewhat** grounded in research and research is evident in the course content). [e.g., when a new topic is introduced the students are provided with links to current research]	**Extensive and authentic** opportunities to engage in and with research (course **is** grounded in research and research is evident in the course content). [e.g., when a new topic is introduced the students are provided with links to current research and in addition, the student has the opportunity to engage in their own inquiry or research]
6. Policy and Politics of Mathematics Teaching	**No** evidence of any exploration of the political aspects of mathematics education.	**Limited** evidence of exploration of the political aspects of mathematics education. [e.g., Regional Curriculum Documents]	**Some** evidence of exploration of the political aspects of mathematics education. [e.g., Regional Curriculum Documents - with **some** journal-type readings which further the discussion about the role of those documents]	**Extensive** evidence of exploration of the political aspects of mathematics education. [e.g., Regional Curriculum Documents - with **lots** of journal-type readings which further the discussion about the role of those documents, and the issues (i.e., high stakes testing)]

7. Equity and Diversity	**No** evidence of any exploration of the equity and diversity considerations in mathematics education.	**Limited** evidence of exploration of the equity and diversity considerations in mathematics education.	**Some** evidence of exploration of the equity and diversity considerations in mathematics education. [e.g., one lesson]	**Extensive** evidence of exploration of the equity and diversity considerations in mathematics education. [e.g., a diversity statement on the syllabi]
8. Affect	**No** evidence of addressing the implications of affect on the teaching of mathematics.	Evidence of **addressing** the implications of affect on the teaching of mathematics.	Evidence of **addressing and challenging** the implications of affect on the teaching of mathematics.	Evidence of **addressing, challenging, and potentially changing** the implications of affect on the teaching of mathematics.
9. Content Knowledge	**No** evidence of exploration of content knowledge at any level.	Engaging in a **selective component** of content knowledge **at the level of** instruction of the students.	Engaging in content knowledge **at the level of** instruction of the students.	Engaging in broader ranges of content knowledge **beyond the level** of instruction of the students.
10. Pedagogical Content Knowledge	**No** evidence of pedagogical discussion.	**Examine** pedagogical strategies. [e.g., limited opportunity for critical analysis]	**Examine** and **develop** pedagogical strategies. [e.g., some opportunity for critical analysis]	**Examine, develop,** and **analyze** pedagogical strategies. [e.g., extensive opportunity for critical analysis]
11. Technology	**No** evidence of technology integration.	**Didactic methods** of———technology investigation and implementation. [e.g., teacher-led only]	**Some** evidence of pre-service teacher investigation and implementation of technology. [e.g., one lesson]	**Extensive** evidence of pre-service teacher investigation and implementation of technology. [e.g., a unit of study or a technology-use statement in the syllabi]

Notes

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