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Article

Michael Scot and the Four Rainbows

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Abstract:
We apply a physical and historical analysis to a passage by the medieval scholar Michael Scot concerning multiple rainbows, a meteorological phenomenon whose existence has only been acknowledged in recent history. We survey various types of physical models to best decipher Scot's description of four parallel rainbows as well as a linguistic analysis of Scot's special etymology. The conclusions have implications on Scot's whereabouts at the turn of the 13th century.

Keywords:
Meteorology; multiple rainbows; optical dispersion; middle-ages

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Introduction

The rainbow is an impressive and fascinating natural phenomenon that is observed when sunlight interacts with raindrops in the air and involves the splitting of white light into its constituent colors. The splitting of light into its colors is also observed when white light passes through a prism. This latter effect has been investigated and explained by Isaac Newton and arises from optical dispersion.

Most of us associate the rainbow with the familiar single bow in the sky. Yet there are occasions in which four, or even more, rainbows are observed. The first, to the best of our knowledge, written record to that effect appears at the beginning of the thirteenth century and is due to Michael Scot, as quoted by Lynn Thorndike:

> It should be known that *four* bows, and maybe more, can be formed at once, at slight distance apart. And when so many are formed, people seeing them are much astounded, and then it is a sign of a small gathering of clouds in air, and those which are there are for the most part *rare and subtle in substance.* And that is a sign of *very little or no rain*, and they do not produce thunder but break into fragments and vanish. And such clouds appear very *zalla* and low and mountainous (*montuose*)*.* Black and thick clouds do not generate so many bows, also produce thunder and rain (…) (Thorndike 1965, 69)

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In the quote above, the term "zalla" is italicized (and will be subject to examination in the section entitled *"Possible Source of Unknown etymology")*. The quote originates from Michael Scotus, the Latinized form for Michael Scot (1175-1235 CE) (Thorndike 1965, Burnett, 1994, 2,101-126). Scot became part of history and legend as the court "astrologer" of Frederick II (Abufalia 1988, Benoist-Méchin 1997), ruler of the Holy Roman Empire. Although well viewed by the papal authorities around 1227, Scot would acquire the sinister reputation of a magician and wizard and would be condemned to the inferno in Dante Alighieri's epic poem, *The Divine Comedy* (albeit "rescued" much later in Sir Walter Scott's poem *Lay of the Last Minstrel)*. Michael Scot and his contemporary Fibonacci were members of the court of Frederick II and would play their part in transmitting much of the scientific knowledge of the Muslims (largely from Moorish Spain) into Europe (largely Italy and Sicily), thereby planting many of the seeds of the Italian "Renaissance" (Haskins 1927, Burnett, 1994, 2,101-126).

In Thorndike's work, reference is made to a 1959 edition of a book by Carl Boyer on rainbows (Boyer 1959), in which it is stated that "the quaternary rainbow arc is not known to appear in nature". Though Thorndike's book on Michael Scot was excellent, Thorndike was not a physicist and was not therefore in a position to evaluate this claim. As a matter of fact, four rainbows may indeed form simultaneously in the sky. Tertiary and quaternary rainbows in particular were spotted in 2011 by Michael Theusner (Theusner 2011) as reported by the BBC (Palmer 2011). If these recently observed quaternary rainbows are indeed related to the four bows described by Michael Scot (as illustrated in Figure 1), then Scot reported, in the 13th century CE, a sighting of a natural phenomenon not fully understood until the 20th century and not fully demonstrated until the 21st century! In particular, as discussed in this article, scholars in Scot's time did not believe in the possibility of formation of more than two bows in the sky. The precociousness of Scot's testimony is the subject of this article as it begs, amongst other things, the question as to the origin of the four observed bows.

Fig. 1 - Four Rainbows, an artistic impression.
Courtesy Zhao Jingying, Taiyuan, China.

This would be not the first time that scholarly experts had misread or underestimated records left by Michael Scot. Haskins for instance had examined a very detailed description in Latin concerning the medical case known as "Mary of Bologna" and dismissed Scot's record as a "calcified fibroid tumor" (Haskins 1927, 274). In the early 1970s however, the exacting detail of Scot's description enabled a new and different medical diagnosis: this was a very rare case of miscarriage or "spontaneous abortion", not followed by immediate expulsion, of *twin embryos*, dead at different dates and calcified (O'Neill 1973, 77-811, O'Neill 1974,125-9). In short, Scot had actually recorded, back in the 13th century, a rare medical case and this has not been fully appreciated until the 20th century! We note in addition that a recent analysis on the Fibonacci numbers (Scott & Marketos 2014) suggests that Scot may have played a role in the formulation of the Fibonacci sequence in Leonardo de Pisa's book "Liber Abaci", a further indication that a conclusive understanding of

Michael Scot and his contributions is yet in the making.

In order to understand the physical origin of Scot's reported observation, we present, in the following, the physics of the rainbow. We also discuss physical models that give rise to multiple rainbows while at the same time keeping in mind the context of the era in which Scot lived and worked. We therefore present, in addition to findings from scientific analysis, cultural, historical and even linguistic evidence, in an effort to unravel the mystery of Scot's sighting.

Primary and Secondary Rainbows

A "primary" rainbow, and a fainter "secondary" rainbow parallel to the first bow are shown in Figures 2 and 3. The primary rainbow results from a single internal reflection of refracted light inside a raindrop. In this case, light is refracted as it enters the raindrop, it is then reflected off the back of the drop, and is finally refracted as it leaves the drop (Figure 4). The color on the outside of the primary rainbow is red, leading through to violet on the inside. The secondary rainbow is formed from rays that are reflected twice within the drop before leaving the drop.

The angle of refraction depends on the frequency of radiation. This effect is known as dispersion and causes sunlight to split into its constituent colors on entering the raindrop. Red light is refracted by a smaller angle than blue light as it enters the drop and red rays turn through a smaller angle than blue rays on leaving the raindrop. Consequently, light in the primary rainbow is spread, with a maximum intensity at an angle of 40° — 42° (About Rainbows – UCAR, 2013) (see Figure 3). The angle of 42° corresponds to what is called an *anti solar point* which is the imaginary point on the celestial sphere exactly opposite the Sun. The anti solar point is the center of rainbows and can be easily identified: on a sunny day, it is located at the shadow of one's head. Secondary rainbows, caused by two total internal reflections of sunrays before they leave the raindrop, appear at an angle of 50 — 53°. As a result of the second internal reflection, the colors of a secondary rainbow are inverted compared to those in the primary bow, with blue on the outside and red on the inside. As some light nevertheless escapes the drop at each internal reflection, the secondary rainbow is fainter than the primary because a) the intensity of the light that is transmitted is smaller in this case (due to the two internal reflections) and b) the secondary rainbow itself is spread over a greater angle in the sky.

A more thorough study of the interaction of sun rays with raindrops requires knowledge of the physics of *reflection, refraction and dispersion.* The law of reflection is easy to understand and was already known in the middle-ages. The path of any ray hitting the drop can be determined using *Snell's law* and simple trigonometry. Dispersion on the other hand arises from the dependence of the angle of refraction on the frequency (wavelength) of radiation.

The law of refraction was first accurately described by Ibn Sahl, of Baghdad, in the manuscript "On Burning Mirrors and Lenses" in 984 CE (Rashed, 1990, Wolf, 1995). This law would not be rediscovered in Europe until the 1600s and then credited (1621 CE) to Willibrord Snell. Snell's law states that:

$$\frac{\sin(\theta_1)}{\sin(\theta_2)} = \frac{n_2}{n_1} \quad (1)$$

where n_1 and n_2 are the indices of refraction of adjacent media, labeled 1 and 2. This is illustrated in Figure 5. We note that Ptolemy, a Greek living in Alexandria, Egypt, in the second century CE (Harland 2007) had found a relationship involving the angle of refraction, which however was inaccurate for angles that were not small. Ptolemy's empirical law was obtained partly as a result of fudging his data to fit theory (Weinstein 1996-2007).

When light travels from a medium with a higher refractive index, such as water or raindrop, to one with a lower refractive index, such as air, if the angle of incidence is large enough, Snell's law requires that the sine of the angle of refraction be greater than one. This is not possible, and the light in such cases is reflected by the boundary, a phenomenon known as *total internal reflection*. The largest possible angle of incidence which still results in a refracted ray is called the critical angle; in this case, the refracted ray travels along the boundary between the two media, with the angle of refraction being equal to 90°.

The process of *total internal reflection* is illustrated in Figure 6 for adjacent media with indices of refraction equal to 1 (e.g. air) and 1.5 (e.g. glass). A similar situation is encountered when a ray of light moves from water to air with an angle of incidence of 50°. The refractive indices of water and air are approximately 1.333 and 1, respectively, and Snell's law (eq. (1)) gives us the relation:

$$\sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1 = \frac{1.333}{1}\cdot\sin(50^\circ) = 1.333\cdot 0.766 = 1.021, \quad (2)$$

which is impossible to satisfy for this angle of incidence. The critical angle θ_{crit} is the value of θ_1 for which θ_2 equals 90°:

$$\theta_{crit} = \arcsin\left(\frac{n_2}{n_1}\sin\theta_2\right) = \arcsin\frac{n_2}{n_1} = 48.6^\circ \ , \quad (3)$$

Fig. 2 - Primary and Secondary Rainbows. Courtesy University of Illinois Guide to Atmospheric Physics.

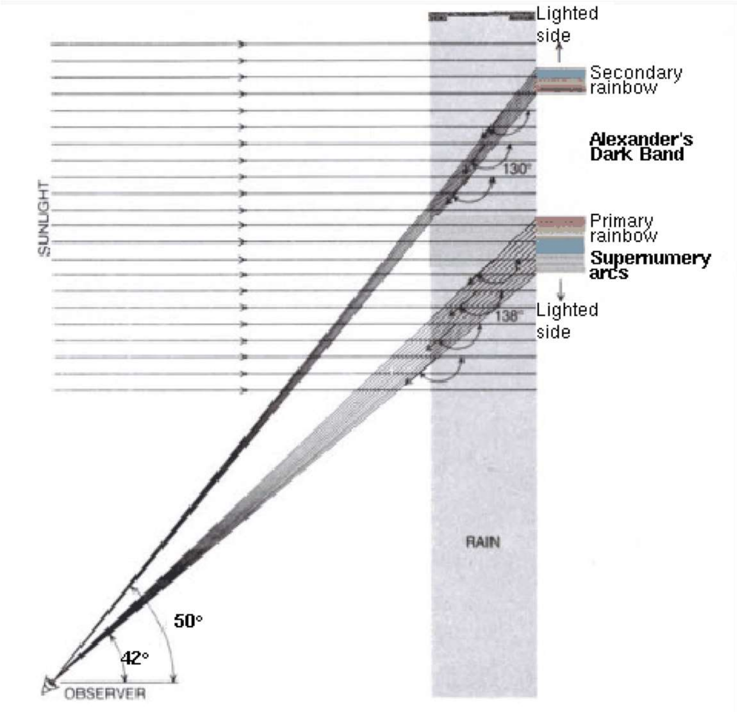

Fig. 3 - Angles of Elevation of Primary and Secondary Rainbows. Courtesy H. Moysés Nussenzveig, Scientific American, 1977.

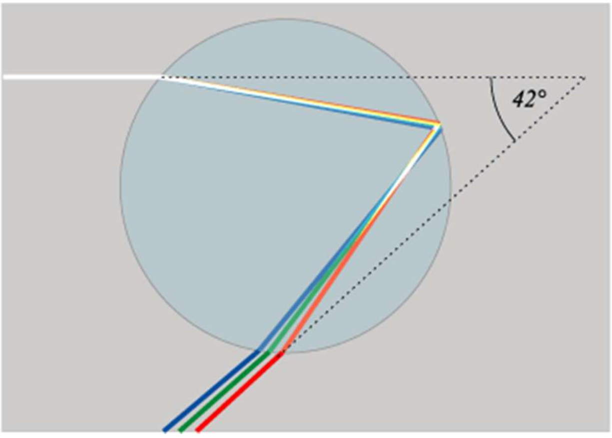

Fig. 4 - Processes that result in the formation of a primary rainbow: dispersion (as light enters and leaves the drop) and total internal reflection. Courtesy of KES47 of Wikipedia Commons, 2010**.**

which is very large even larger than the anti solar angle of 42°. Thus, contrary to popular belief, the light at the back of the raindrop does *not* undergo total internal reflection and some light does emerge from the back. (This is important to remember when dealing with successive reflections and consequently multiple rainbows.) It should be noted that light coming out the back of the raindrop does not create a rainbow between the observer and the sun because spectra emitted from the back of the raindrop do not have a maximum of intensity, as the other visible rainbows do, and thus the colors blend together rather than forming a rainbow.

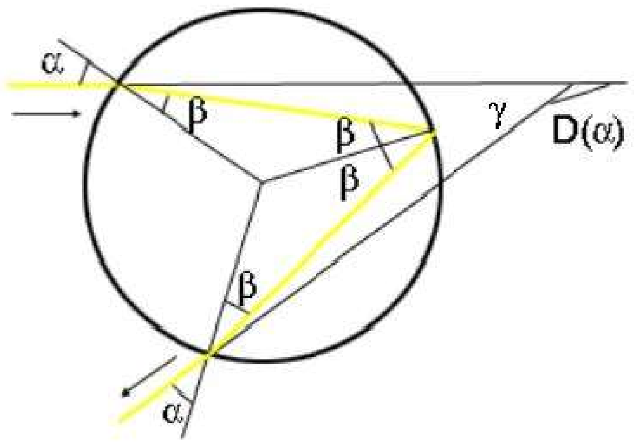

Fig. 7 - Angle of deviation

Interestingly enough, in 1266 CE a contemporary of Scot, Roger Bacon, measured the angle of the rainbow cone for the maximum elevation of the rainbow as 42° (Hackett 2013). In this measurement, probably achieved with an astrolabe, Bacon advocates the skillful use of instruments in an experimental science. However, Bacon's knowledge of the rainbow was understandably limited. For instance, like Aristotle, he attributed the rainbow solely to reflection and not refraction. The anti solar angle of 42° shown in Figure 3, which matches the total angle (later denoted γ in Figure 7), between incident and reflected waves shown in Figure 4, can be derived from the deviation $D(\alpha)$. According to the ideas of Rene Descartes (Descartes 2001, Osler, 2008,Tipler, 2004) as illustrated in Figure 7, $D(\alpha)$ is given by the formula:

$$D(\alpha) = (\alpha - \beta) + (180^\circ - 2\beta) + (\alpha - \beta) = 180^\circ + 2\alpha - 4\beta\,. \qquad (4)$$

From Snell's law of eq. (1):

$$\frac{\sin\alpha}{\sin\beta} = \frac{n_{water}}{n_{air}} \qquad (5)$$

where n_{air} is equal to 1.

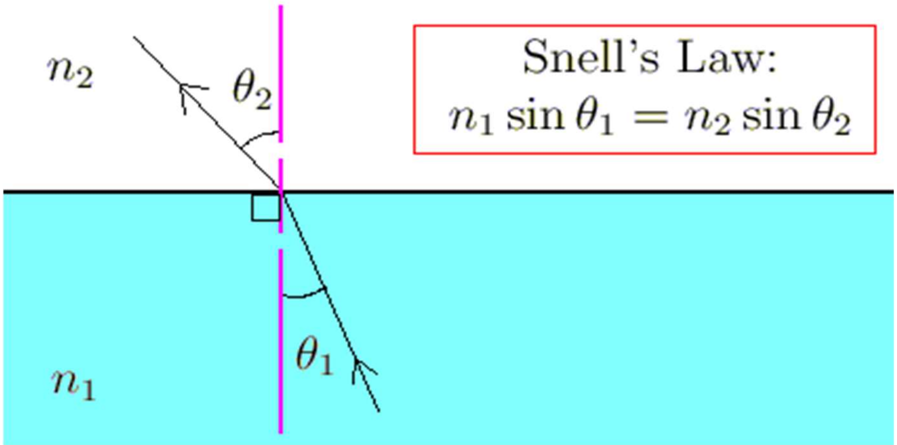

Fig. 5 – Snell's Law of Refraction. Courtesy math.ubc.ca

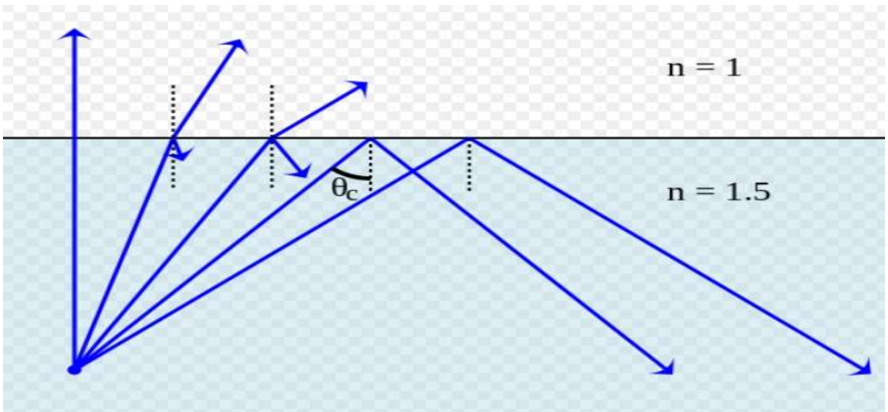

Fig. 6 – Demonstration of refraction and total internal reflection, when the angle of incidence exceeds a critical angle. Courtesy Lasse Havelund, Wikipedia Commons, 2009.

Solving for β, one obtains:

$$\beta = \arcsin\frac{\sin\alpha}{n_{water}} \;,$$

which is then substituted back into eq. (4). The rainbow is produced by rays around this minimum deviation. Solving $dD(\alpha)/d\alpha = 0$ for α to find α_m, the value of the angle α for which D is a minimum, we obtain:

$$\alpha_m = \arccos\sqrt{\frac{n_{water}^2 - 1}{3}} \;.$$

Taking the index n_{water} = 1.33 for a particular frequency in the red, we obtain α_m = 59.58°, β = 40.42°, and $D(\alpha)$ = 137.48°. Now if an emerging rainbow ray from a droplet meets one's eye, this means that this ray makes an angle γ = 180° - 137.48° = 42.52° which is indeed the angle of 42° at the anti-solar point as shown in Figure 4.

The rainbow's spectacular aspect, its colors, were explained by Newton. In 1666 CE, Newton showed that white light being refracted in a prism is split up in its constituent colors. The color scattering is due to the index of refraction being dependent on the wavelength (and hence the color) of radiation. Each color in the sunlight thus produces its own bow. A rainbow is a collection of these bows, each slightly displaced from the rest. Newton worked out the angles of the red bow, 42° 2′ and the violet bow, 40° 17′. This gives a rainbow spread of 1° 45′. This would have been the width of the rainbow if the sun rays were parallel. As a matter of fact, the sun disk has a diameter of half a degree. Taking this into account, Newton concluded that the width of the rainbow should be 2 degrees and 15 minutes, a value that agreed nicely with Newton's own measurements. Note that although Newton's original used a corpuscular theory rather than a wave theory, the results of his model nonetheless carry through to a wave theory (Blay 2001, Shapiro, 2002) and his corpuscular model is "rescued" by today's modern particle-wave duality and the quantization of electromagnetic light waves known as photons (Tipler 2004).

Types of Multiple Bows

In addition to the primary and secondary rainbows, four bows may also form in the sky. These are classified as follows:

1. Reflected Double Rainbows
2. Supernumerary Rainbows
3. Quaternary Rainbows

In an effort to associate the four bows reported by Scot with one of the cases above, we will first review each of these cases in the following sub-sections.

Reflected Double Rainbows

A double rainbow (consisting of a primary and a secondary rainbow) and reflections on a sufficiently large shallow body of water, such as a lake or the water near a calm seashore, can produce four bows. In the *reflected rainbow*, the sunlight is first deflected by the raindrops, and then reflected off the body of water, before reaching the observer. A *reflection rainbow* is produced when sunlight reflects off a body of water before reaching the raindrops. It should be noted however that, in either case, the four bows are never parallel, as seen in Figure 8. Due to the combination of requirements, a reflected double rainbow is not very common. The reflection rainbow appears above the horizon. It intersects the normal rainbow at the horizon, and its arc reaches higher in the sky, with its center as high above the horizon as the normal rainbow's center is below it. Six (or even eight) bows may also be distinguished (Atmospheric Optics 2013, Nordvik, 2007). Reflected double rainbows have been cited and are more common than the tertiary or quaternary rainbows that will be discussed later.

Fig. 8 – Reflection rainbow: bows reflected on a body of water.
Courtesy of Lawlnut, i.imgur.com, 2012.

The Scottish Western Isles are favorable for the formation of reflection bows. The prevailing warm south westerlies from the Atlantic Ocean bring frequent showers of fine rain interspersed by skies of exceptional purity whose sunlight is reflected in the many bays and inlets. Since Scot originated from South Scotland, it is possible that he was aware of reflection rainbows. In his description, he made a point of emphasizing that the four bows are "at slight distance apart" indicating they are likely *parallel*. This is not the case for reflection rainbows, as the generation of four bows from reflection rainbows would involve a reflected double rainbow making up two sets of parallel bows that *intersect* each other as shown in Figure 8. Scot's precise and meticulous descriptions, as demonstrated in the case of Mary of Bologna suggests that Scot was aware of the distinction. This excludes the possibility that reflected double rainbows are the origin of Scot's observation.

Supernumerary Rainbows

A supernumerary rainbow also known as a "stacker rainbow" is not observed frequently. It consists of several faint rainbows on the inner side and within the primary rainbow, and very rarely also outside the secondary rainbow. It is not possible to explain supernumerary rainbows using classical geometric optics, as these are caused by the interference of light waves, a phenomenon that has been investigated in detail by Thomas Young. In 1803, Thomas Young showed that waves from two wave sources (e.g. two holes in a pier in a bowl of water) interfere constructively and reinforce each other, creating crests or troughs, or interfere destructively and cancel out. Young pointed out that the supernumerary bows could be caused by constructive and destructive interference of sunrays which have followed different paths through the raindrop, if the difference between the distances traveled by these rays is equal to an odd number of half wavelengths (destructive interference) or an even number of wavelengths (constructive interference).

Fig. 9 – Over Niagara Falls.Courtesy A. Bierstadt (1830-1902).

Fig. 10 – In Fife, Scotland. Corinne Mills, 2011.

Examples of Supernumerary Rainbows

Figures 9 and 10 provide examples of supernumerary rainbows over the Niagara falls and in the region of Fife in southern Scotland. Since Scot came originally from southern Scotland, perhaps even from the area of Fife itself, which are regions abundant in rainfall, it is possible that he may have observed supernumerary rainbows and that the four bows in his description perhaps can be classified as a supernumerary rainbow.

Tertiary and Quaternary Rainbows

Unlike primary and secondary rainbows, that can be observed in a direction opposite to the sun, it is also possible (but rare) to observe two faint rainbows in the same side of the sky as the sun. These are the *tertiary* and *quaternary* rainbows, appearing on the opposite side of the sky to the familiar rainbow arc, at about 40° from the sun (for tertiary rainbows) and 45° (for quaternary rainbows). A tertiary rainbow is formed by light that has suffered three total internal reflections inside the raindrop, whereas a quaternary rainbow by light that has suffered four total internal reflections within the rain drop (Figure 11). It is difficult to observe these types of rainbows with the naked eye not only because of the sun's glare, but also because the intensity of the n^{th} bow decreases dramatically as n increases.

Theoretical possibilities for multiple and higher-order rainbows were described by Felix Billet (1808-1882 CE) who depicted angular positions up to a 19th-order rainbow, a pattern he called a "rose of rainbows" (Billet 1868, Walker 1977, 138-144, 154). In the laboratory, it is possible to observe higher-order rainbows by using extremely intense and well collimated light produced by lasers. Up to the 200th-order rainbow was reported by Ng et al. in 1998 using an argon ion laser beam (Ng *et al*. 1998).

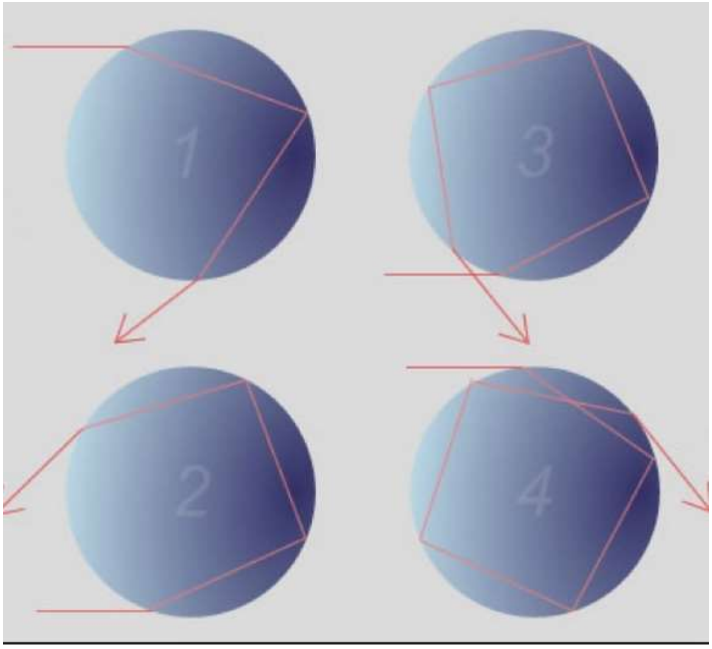

Fig. 11 – Formation of four rainbows from one, two, three and four successive total internal reflections in water droplets suspended in air. Rainbows formed from one and two total internal reflections appear on the opposite side of the Sun, those formed from three and four total internal reflections on the same side as the Sun. Courtesy Robbie Gonzalez, io9.com, 2011.

Raymond Lee (Lee & Fraser 2001, chap. 8), a meteorologist at the US Naval Academy, combed 250 years of scientific literature for recorded evidence of tertiary rainbows: he found just five examples. The conditions under which those five sightings occurred and a recipe for spotting tertiary rainbows has been published in Applied Optics (Grossmann 2011). It has been suggested that such rainbows could be spotted against dark clouds after a storm. Evenly sized drops were also a requirement.

The *Arbeitskreis Meteore*, the German association for the observation of atmospheric phenomena, went hunting for the tertiary rainbow and Michael Grossmann found one following a storm in Kaempfelbach, in south-western Germany (Grossmann 2011). Because the effect is so faint, a number of shots had to be taken and superimposed. A digital enhancement known as "unsharp masking", was also required to reveal the tertiary rainbow.

Soon after, another rainbow hunter, Michael Theusner (Theusner 2011), caught another tertiary rainbow and its adjacent quaternary counterpart near Bremerhaven in northern Germany - after processing the images in the same way. The rare conditions that lend themselves to a nearly visible tertiary or quaternary rainbow, along with the processing required to make them apparent, means that amateur sky gazers are unlikely as ever to catch sight of one.

Tertiary and quaternary rainbows are so elusive because the intensity of the n^{th} bow decreases dramatically as n increases. A calculation within the Descartes model using Fresnel equations indicates that a secondary rainbow is about 2.4 times less intense than the primary rainbow (Calvert 2003). However, dispersion and the wave nature of light are not considered in this calculation. A more refined model using the theory of diffraction was derived by the Mathematician and Astronomer George Biddell Airy in the 1820s. Airy was able to express the intensity of the scattered light in the rainbow region in terms of a new mathematical function, then known as the *rainbow integral* and today called the Airy function and explained the dependence of the intensity of the colors of the rainbow on the size of the water droplets (Airy 1838, Airy, 1849).

Modern descriptions of the physics of the rainbow are based on Mie scattering (solutions to Maxwell's equations describing the scattering of electromagnetic radiation by a sphere), a body of work published by Gustav Mie in 1908 (Mie 1908). Neither the mathematical form of the Airy function nor the more complex models used to explain intensities of multiple rainbows will concern us here, as these are far beyond the means of Scot and his period. It suffices that the $(n+1)^{th}$ bow is less intense than the n^{th} bow and consequently the observation of tertiary and quaternary rainbows is *extremely* rare indeed.

Note that the four bows, consisting of two sets of two bows on each side of the sun, as photographed by Theusner, differ from Scot's description of four bows at slight (visual) distance from each other. The same applies for the reflected double rainbow. The supernumerary rainbow might seem to come closest to Scot's description but it is usually not so spread out in the sky. A historical and cultural analysis is needed here.

Known Rainbow Models at the Time of Michael Scot

Around 300 BCE, in his *Meteorology* (Aristotle 1984), Aristotle presented the first explanation for the formation of the rainbow. He attributes its formation to clouds on a hemisphere resting on the circle of the horizon reflecting sunlight to the observer where the angle is equal (to some constant angle – see Figure 13) and was the first to explain the rainbow's circular shape and the fact that the rainbow is not located at a definite place on the sky, but is seen in a certain direction.

The angular gap between the primary and secondary rainbow, illustrated in Figure 3, is Alexander's (dark) band, named after Alexander of Aphrodisias who first described it in 200 CE (Lee & Fraser 2001, 110-111). The Aristotelian theory of the rainbow made no allowance for refraction. The primary bow was believed to be caused by reflection, at a dewy cloud, of rays of sight from the eye which then were bent back towards the sun. The fainter bow was assumed to be made in the same manner. Reflection in the latter case takes place at a portion of the cloud much higher than that causing the primary bow. As the assumed reflection causing the second bow took place more obliquely at a greater distance from the eye, it seemed natural that its colors should appear paler. A third bow would have to be caused by a reflection from clouds placed at a still greater altitude.

Fig. 12 – Example of Quadruple Rainbow
Courtesy Robbie Gonzalez, io9.com, 2011.

From Boyer (Boyer 1987, 141), (Aristotle [Works of Aristotle] 1931)

> Three rainbows or more are not found because even the second one is fainter, so that the third reflection can have no strength whatsoever and cannot reach the Sun at all.

Consequently, tertiary and quaternary rainbows simply do not exist in Aristotle's model. According to Raymond L. Lee and Alistair B. Fraser, "Despite its many flaws and its appeal to Pythagorean numerology, Aristotle's qualitative explanation showed an inventiveness and relative consistency that was unmatched for centuries." (Lee & Fraser 2001, 109).

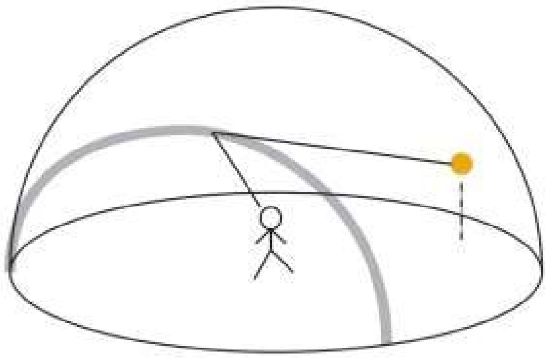

Fig. 13- Aristotle's rainbow model: clouds on the sky hemisphere reflect rays of sight. Courtesy of "The Aristotelian Rainbow: From Philosophy to Computer Graphics" by Jeppe Revall Frisvad, Niels Jørgen Christensen, and Peter Falster, Technical University of Denmark, ACM, 2007.

Scot had recovered the knowledge of Aristotle's rainbow in part thanks to Muslim scholars of the middle-ages such as Averroes (see e.g. (Topdemir 2007)) and Avicenna. Carl Benjamin Boyer described Avicenna's ("Ibn Sina") theory on the rainbow as follows:

> Independent observation had demonstrated to him that the bow is not formed in the dark cloud but rather in the very thin mist lying between the cloud and the sun or observer. The cloud, he thought, serves simply as the background of this thin substance, much as a quicksilver lining is placed upon the rear surface of the glass in a mirror. Ibn Sina would change the place not only of the bow, but also of the color formation, holding the iridescence to be merely a subjective sensation in the eye. (Boyer 1954)

Significant developments in the middle-ages concerning scientific explanations for the rainbow include the contributions of the Persian physicist and polymath Ibn al-Haytham (also known as Alhazen; 965-1039 CE) and especially Kamal al-Din al-Farisi (1267-1319 CE), who lived later than Scot though. Farisi gave the first satisfactory explanation of the rainbow and had "proposed a model where the ray of light from the sun was refracted twice by a water droplet, one or more reflections occurring between the two refractions", also verified experimentally (O'Conner & Robertson 1999). Nonetheless, rainbow models at the time of Scot address the causes of the bows themselves or the source of their colors but never question the maximum possible number of bows in the sky i.e. two at most!

It is therefore seen that Scot's observation of four bows is outside of the body of thought concerning rainbows in both the ancient Greek as well as Muslim scholarships in his time. In such a case, where could his reported observation of four bows originate from? Presumably, from the places Scot lived and worked, Scotland or the wider area around Toledo, Spain.

Multiple Bows and the Tuareg

To the best of our knowledge, occurrences of four bows are not very common. If one considers the wider area in which Scot lived, such descriptions are only encountered in the Tuareg, a Berber people with a traditionally nomadic pastoral lifestyle and the principal inhabitants of the Saharan interior of North Africa. Mahmoudan Hawad (born 1950), a contemporary Tuareg poet and author (Chaker & Claudot-Hawad 1989) has observed parallel bows in the Sahara-Sahel region of Aïr, a triangular granitic mountainous region located in north central Niger, as recounted by his wife Hélène Claudot-Hawad (Claudot-Hawad 2002). He had witnessed multiple rainbows during his younger days in this region and was aware of Tuareg lore on the subject. These parallel rainbows were observed after thunder where the rain is able to appear at a distance. The Sahara-Sahel has a range of microclimate changes caused by the close proximity of deserts, oases, and green mountainous regions. Today, while the Aïr mountains are largely bare of vegetation, the dry wadi river valleys (known by the Hausa term "Kori") dissect the mountains, channel and hold rainwater in "gueltas" (stone pools) as shown in Figure 15, creating oases. Hot springs are found in the mountains, as are ancient rock carvings, a testimony to a much lusher vegetation in the distant past. The sight of rainfall at a distance is possible from these mountains. In the desert, the rain can appear in the form of numerous flash floods of intense and short duration.

Fig. 14 - Tuareg in Sahara Desert. Courtesy PhotoBucket.

The Tuareg have been in North-Africa for thousands of years. William Langewiesche claimed their ancestors were the warriors with chariots pictured in Sahara rock art (Langewiesche 1997) which is abundant in the Aïr region. In the past few centuries the Sahara-Sahel region has faced drought (Brahic, 2012). Currently the Sahara Desert is apparently increasing in size and the region faces serious ecological issues (Schmidt, 2001). In the time of Scot however, one would expect a greater abundance of rain and a higher frequency of rainbow formation.

Given the endless precariousness of rain water, it is no coincidence that the rainbow, which is associated with rain, appears in the Tuareg mythology. It would further make sense for the Tuareg to discern patterns and divination techniques based on rainbows, in an attempt to make predictions of rainfall.

In the Tuareg language, the word for the rainbow is "tezzel ader" which means "(she) stretches the leg". In their cosmogony, the rainbow is the very picture of the metamorphosis ("tebedya"). It appears when, after lightning and thunder, rain cannot fall. This abortion of the storm is dangerous, disrupting the harmony between the earth and the sky. While turning into a rainbow, the multicolored snake "stretches the leg" above an anthill (symbol of the world below) and the wasted energy of the storm creates the curve of a transient universe. This third ephemeral world is capable of pacifying the irregularity instituted by the absence of rain, a negation of the exchange relations between the two antagonistic and complementary parts of the universe, earth and sky. The absence of rain resembles the condition of "very little or no rain" in the description of Scot. Further, cloud formations in these regions match Scot's description of "low mountainous clouds".

Fig.15 – A *guelta* near Timia, in central Aïr, provides water throughout the year in an otherwise dry region. Courtesy Wikipedia Commons, 2006.

Possible Source of Unknown Etymology

At this stage, we are left to come to grips with the meaning of the word *"zalla"* mentioned by Michael Scot in his description of the multiple rainbow. We also mention other unknown terms by Michael Scot, in particular, two words to describe musical string instruments namely *ineba* and *senphonium* which "are not found in dictionaries and seem peculiar to Scot" (Thorndike 1965, 12) as part of a list which included the viola, psaltery, lute and harp. Michael Scot had his own etymologies, a natural complement to his translation activities. Nobody operates in a vacuum and it is unlikely that Michael invented these words entirely on his own and it therefore becomes important to understand his historical and geographical location, namely the Toledo School of Translation in the 12th and 13th centuries (eq. see (Kann 1993, Universidad Castilla 2012, Wightman 332, 1953)).

According to J. Wood-Brown, Michael Scotus was helped in his translations by a Jew named "Andrew" (Brown 1897)[2]. Although the name "Andrew" is in doubt, there can be no doubt that Michael Scotus was helped by Jews who knew Hebrew, Arabic and Spanish when he made his translations of scholarly works from Arabic to Latin while in Toledo. Under the leadership of King Alfonxo X of Castille (1221-1284 CE - known as "the wise"), Sephardic Jewish scientists and translators acquired a prominent role in the

[2] It has been conjectured that Andrew was a convert to Christianity. It is curious at any rate that the name given him was that of Scotland's patron saint.

School. They were highly valued by the King because of their intellectual skills and mastery of the two languages most used in the translations: Arabic and Spanish (Muñoz Sendino 1949, 15).

Alfonso's nephew Juan Manuel wrote that the King was so impressed with the intellectual level of the Jewish scholars that he commissioned the translation of the Talmud as well as the Kabbalah (Castro y Calvo 1947, 2). Amongst these Jewish scholars, we cite the personal physician of King Alfonso himself, Yehuda ben Moshe ha-Kohen (Proctor 40, 1945,González, 1998) and especially members of the Ha – Levi family, such as Judah Halevi - actually Yehuda Ben Shemuel Ha-Levi (1075-1141 CE) a Jewish physician and poet (see e.g. (Kaplan 1993, 405-407)) - as well as Meir ben Todros Ha-Levi Abulafia (c. 1170-1244 CE) (Kohler *et al.* Jewish Encyclopedia, 1906) a major Sephardic Talmudist and authority on Jewish religious law and contemporary of Michael Scot. When Michael Scot was in Toledo, the school of translation was under the direction of the Archbishop Rodrigo Jimenez de la Rada who played a key role in the war against the Almohads and at the battle of Las Navas de Tolosa in 1212 CE (Pick 2004).

Kabbalah and its numerology system called "Gematria" was very popular in Michael's time and place. This is an ancient system which consists in assigning a number to each Hebrew letter and them summing these numbers for a given word. It is believed that identical numerical values bear some relation to each other or bear some relation to the resulting number itself. For clarification, it is not, by any means, our goal to convince the reader of the validity of Kabbalah, only that it was used in many studies in locality and time of Michael Scotus and is relevant to his etymology. Quite the contrary, we agree with the assessment of the Mathematician Barry Simon that e.g. there is, as yet, no real proof of a so- called "Biblical code" (Simon 1998). Rather, Hebrew as a language, has particular linguistic properties: when viewed as a mathematical basis for a language, it is *overcomplete.* For example, the words "Michael" and "Samuel" both end in "-el" which means God and so each Hebrew letter has a meaning in itself. Thus, when the original Hebrew words are constructed as compounded symbols, where each individual letter has meaning, it is not surprising that the Gematria system can produce a consistent complementary meaning to the word itself. In this context, Kabbalah with Gematria are considered as linguistic *etymological tools***,** nothing more. This approach is not so unorthodox if one considers that Gematria has been used for coding (e.g. Atbash) and encryption. Concerning Michael's appreciation for music, according to "Music and Kabbalah" by Matityahu Glazerson (Glazerson 1996, 23)

> When a person has a connection between the physical world and the spiritual world, he has a desire to sing. Singing is the result of the natural world (the number seven, the **ז**) joining with its higher root (the **א**). It is written in the book *Livnat HaSapir* (2 Kings, 3:15) that the reason a baby is pacified when he is sung to is because the singing reminds him of the root of his **נשמה** (*neshama*, soul), i.e. the spiritual world from whence he came.
>
> The **א** represents the **כתר** (*keter*, crown), the highest sphere and the source of all the spheres. **כתר**, in Kabbalah, is called **עליון פלא** (*peleh elyon*, the sublime wonder). The word **פלא** contains the same letters as the word **אלף**. It is interesting to note that the numerical value of the word **מוסיקה** (*musika*, music: 40 + 6 + 60 + 10 + 100 + 5) is 221, which is the same numerical value of the word **ארך**, the term used for the sphere **כתר**. Even though **מוסיקה** is not actually a Hebrew word, our Sages also sometimes gave numerical values to foreign words. The reason is based on the principle that all languages are derived from and have a connection with Hebrew, the holy language.

Such notions are not accepted by many today and an analysis based on Jewish Gematria may seem unorthodox but it would certainly be acceptable by Michael Scot in his time and place especially as he was surrounded by languages such as Hebrew and Arabic in the city of Toledo. He appreciated etymology, numerology as well as music. Accordingly, our analysis for the three words of Michael Scot is as follows:

senphonium: looks very much like a Latinized version of an old Hebrew word "Sumponia" or "Cumponyah" **סומפניא** and corresponds to the Greek word "symphonia" meaning "symphony" (in modern Hebrew **סימפוניה**) or "symphonos" meaning "harmony". However, Michael Scot used this word to describe a string instrument, possibly a lyre (Thorndike 1965,12). The Hebrew word "Sumponia" is mentioned in the Book of Daniel (Dan 3:5), and corresponds to a bag-pipe:

> "that when you hear the sound of the horn, pipe, lyre, trigon, harp, bagpipe, and every kind of music, you are to fall down and worship the golden image that King Nebuchadnezzar has set up."

However, there is another view that it is not a musical instrument, but rather the collective of instruments in harmony (Encyclopedia Judaica v. 12, 563). If we apply Glazerson's reasoning with Jewish Gematria, the Hebrew letters of "Sumponia" have respectively the numbers from right to left (i.e. from ס to א in סומפניא) are respectively 60, 6 , 4 0 , 8 0 , 50 ,10 and 1 which add up to 247. This is the same number as for "zemer" זמר which means singer and according to Glazerson relates to "neshama" (soul) as mentioned in the earlier quote and represents the "highest part of the spirit within man" (Glazerson 1996, 55). With both definitions as an instrument and harmony, this seems a likely source for Michael Scot's word "senphonium".

ineba: This word is much more difficult as it is not a Spanish word nor a Hebrew or Arabic word *per se.* Nor can we find it amongst the Berber Languages like Tamazight nor Tamasheq (the ancient language of the Tuareg). It is none of the Hebrew words string instruments for lute or harp such as e.g. "nevel" נבל nor "kinnor" כנור. It is thus a challenge. The Hebrew closest to it is "Annaba" אנאבה which also sounds very much like the Arabic word, a portal city in modern-day Algeria (which used to be called "Hippo Regius", the birthplace of Pope Augustine, a major influential Church authority in Michael's time and thereafter). The vowels "i" and "e" do not register in Jewish Gematria indicating a Latinization process. If we apply Jewish Gematria on the remaining letters "n", "b" and the ending of the word ב, נ and ה, we get respectively 50, 2 and 5 which add up to 57. Michael Scot was fascinated by multiples of 7 but this number falls short of 77 which represents the "perfection of the number seven" according to Matityahu Glazerson (Glazerson 1996, 22) as found multiple times in the Bible. Michael Scot certainly knew Latin and would have realized that "ineba" would cognate with the Latin word for inebriation. This suggests an instrument used for light-hearted situations rather than solemn religious music. Though decidedly we have no actual proof, it nonetheless becomes tantalizing to think that Michael Scot may be actually describing the *guitar* which reached Europe by way of Spain and existed in various forms in Spain and throughout North-Africa. However, this remains a suggestion which we do not insist on, especially as it does not appear in the description of rainbows. This example is shown only to see how far we can push this analysis even for such an elusive example.

Zalla: "Zalla" is very similar to the Arabic "Allah" which not only means God but also the "whiteness of consciousness or light it is sometimes symbolized as clouds". The term Allah is derived from a contraction of the Arabic definite article al- "the" and "ilāh" "deity, god" ,to "al-ilāh" meaning the sole deity. This is similar to the Hebrew word "Elohim" i.e. deity and also means sky. A word which phonetically sounds similar is the Hebrew word "tzillah" צלה which means shadow, shade, umbra, or darkness which translates into the Arab word "vallah". Thus, it seems, the lightness of the cloud can be mitigated, tuned and even negated linguistically. It's gematria value (from right to left) is 90, 30, 5 which add up to 125 and this number has a special meaning in Kabbalah though we are no longer dealing with music. There are the 125 "spiritual degrees that complete the correction of one's soul" which are associated with "Rashbi", the author Kabbalah's chief work i.e. the Zohar (Laitman 2008, 9). Michael's word "zalla" appears elsewhere (Thorndike 1965, 69) where he attributes colors of the rainbow to the color or darkness of a cloud following a quasi-Aristotle reasoning:

> "These variations occur according to the varied dispositions of the clouds, which receive such impression from the sun's rays, wherefore a fiery cloud makes a red color; a thin one of little substance, white; a *zalla* cloud, purple or blue and black or quasi-green or black like oil."

Thus, the "zalla" cloud color seems to widely range from blue and green to black. The colors described here match the colors of the multicolored snake in the mythological Tuareg description.

Verdict

We finally present our answer to the question: what kind of multiple rainbow was Scot describing? Our answer is that it is a description from some form of weather divination lore generated and influenced by multiple eye-witness accounts of multiple rainbows over the ages. The observed phenomena were likely supernumerary rainbows and possibly the occasional reflection bow and quaternary bow. What leads us to this conclusion is as follows. Firstly, he was clearly reporting accounts from other people not his own personal observations. Secondly, just as the properties of the biblical manna do not perfectly match its proposed identifications such as the resin of the Tamarisk tree (Jewish Encyclopedia 1906) or the honeydew of certain insects, etc.…, Scot's description does not fit perfectly into any of the three given categories of multiple rainbows. In form, Scot's parallel rainbows greatly resembles supernumerary rainbows but these are not localized around e.g. waterfalls, rather they located in the sky much like a quaternary rainbow. Scot's description is thus from third-party or rather mythological sources though, as is often the case, with a basis in reality.

Discussion and Implications

Michael Scot's record of multiple rainbows does not originate from the Muslim scholarship of his time, in which knowledge on rainbows was hardly more advanced than that of Aristotle, but rather from observations either in Scotland, where he lived at a young age, the or in the Sahara-Sahel and in particular the Aïr region, where the mountainous settings and weather conditions favor a more frequent realization of multiple rainbows. Such conditions do not materialize in Christian Europe which our conventional knowledge associates with Scot: Spain, Sicily and the rest of Italy, France and even Germany - given that Germany was part of the Holy Roman Empire under the jurisdiction of his patron Frederick Hohenstauffen II. If this specialized and rare piece of information does not originate from Scotland, it is highly possible that access to it has been gained through contact with the Tuareg. These desert roaming nomads are the only tribe in the wider region in which we place Scot that have a culture in which the rainbow has a central role. The Tuareg have a mythology surrounding the rainbow. Established in a wide region in North Africa over thousands of years, with a perennial concern for rain water, the Tuareg have further developed a system by which to make predictions based on the rainbow and have a "divination" technique based on multiple rainbows. Mythologies concerning rainbows exist in many cultures but only Hawad's description of Tuareg lore matches the description of Scot.

So, how did the Tuareg knowledge concerning the four bows reach Scot? The Sahelian kingdoms were thriving during the middle-ages because their wealth came from controlling the Trans-Saharan trade routes across the desert, especially the slave trade within the Islamic world. Their power came from having large pack animals like camels and horses that were fast enough to keep a large empire under central control and were also useful in battle. Camels could travel in valleys, often dry river-beds called "wadis" (or "oueds") and ultimately at a much faster rate than horses could travel over long distances in Europe. These Sahelian kingdoms supported several large trading cities in the Niger Bend region, including Timbuktu, Gao, and Djenné. The distance between Morocco and Béjaïa (Algeria) where Fibonacci resided could be bridged in a matter of months.

Given in part the connection between Michael Scot and Leonardo Fibonacci concerning the Fibonacci numbers (Scott & Marketos 2014) and the known historical record that Leonardo Fibonacci had been stationed in Béjaïa, located on the coast of modern-day Algeria, as part of a Pisan trade colony, with sea trade routes all around the Mediterranean, the outcome of this analysis implies that Michael Scot would have ventured into North-Africa, at least as far as Morocco (which is not that far from Toledo) and quite possibly southward where he would have learned about multiple rainbows and the lore surrounding them from actual contact with the Tuareg people. There does not appear any other way by which Michael could have obtained this very specialized and extraordinary bit of information as it does not appear in Muslim scholarship or the models of Aristotle at any time. Mythologies surrounding rainbows exist in many cultures but only Hawad's description of Tuareg lore matches the description of Michael Scotus. Moreover, quaternary rainbows are *extremely rare* and only a people established in a given region over thousands of years with a perennial concern for rain water would develop a belief system by which to make predictions based on e.g. the number of parallel bows in the sky. Nor is there any other group apart from these desert roaming nomads that have such a mythology surrounding the rainbow. Michael Scot might have been aware of supernumerary or reflected rainbows from Scotland but only the Tuareg have a “divination” technique based on multiple rainbows and were relatively near his whereabouts in Toledo, Spain (or Sicily for that matter).

Possibly, a trek to North-Africa would have also included a meeting with Leonardo Fibonacci concerning quite possibly the translations by Gérard of Cremona of the works of Al-Khwârizmî and Abû-Kâmil used by Fibonacci himself for his book *Liber Abaci* (Scott & Marketos 2014). Although, Béjaïa lies north of the Aïr mountains, we do not claim that Michael Scot actually reached this region, nor Béjaïa itself, only that he had communications with Leonardo de Pisa and the Tuareg tribesmen (which means he had to reach Morocco at least).

Apart from the question of where, there is also the question of when. Even though Leonardo Fibonacci and Michael Scot were part of the court of Frederick II, each of them was much older than Frederick and of the same generation with respect to each other. Michael Scot was not part of Frederick’s court until after 1223-4 CE being the dates Pope Honorious tried to get Michael a position in Cashel, Ireland (which Michael subsequently refused). Note that the chronology of J. Wood Brown (Brown 1897) is incorrect: Michael Scot was not in Sicily before traveling to Spain: it was the other way around. The first official translation and recorded date of Michael Scot was in 1217 CE (Thorndike 1965) and a conjecture made by Charles Burnett suggests that he might have been in Spain as early as 1200 CE or maybe even earlier as a young man (Burnett 1994,101-126). The first version of Leonardo’ Liber Abaci was written in 1202 CE while the second version of Leonardo Fibonacci’s famous book was dedicated to Michael Scot in 1227 CE. Thus, both scholars, Michael Scot and Leonardo Fibonacci were active long before they were part of the court of Frederick II and could have met any time between 1200 CE and 1217 CE.

The French writer and historian Henri Daniel-Rops once said that history is always a conjectural science. For example, it is amazing how much science and applied science in archeology can debunk some of the most conventional notions. Even though, there is no historical record, the record of multiple rainbows by Michael Scot indicates that his reputed thirst of knowledge starting in early boyhood (Thorndike 1965) would have made him venture into “enemy territory”, namely the world under Islamic jurisdiction around the time of the Crusades. The outcome of the present work suggests an even greater penetration into the world under Islamic jurisdiction on the part of Michael Scot than previously thought. The contradictory nature of the Tuareg observation of multiple rainbows and the known Science in his time could very well have prompted Michael Scot to realize that the knowledge of the ancient Greeks and the Muslims was imperfect and that experimentation was needed.

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References

"About Rainbows-UCAR". The National center for atmospheric research & the UCAR Office of Programs, publisher Eo.ucar.edu, (2013-08-19). http://eo.ucar.edu/rainbows/

Abufalia, D. 1988. *Frederick II. A Medieval emperor.* Toronto: Oxford Univ. Press.

Airy, G.B. 1838. On the intensity of light in the neighbourhood of a caustic. *Transactions of the Cambridge Philosophical Society* 6 (3): 379-403.

Airy, G.B. 1849. Supplement to a paper, On the intensity of light in the neighbourhood of a caustic. *Transactions of the Cambridge Philosophical Society* 8: 595-600.

Aristotle. 1931. *Works of Aristotle*, 3 vols. Translated by E. W. Webster. Ed. By W. D. Ross. Oxford: Clarendon Press.

Aristotle. 1984. 350 B.C.E., *Meteorology,* Translated by E. W. Webster. In *The Complete Works of Aristotle.* Edited by J. Barnes. N.J.: Princeton University Press.

"Atmospheric Optics: Refection rainbows formation." Atoptics.co.uk, Retrieved 2013-08-19. http://www.atoptics.co.uk/rainbows/reflform.htm

Blay, Michel. 2001. *Lumières sur les couleurs. Le regard du physician.* Paris: Ellipses.

Benoist-Méchin, Jacques. 1997. Le rêve le plus long de l'Histoire IV: Frédéric de Hohenstaufen ou le rêve excommunié. Paris: Librairie académique Perrin, 2nd ed.

Billet, Felix. 1868. Memoire sur les dix-neuf premiers arcs-en-ciel de l'eau. (Memoir on the first nineteen rainbows). *Annales scientiques de l'Ecole Normale Supérieure* 1 (5): 67-109, Retrieved 2008-11-25.

Boyer, Carl Benjamin. 1987 [1959]. *The rainbow: from myth to mathematics.* New Jersey: Princeton Univ. Press. ISBN-0-691-02405-7.

Boyer, Carl Benjamin. 1954. Robert Grosseteste on the Rainbow. *Osiris* 11: 247-258 (248).

Brahic, Catherine, Africa trapped in megadrought cycle. *New Scientist*. Retrieved 17 December 2012.

Brown, J. Wood. 1897. *An inquiry into the life and legend of Michael Scot*. Edinburgh: Edited by D. Douglas. (Note: While this book presented an unprecedented amount of information concerning Michael Scot in its time, it must be used with caution.)

Burnett, C. 1994. Michael Scot and the transmission of scientific culture from Toledo to Bologna via the court of Frederick II Hohenstaufen. *Micrologus II: Natura, scienze e societa medievali (Nature, Sciences and Medieval Societies), Le scienze alla corte di Federico II (Sciences at the Court of Frederick II)* 2: 101-126.

Burnett, C. 1996. *Magic and divination in the Middle Ages: Texts and techniques in the Islamic and Christian Worlds*. Aldershot, Hampshire: Variorum.

Calvert, J.B. 2003. The rainbow. http://mysite.du.edu/~jcalvert/astro/bow.htm

Castro y Calvo, J. M. 1947. *Juan Manuel, libro de la caza.* Barcelona: Consejo superior de investigaciones Cientificas. 2., Otrosí fizo tras-lador toda le ley de los judíos et aun el su Talmud et otra sciencia que han los judies muy escondida, a que llaman Cabala. Et esto fizo porque parezca manifestamente por la su ley que toda ella es figura de esta ley que los cristianos habernos; et que también ellos como los moros están en gran error et en estado de perder las almas. https://en.wikipedia.org/wiki/Toledo_School_of_Translators

Chaker, S. and Claudot-Hawad, H. 1989. *Encyclopédie berbère*. In association with UNESC0, Édisud, Arzuges, Aix-en-Provence, France, A262, 861-862. https://encyclopedieberbere.revues.org/ http://en.wikipedia.org/wiki/Hawad

Claudot-Hawad, H., Private email Communication, Fri, 18 Oct 2002.

Descartes, R. 2001 [1637]. *Discourse on method, optics, geometry, and meteorology.* Translated by P. J. Olscamp. Indianapolis: Bobbs-Merrill.

Encyclopedia Judaica. Article Music. 12: 554-678.

Glazerson, Matityahu. 1996. *Music and kabbalah.* Lanhan, Maryland: Jason Aronson, Inc. ISBN-10: 1568219334, ISBN-13: 978-1568219332.

González, Serafin Vegas. 1998. *La escuela de traductores de toledo en la historia del pensamiento*, Toledo: Ayuntamiento de Toledo.

Grossmann, Michael, Elmar Schmidt and Alexander Haumann. 2011. Photographic evidence for the third-order rainbow. *Applied Optics (The Optical Society)* 50 (28): F134-F141.

Hackett, Jeremiah. 2013. Roger Bacon. *Stanford Encyclopedia of Philosophy, Library of Congress Catalog Data*. ISSN 1095-5054. http://plato.stanford.edu/entries/roger-bacon/

Harland, David. 2007. Cassini at saturn: Huygens results. p. 1. ISBN 0-387-26129-X

Haskins, C. 1927. *Studies on the history of medieval science.* Cambridge Mass.: Harvard University Press, 2nd ed.

Kann, Christoph. 1993. Michael Scotus. In Bautz, Traugott. Biographisch-Bibliographisches Kirchenlexikon *(BBKL)* (in German), Herzberg: Bautz, 5 cols. 1459-1461. ISBN 3-88309-043-3.

Kaplan, Gregory B. 1993. Review of: The Compunctious poet: Cultural ambiguity and Hebrew poetry in Muslim Spain by Brann, Ross. Johns Hopkins UP. *Hispanic Review* 61: 3, 405-407. http://www.jstor.org/stable/475075

Abulafia, Abraham ben Samuel. 1906. By Kohler, K., Bloch, P., Kayserling, M. and Wendland, Paul, *The Jewish Encyclopedia*. Funk and Wagnalls, New York: LCCN 16-14703. http://www.jewishencyclopedia.com/articles/699-abulaa-abraham-ben-samuel https://en.wikipedia.org/wiki/Meir_Abulafia

Langewiesche, William. 1997. *Sahara unveiled: A Journey across the desert*. New York, NY: Vintage Books. ISBN-10: 0679750061 ISBN-13: 978-0679750062

Laitman, Michael. 2008. *The Zohar: Annotations to the Ashlag commentary*, Bnei Baruch, Laitman Kabbalah, Oct. 8 (9), ISBN-10: 1897448090, ISBN-13: 978-1897448090 http://www.kabbalah.info/engkab/mystzohar.htm

Lee, Raymond L. Jr. and Alistair Fraser. 2001. *The rainbow bridge*. Philadelphia, Pennsylvania: The Pennsylvania University Press. Chapter 8.

Mie, G., 1908. Beiträge zur optik trüber medien speziell kolloidaler metallösungen (Contributions to the optics of turbid media, especially of colloidal metal solutions), *Annalen der Physik,* 4th series, 25 (3): 377-445.

Muñoz Sendino, José. 1949. La escala de Mahoma. *Madrid: Ministerio de asuntos exteriores*, 15. http://en.wikipedia.org/wiki/Toledo_School_of_Translators

Ng, P.H., M. Y Tse, and W. K. Lee. 1998. Observation of high-order rainbows formed by a pendant drop. *Journal of the Optical Society of America B* 15 (11): 2782-2787. Doi:10.1364/JOSAB.15.002782 http://josab.osa.org/abstract.cfm?URI=josab-15-11-2782

Terje O. Nordvik. 2007. Six rainbows across Norway. APOD (Astronomy Picture of the Day). Retrieved 2007-06-07. http://apod.nasa.gov/apod/ap070912.html

O'Neill, Ynez Viol. 1973. Michael Scot and Mary of Bologna. A medieval gynecological puzzle. *Clio Med,* 8(2); 87-111. PMID: 4135131

O'Neill, Ynez Viol. 1974. Michael Scot and Mary of Bologna. an addendum. *Clio Med*, 9(2): 125-9. PMID: 4134846

O'Connor, J.J. and Robertson, E.F., Kamal al-Din Abu'l Hasan Muhammad Al-Farisi. 1999. *MacTutor History of Mathematics archive.* University of St Andrews. Retrieved 2007-06-07. http://www-gap.dcs.st-and.ac.uk/history/Biographies/Al-Farisi.html

Osler, Margaret J. 2008. *Descartes's optics: Light, the eye, and visual perception*. Edited by J. Broughton and J. Carriero. Oxford: Blackwell Publishing Ltd, pp. 124-141. http://dx.doi.org/10.1002/9780470696439.ch8

Palmer, Jason, Quadruple rainbow caught on film for the first time. BBC news, science and environment, 2011. http://www.bbc.co.uk/news/science-environment-15197774

Pick, Lucy, 2004. *Conflict and coexistence. Archbishop Rodrigo and the Muslims and Jews of Medieval Spain*. Ann Arbor: University of Michigan.

Procter, Evelyn. 1945. *The Scientific Works of the Court of Alfonso X of Castile: The King and His Collaborators*. MLR, 40.

Rashed, Roshdi. 1990. A pioneer in anaclastics: Ibn Sahl on burning mirrors and lense. *Isis* 81 (3): 464-491.

Shapiro, Alan E. 2002. *Newton's optics and atomism.* Edited by B. I. Cohen and G. E. Smith. The Cambridge Companion to Newton. Cambridge: Cambridge University Press.

Schmidt, Laurie J. 2001. From the Dust Bowl to the Sahel. *NASA,* 18. http://en.wikipedia.org/wiki/Sahel

Scott, T.C. and Marketos, P. 2014. On the origin of the Fibonacci sequence. *MacTutor History of Mathematics archive*. University of St Andrews. http://www-history.mcs.st-andrews.ac.uk/Publications/fibonacci.pdf

Simon, Barry. 1998. Barry Simon on Torah Codes. http://torahcode.us/torah_codes/code_history/simon.html

Theusner, Michael. 2011. Photographic observation of a natural fourth-order rainbow. *Applied Optics* 50 (28): F129-F133. http://dx.doi.org/10.1364/AO.50.00F129

Thorndike, Lynn. 1965. *Michael Scot.* London: Thomas Nelson and Sons.

Tipler, P. A. and G. Mosca. 2004. *Physics for scientists and engineers*. San Francisco: W. H. Freeman. ISBN 0-7167-4389-2.

Topdemir, Hüseyin Gazi. 2007. Kamal Al-Din Al-Farisi's explanation of the rainbow. *Humanity & Social Sciences Journal* 2 (1): 7585 (77). Retrieved 2008-09-16. http://www.idosi.org/hssj/hssj2(1)07/10.pdf

Universidad Castilla. 2012. https://en.wikipedia.org/wiki/Toledo_School_of_Translators http://www.uclm.es/escueladetraductores

Walker, Jearl D. 1976. Multiple rainbows from a single drop of water and other liquids. *American Journal of Physics* 44 (5): 421-433.

Walker, Jearl D. 1977. The Amateur scientist – How to create and observe a dozen rainbows in a single drop of water. *Scientific American* 237: 138-144, 154. Doi:10.1038/scienticamerican0777-138. Retrieved 2011-08-08. https://www.scientificamerican.com/article/the-amateur-scientist-1977-07/

Weinstein, Eric. 1996-2007. Ptolemy (ca. 100 - ca. 170), *Eric Weinstein's World of Scientific Biography*. http://scienceworld.wolfram.com/biography/Ptolemy.html

Wightman, William P. D. 1953. *The growth of scientific ideas*. New Haven: Yale University Press. ISBN 1-135-46042-6.

Wolf, K.B. 1996. Geometry and dynamics in refracting systems. *European Journal of Physic* (16) 1420.